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# IDENTIFIABILITY AND IDENTIFICATION OF THE AIRCRAFT LONGITUDINAL MOTION WITH THE MAXIMUM LIKELIHOOD METHOD

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*The peculiarities of the identification problem of the longitudinal motion of one aircraft class according to flight test records in the presence of intensive noises are considered. It is shown, that for the given aircraft class with the given input signals only identification of the short period component of the longitudinal motion is possible. The identification of the short period motion of aircraft using the maximum likelihood method is executed.*

## Introduction

Identification of the aerodynamic characteristics of the airplane lateral motion in the presence of noises, recorded in the flight test data, based on the Kalman filtration and maximum likelihood function as the identification criterion was considered in the papers [1; 2].

The identification of the aerodynamic characteristics of aircraft longitudinal motion on the basis of the flight test data has its own peculiarities. As is known the longitudinal motion consists of the short period and the phugoid motions. Division of the longitudinal motion into short and long period components is especially important for big airplanes, because their long periods are 100 times more, than their short ones. Such systems are known to be called “stiff” systems.

Attempts of this problem solution are given in [3; 4], however, in these papers the attention to the identifiability problem has not been paid. The previous analysis of the dynamic properties of a real plant will allow to determine the identification algorithm and to estimate its mathematical models (MM). This article is devoted to the analysis of the peculiarities of the aircraft longitudinal motion identification and to building the corresponding procedure.

## Statement of the problem

The identification problem of the longitudinal motion of the airliner such as DC-8 [5] in cruising flight based on the records of variables which characterize the longitudinal motion, namely, longitudinal and vertical speeds $u$ and $w$ respectively, pitch angle $\vartheta$ and pitch rate $q$, is considered. These variables are the components of state vector

$$\mathbf{x}=[u, w, \vartheta, q]^T,$$

the control input is the deflection of the elevator $\delta e$. Records of these variables are contaminated with noises of different nature. We shall consider that the received records of the flight test contain only uncorrelated stochastic noises with zero expectation.

Linearized MM of the aircraft longitudinal motion is described with the well known linear state space equations [5].

It is necessary to determine parameters of the state space model, i.e. elements of state, control, observation and direct transfer matrices on the basis of flight test data.

The presence of the measurement noises in the flight records results in the biased estimations of parameters of the dynamic model, therefore the important task at data processing is the minimization of errors, which are due to these factors. At the same time the number of parameters to determine is rather great. It is explained by the following circumstance: the aircraft model has to be identified in state space, and the parameters of this model are aerodynamic derivatives of linearized model of aircraft dynamics for a selected mode of flight.

## The model of the longitudinal channel and its properties

As it is known [5], the linearized model of the aircraft longitudinal motion is described with the following state space equations with constant coefficients:

$$\begin{aligned}\dot{\mathbf{x}}&=\mathbf{A}\mathbf{x}+\mathbf{B}\mathbf{u}+\boldsymbol{\omega}_1;\\ \mathbf{y}&=\mathbf{C}\mathbf{x}+\mathbf{D}\mathbf{u}+\boldsymbol{\omega}_2,\end{aligned}\qquad(1)$$

where $\mathbf{A}$, $\mathbf{x}$ are the state matrix and the state vector respectively; $\mathbf{B}$, $\mathbf{u}$ are the control matrix and the input vector respectively; $\mathbf{C}$, $\mathbf{y}$ are the observation matrix and the observation vector respectively; $\mathbf{D}$ is the matrix of the direct transfer from control input to output; $\boldsymbol{\omega}_1$ and $\boldsymbol{\omega}_2$ are uncorrelated white noises with intensities $\mathbf{Q}$ and $\mathbf{R}$ respectively.

For the aircraft longitudinal motion these matrices and vectors are the following [5]:

$$\mathbf{A}=\begin{bmatrix} x_u & x_w & -g & 0\\ z_u & z_w & 0 & V_0\\ 0 & 0 & 0 & 1\\ m_u & m_w & 0 & m_q\end{bmatrix};\quad \mathbf{x}=\begin{bmatrix}u\\ w\\ \vartheta\\ q\end{bmatrix};$$
$$\mathbf{B}=\begin{bmatrix}0\\ z_{\delta e}\\ 0\\ m_{\delta e}\end{bmatrix};\quad \mathbf{u}=[\delta e];\quad \mathbf{y}=\begin{bmatrix}u\\ w\\ \vartheta\\ q\end{bmatrix},\qquad(2)$$

where $x_u$, $x_w$ and $z_u$, $z_w$, $z_{\delta e}$ are the corresponding partial derivatives of the longitudinal and vertical forces respectively; $m_u$, $m_w$, $m_q$, $m_{\delta e}$ are the corresponding partial derivatives of the pitch moment.
The vector of unknown parameters for the identification of the aircraft longitudinal motion has the following form:

$$\boldsymbol{\theta} = \left[x_u,\ x_w,\ z_u,\ z_w,\ m_u,\ m_w,\ m_q,\ z_{\delta e},\ m_{\delta e}\right]^T . \quad (3)$$

According to [5] the nominal values of elements of parameter vector are shown in tab. 1.

