UDC 656.7.052(045)

A.V. Gusynin

APPLICATION OF DIFFERENTIAL TRANSFORMATIONS FOR THE MODELING MOTION OF AEROSTATIC VEHICLES AND FOR THE SYNTHESIS OF CONTROL ALGORITHMS

NAU Aerospace Institute, e-mail: gusynin@hotmail.com

The approach to simulation of flight dynamics and numerically-analytical method of airship control algorithms are offered. It's based on differential transformations of initial mathematical model of airship motion. The given approach allows for elimination of viewing time function for their differential spectra in the image field. It gives the possibility to reduce a problem of closed algorithm synthesis of vehicle control to the solution of non-linear equation system concerning control variable.

Introduction

Among significant problems of lighter-than-air (LTA) vehicles operation the poor efficiency of their low-speed control, especially during takeoff and landing is selected. In this connection frequent there are difficulties at their ground maintenance and the relevant restrictions for safe vehicle operation are inlet with gusts present. The use of the power-plant with thrust vector control on LTA vehicle essentially dilates aircraft performance characteristics, simplifies taking-off and landing, improves ground maintenance, safety and the pliability of operation also at active gusts [1; 2].

Automation of LTA vehicle motion control, qualitative improvement of the board and ground equipment are stipulated by expansion of a circle of problems, which such vehicles face, and growing requirements to quality of their solution. The control of LTA vehicle landing procedure is a complex procedure and is characterized by different modes of simultaneous operation of propellers and aerodynamic controls (elevators and rudders), practically by sudden change of vehicle mass at the moment of ballast drop. Other feature is the presence of restrictions on permissible descent vehicle velocity at the moment of touchdown, necessity of the registration of the external environment (gusts) impact. The rated path of the LTA vehicle auto-landing is created in view of all above features and restrictions. It brings about the necessity of the solution of a multiparameter problem on the choice of the relevant control algorithms and their use in real time.

In this paper for an evaluation of the different concepts of automatic control of a modern LTA vehicle at a landing stage (the airship of a classical type is considered), the approach to modeling a vehicle movement and a numerical - analytical method of control algorithm synthesis, based on application of the mathematical apparatus of differential transformations is offered. According to the accepted approach the path of airship auto-landing is divided into some segments, whose boundaries correspond to the moments of control switching, changes of mass at ballast drop, moments of a path output or control on the relevant restrictions. Thus it is supposed, that the data time frames are given and within them a vehicle motion parameters have no jump changes. The resulting path is recovered on sites with a docking of regional requirements. A similar approach to vehicle control algorithms synthesis on the basis of differential transformations was already applied in solution of a problem of control of aerospace system get to the orbit [3].

As the used mathematical apparatus of differential transformations is practically new in application to problems of airship movement control, we shall briefly consider the basic properties of differential transformations.

Differential transformations and their basic properties

The method of differential transformations is offered by academician G.E. Puhov [6]. As apposed to known integrated Laplace and Fourier transformations, it is based on translation of the originals into the images field through the differentiation operation. At mathematical simulation of physical processes and objects featured by differential and integral equations, the differential transformations allow to replace operations of integration and differentiation by equivalent algebraic operations.

The differential transformations allow to replace in the mathematical model of object dynamics the functions $x(t)$ of continuous argument by their spectral models in the form of discrete functions $X(k)$ of integer argument $k = 0,1,2,...$. The differential transformations are the functional transformations of the type [6]:

$$\underline{x(t)} = X(k) = \frac{h^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0} \Leftrightarrow$$
$$\Leftrightarrow x(t) = \sum_{k=0}^{k=\infty}\left(\frac{t}{h}\right)^k X(k), \tag{1}$$

where $x(t)$ is the original, which represents the material analytical function of material argument; $x(t)$ and $X(k)$ are equivalent labels of the differential image of the original representing discrete function of integer argument k, which is termed as a differential spectra of function $x(t)$ in the point $t = t_0$; h is the scale stationary value having dimensionality of argument t, is usually chosen equal to the segment $0 \le t \le h$, on which the function $x(t)$ is considered; the line below is the figure of transformation; $\Leftrightarrow$ is the figure of correspondence between the original $x(t)$ and its differential image $X(k)$.

The expression to the left of figure $\Leftrightarrow$ in (1) defines direct transformation permitting by the original $x(t)$ to find the image $X(k)$, and on the right – inverse transformation, which recovers the original $x(t)$ by the images $X(k)$ as a Taylor series with the center in the point $t = 0$.