## Solution of the problem

As measurements of the state vector components are contaminated with the considerable noises it is desirable to use the maximum likelihood method for the parametric identification of the aircraft state space model [3; 6 – 8].
This method results in the estimations, unbiased asymptotically, with the minimal variance in the case of Gaussian noise by selection of parametrical model which corresponds to the maximal value of likelihood function, that is

Table 1

**Nominal values of parameters**

| Parameters | $x_u$, 1/s | $x_w$, 1/s | $z_u$, 1/s | $z_w$, 1/s | $m_u$, 1/(m·s) | $m_w$, 1/(m·s) | $m_q$, 1/s | $z_{de}$, $m/(s^2 \cdot rad)$ | $m_{de}$, $1/s^2$ |
|---|---|---|---|---|---|---|---|---|---|
| Nominal values | -0,0140 | 0,0043 | -0,0735 | -0,8060 | -0,0026 | -0,0364 | -0,9240 | -10,5489 | -4,5900 |

$$\boldsymbol{\theta}_{\text{opt}}(\mathbf{Y}_m) = \arg\max_{\boldsymbol{\theta}\in\boldsymbol{\theta}_s} L(\mathbf{Y}_m, \boldsymbol{\theta}),$$

where $\boldsymbol{\theta}, \boldsymbol{\theta}_{opt}$ are a vector of unknown parameters and its estimation; $\mathbf{Y}_m$ is the measured output vector; $L$ is the likelihood function.
For convenience it is expedient to find not the maximum of likelihood function, but the minimum of the negative logarithm of this function, that is

$$J(\boldsymbol{\theta}) = -\ln\{L(\mathbf{Y}_m, \boldsymbol{\theta})\} = = 0{,}5\{\sum_{i=1}^{N}(\mathbf{Y}_m - \hat{\mathbf{Y}})^T \mathbf{R}_{in}^{-1}(\mathbf{Y}_m - \hat{\mathbf{Y}}) + + N\ln|\mathbf{R}_{in}| + mN\ln(2\pi)\}, \quad (5)$$

where $\hat{\mathbf{Y}}$ is the output vector of model; $\mathbf{Y}_m - \hat{\mathbf{Y}}$ is the vector of innovations; $|\mathbf{R}_{in}|$ is the Frobenius norm of innovation's matrix; $N$ is the number of measurement points (it depends on the length of realization); $m$ is the size of output vector $\hat{\mathbf{Y}}$ (it depends on the number of measured values).

Before starting of the identification procedure it is expedient to investigate identifiability of the plant.
For this purpose it is necessary to check up whether the system (2) with vector of parameters (3) satisfies the identifiability condition, which is the following [3; 6]:

$$\text{rank}\left[\frac{\partial \mathbf{Q}_{id}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}\right] = k, \quad (4)$$

$$\mathbf{Q}_{id}(\boldsymbol{\theta}) = \left[(\mathbf{CB})^T \quad (\mathbf{CAB})^T \quad \cdots \quad (\mathbf{CA}^{2n-1}\mathbf{B})^T\right];$$

where $k$ is the size of vector of unknown parameters.
As a result of the check it has been revealed, that the system (2) is uniquely determined by parameter vector (3) as the Jacobian matrix (4) has a full rank $k$.
Nevertheless for practical application it is necessary to make the numerical estimation of the identifiability of specific MM parameters. It can be made on the basis of the singular values analysis of the Jacobian matrix

$$\mathbf{Q}_j = \frac{\partial \mathbf{Q}_{id}(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}}.$$

Relation between their numerical values will show how many parameters of the identified model are weakly identifiable.
The eigenvalues of the longitudinal motion model of airliner DC-8 are $-0{,}8662 \pm 3{,}0237i$ and. The frequency of the short period component of longitudinal motion is 3,1453 rad/s, and the frequency of the phugoid component is 0,0240 rad/s. These frequencies differ more than by two orders; therefore it is necessary to carry out identification of the longitudinal channel for the given aircraft in two stages: identification of the short period component of longitudinal motion on short length of realization and identification of the phugoid component on the greater length of realization.

Identification procedure of aircraft in the state space on the basis of the maximum likelihood method is based on application of the optimal Kalman observer of aircraft dynamics and optimization procedure together with logarithmic likelihood function as a cost function. The block diagram of this procedure is presented in fig. 1.
Digital data $\mathbf{U}_m$ and $\mathbf{Y}_m$ received as a result of the flight test are processed in the block Data processing. The Kalman gain matrix (KGM) and the aircraft state space model (ASSM) represent the optimum observer. The parametrical identification algorithm (PIA) represents iterative procedure of parametrical optimization which arranges the model parameters using the error $\mathbf{E}$.

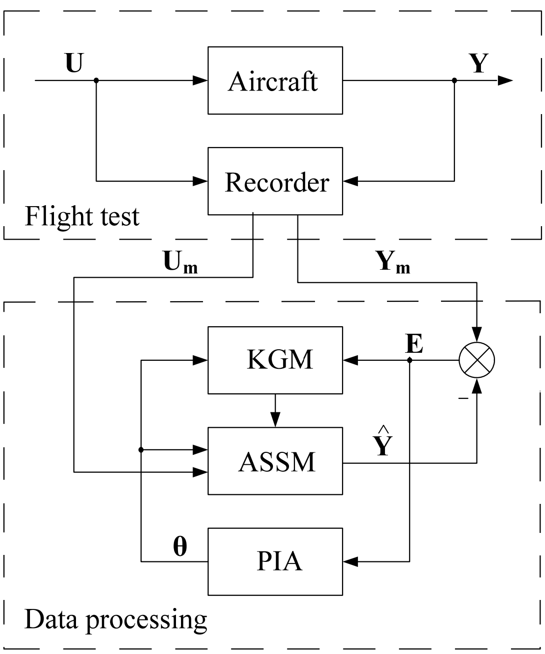



Fig. 1. The block diagram of the identification algorithm

The convergence of identification procedure depends on the sensitivity functions.

The gradient of logarithmic likelihood function with respect to the vector of parameters is equal to:

$$\nabla_\theta J = \sum_{i=1}^{N} (\nabla_\theta (\mathbf{Y}_m - \hat{\mathbf{Y}}))^T \mathbf{R}_{in}^{-1} (\mathbf{Y}_m - \hat{\mathbf{Y}});$$

$$\nabla_\theta (\mathbf{Y}_m - \hat{\mathbf{Y}}) = \nabla_\theta \hat{\mathbf{Y}}.$$

The sensitivity of each parameter can be approximated as follows:

$$\frac{\partial \hat{\mathbf{Y}}_i}{\partial \theta_k} = \frac{\hat{\mathbf{Y}}_i(\theta_k + \delta\theta_k) - \hat{\mathbf{Y}}_i(\theta_k)}{\delta\theta_k},$$

where $\delta\theta_k$ is a small disturbance of the $k$–th parameter.

The value of the disturbance $\delta\theta_k$ must be small enough to ensure the linear variation in response.

The records received as a result of simulation in Simulink package of the cruising flight, that was 20 seconds long, were used for the identification procedure. It has been revealed, that unknown parameters, which correspond to the phugoid component of longitudinal motion of the given aircraft, are poorly sensitive as compared to unknown parameters which correspond to the short period component. As it is known, the phugoid component of longitudinal motion should be identified on a greater length of realization, than the short period component. So the research of sensitivity function on the basis of simulated flight test data of the cruising flight, whose duration was 200 s, has been carried out. Unknown parameters corresponding to the phugoid component of longitudinal motion of the given aircraft were found to grow less sensitive as a result of the realization length increase. Owing to the long period parameters insensibility, its identification cannot be made with sufficient accuracy. This conclusion can be confirmed also by the analysis of the singular values of the Jacobian matrix $\mathbf{Q}_j$. The range of these values is from 0,0374 to 32693,0022. The first four smallest singular values indicate, that the corresponding parameters of state space model are weakly identifiable.

**Identification algorithm of the short period component of aircraft longitudinal motion**

The matrices and vectors which describe the short period component in state space (1) are the following [5]:

$$\mathbf{A} = \begin{bmatrix} z_w & V_0 \\ m_w & m_q \end{bmatrix}; \quad \mathbf{x} = \begin{bmatrix} w \\ q \end{bmatrix};$$
$$\mathbf{B} = \begin{bmatrix} z_{\delta e} \\ m_{\delta e} \end{bmatrix}; \quad \mathbf{u} = [\delta e]; \quad \mathbf{y} = \begin{bmatrix} w \\ q \end{bmatrix}. \tag{6}$$

The identification problem of the short period component of the aircraft longitudinal motion is to estimate vector of model parameters (6)

$$\boldsymbol{\theta} = [z_w,\ m_w,\ m_q,\ z_{\delta e},\ m_{\delta e}]^T. \tag{7}$$

As a result of research of the identifiability (4) of the short period component it has been discovered, that this component of longitudinal motion is completely identified.

As has been specified earlier, the most effective identification method of aerodynamic derivatives which determine aircraft dynamics in state space is the maximum likelihood method. Identification procedure in state space by the maximum likelihood method is based on the application of optimal Kalman observer of aircraft dynamics and optimization procedure together with logarithmic likelihood function as criterion [3; 4; 6].

At each step of the optimization procedure the steady state Kalman filter is used for optimal observation of measured responses of the aircraft motion model.

The first step of this procedure consists in determining the covariance matrices of the state variables $\mathbf{Q}$ and measurements $\mathbf{R}$. Then it is necessary to determine the covariance matrix of innovations $\mathbf{R}_{in}$ of Kalman filtering

$$\mathbf{R}_{in} = \mathbf{CPC}^T + \mathbf{R}, \tag{8}$$

where $\mathbf{P}$ is the Riccati equation solution.

Several methods for iterative determination of this matrix [4] are discussed. The problem is that $\mathbf{R}_{in}$ in (8) depends on the covariance matrix $\mathbf{P}$ of the state variables, which can be defined later as the Riccati equation solution that in its turn depends on the matrix $\mathbf{R_{in}}$. In the given work the authors propose the simplest way for this problem solution. As the first step before Riccati equation solution, the matrix $\mathbf{R}_{in}$ can be approximated as:

$$\mathbf{R}_{in} = \mathbf{CQC}^T + \mathbf{R}\,. \tag{9}$$

This approximation can be successfully used for Kalman filtering at each step of the optimization procedure.

As offered in [4], better results (in comparison with the results which refer to the purely discrete case) can be received using the first order approximation of the continuous Riccati equation instead of the discrete one:

$$\mathbf{AP} + \mathbf{PA}^T - (1/d)\mathbf{PC}^T(\mathbf{R}_{in})^{-1}\mathbf{CP} + \mathbf{Q} = 0\,, \tag{10}$$

where $d$ is sampling interval.

Solution of this equation gives the state variables covariance matrix $\mathbf{P}$, which is then used for determination of Kalman gain matrix $\mathbf{K}$

$$\mathbf{K} = \mathbf{PC}^T(\mathbf{R}_{in})^{-1}\,. \tag{11}$$

The updated state space vector can be calculated as follows:

$$\mathbf{X}_s(i/i-1) = \mathbf{FX}_s(i-1) + \mathbf{B}_d\mathbf{U}(i-1)\,, \tag{12}$$

where $\mathbf{F} = \mathbf{I} + d\mathbf{A}$; $\mathbf{B}_d = d\mathbf{B}$ are matrices of the state space description for the discrete system;

$$\hat{\mathbf{Y}}(i) = \mathbf{CX}_s(i/i-1) + \mathbf{DU}(i-1)\,; \tag{13}$$

$$\mathbf{X}_s(i) = \mathbf{X}_s(i/i-1) + \mathbf{K}(\mathbf{Y}_m - \hat{\mathbf{Y}})\,. \tag{14}$$

The equations (8)–(14) describe steady-state Kalman filtering procedure.

Logarithmic likelihood function as a cost function for the optimization procedure can be written as (5).

The problem is to find the optimal value of the parameters vector $\boldsymbol{\theta}_{opt}$ with the components determined by expression (7), which delivers the minimal value of the cost function $J(\boldsymbol{\theta})$:

$$\boldsymbol{\theta}_{opt} = \arg\min_{\boldsymbol{\theta}\in\boldsymbol{\theta}_s} J(\boldsymbol{\theta})\,.$$

Here the quasi-Newton method of optimization as one of the most reliable from the view point of convergence is accepted [9].

The optimization algorithm can be described shortly in the following way:

$$\boldsymbol{\theta}(i+1) = \boldsymbol{\theta}(i) - \gamma\mathbf{H}^{-1}(i)\nabla\mathbf{J}(\boldsymbol{\theta}(i))\,,$$

where $\gamma$ is the scalar parameter which determines the step size; $\nabla\mathbf{J}(\boldsymbol{\theta}(i))$ is the vector of gradient, which is determined by partial derivatives $\partial\mathbf{J}/\partial\theta_k(i)$ in our case $k = 1,...,5$; $\mathbf{H}(i)$ is the matrix of the second derivatives $\mathbf{H}(i) = \nabla^2\mathbf{J}(\boldsymbol{\theta}(i))$.

It is recommended to apply scaling of the optimizable function and the parameters vector for convergence improvement.

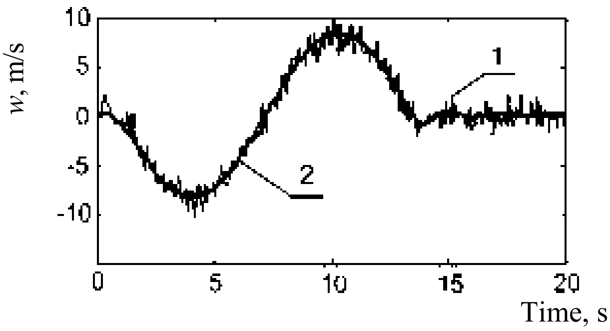
Scaling of optimizable function allows to prevent a divergence of optimization procedure at its beginning, and scaling of parameters vector allows to improve its convergence at the end.

**Results of parametrical identification of the short period component**

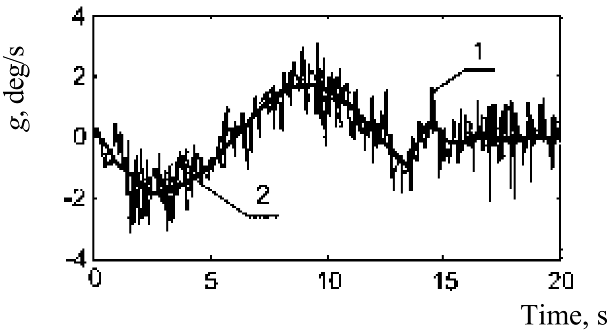
Records of the elevator deflections, vertical speed and pitch rate have been received as a result of simulating cruising flight which was 20 s long in Simulink package taking into account the noises of real sensors.

Identification of the short period component was made according to the offered identification algorithm with different input control signals (elevator deflections): the sinusoidal and the trapezoidal waveform [3; 6] signals. The results of identification at input signals used do not differ essentially. Initial, nominal and estimated values of unknown parameter vector at sinusoidal input signal are presented in tab. 2.

Comparison of the measured and calculated output signals is presented in fig. 2.




*a*




*b*

Fig. 2. Comparison values of the measured and estimated output data:
*a* – vertical speed; *b* – pitch rate;
*1* – measured output data; *2* – estimated output data

*Table 2*

**The results of parametrical identification**

| Value | Parameter | | | | |
|---|---|---|---|---|---|
| | $z_w$, 1/s | $m_w$, 1/(m·s) | $m_q$, 1/s | $z_{de}$, m/(s$^2$·rad) | $m_{de}$, 1/s$^2$ |
| Initial values | -0,70 | -0,07 | -0,84 | -17,19 | -2,70 |
| Nominal | -0,8060 | -0,0364 | -0,9240 | -10,5489 | -4,5900 |
| Estimated | -0,8626 | -0,0328 | -0,8719 | -12,4314 | -4,0590 |

## Conclusion

Before starting the identification of the aircraft dynamics it is necessary to analyze the plant for identifiability and sensitivity to change of the parameters. As has been proved, it is expedient to carry out identification of the short period component only.

Identification of dynamics of the short period of aircraft motion using Kalman filtration and the maximum likelihood function as criterion of identification is sure to yield good results.

## Literature

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Ідентифікованість та ідентифікація поздовжнього руху літальних апаратів методом максимальної правдоподібності

Розглянуто особливості задачі ідентифікації поздовжнього руху одного класу літальних апаратів за записами льотних випробувань в умовах інтенсивних завад. Показано, що для цього класу літальних апаратів при даних вхідних сигналах можлива ідентифікація тільки короткоперіодичної складової поздовжнього руху. Використовуючи метод максимальної правдоподібності, виконано ідентифікацію короткоперіодичного руху літальних апаратів.

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Идентифицируемость и идентификация продольного движения летательных аппаратов методом максимального правдоподобия

Рассмотрены особенности задачи идентификации продольного движения одного класса летательных аппаратов по записям летных испытаний в условиях интенсивных помех. Показано, что для этого класса летательных аппаратов при данных входных сигналах возможна идентификация только короткопериодической составляющей продольного движения. Методом максимального правдоподобия выполнена идентификация короткопериодического движения летательных аппаратов.