The transformations (1) are termed as differential Taylor transformations or T – transformations, differential images $X(k)$ – differential T – spectra, and value $X(k)$ at concrete values of argument are termed discretes. For example, $X(0)$ is zero discrete, $X(1)$ is the first discrete etc.

Let's give some basic properties of differential transformations [4].

The initial value of the original $[x(t)]_{t=0} = x(0)$ equals zero discrete $[X(k)]_{k=0} = X(0)$ of basic image $X(k)$, i.e. $X(0) = [X(k)]_{k=0} = X(0) = [x(t)]_{t=0}$.

The value of the original $x(t)$ in point $t = h$ is equal to the total off all discretes of the basic image $X(k)$:

$$x(t = h) = X(0) + X(1) + X(2) + ... = \sum_{k=0}^{k=\infty} X(k).$$

If at $t \succ 0$ original $x(t)$ is presented by a Taylor series, its value in point $t = -h$ will be determined by expression:

$$x(-h) = X(0) - X(1) + X(2) - X(3) + ... = \sum_{k=0}^{k=\infty} (-1)^k X(k)$$

Let's consider now a number of mathematical operations above T-functions. As follows from expression (1) the algebraic total of the originals there corresponds to the algebraic totals of their T-images:

$$x(t) \pm y(t) \Leftrightarrow X(k) \pm Y(k),$$

$$x(t_v + \tau) \pm y(t_v + \tau) \Leftrightarrow X_v(k) \pm Y_v(k).$$

Operations of multiplication of the original $x(t)$ by the constant value C corresponds to multiplication of the images $X(k)$ and $X_v(k)$ by it:

$$Cx(t) \Leftrightarrow CX(k);$$

$$Cx(t_v + \tau) \Leftrightarrow CX_v(k).$$

The product of two functions $x(t)$ and $y(t)$ corresponds in the field of T-images to the product, which is designated by the symbol *.

$$x(t)y(t) \Leftrightarrow X(k) * Y(k) == \frac{h^k}{k!}\left[\frac{d^k x(t)y(t)}{dt^k}\right]_{t=0} =$$

$$= \sum_{l=0}^{l=k} X(k-l)Y(l).$$

The operation of differentiation in the field of the originals corresponds to the following expression in the field of images:

$$y(t) = \frac{dx(t)}{dt} \Leftrightarrow Y(k) = DX(k) = \frac{k+1}{h} X(k+1),$$

where the figure D means T-derivative.

The images of higher derivatives from $x(t)$ per t are similarly determined:

$$\frac{d^m x(t)}{dt^m} \Leftrightarrow D^m X(k) = \frac{(k+m)!}{k! h^m} X(k+m),$$

where m is natural number; D^m is the figure of T-derivative of the m order.

Substituting the power series

$$x(t) = \sum_{k=0}^{k=\infty} \left(\frac{t}{h}\right)^k X(k).$$

In the expression of particular integral from $x(t)$ per t, we shall receive:

$$\int_{t_a}^{t_b} x(t)dt = \int_{t_a}^{t_b} \left[\sum_{k=0}^{k=\infty} \left(\frac{t}{h}\right)^k X(k)\right] dt = h \sum_{k=0}^{k=\infty} \left[\left(\frac{t_b}{h}\right)^{k+1} - \left(\frac{t_a}{h}\right)^{k+1}\right] \frac{X(k)}{k+1}. \qquad (2)$$

In that specific case, when $t_a = 0$ and $t_b = h$, we have:

$$\int_0^h x(t)dt = h \sum_{k=0}^{k=\infty} \frac{X(k)}{k+1}. \qquad (3)$$

Expressions (2) and (3) allow discovering in given limits a particular integral from the original $x(t)$ on discretes of the image $X(k)$.

The mathematical models, converted through differential transformations, are termed as spectral models.

Target setting of terminal control

All controllable airship landing process is conditionally divided into r given time frames, inside which the parameters of the vehicle have no sudden changes:

$$T_i = t_i - t_{i-1},\ i = \overline{1,r},\ \sum_{i=1}^{r} T_i = T,$$

where T is airship's landing time from the beginning of descending to touchdown with ground.

Further we shall figure, that all these changes in the form of given springs happen at boundaries of selected intervals.

Mathematical model of an airship motion at the landing stage we shall present as the vector differential equation:

$$\frac{dx_i}{dt} = f_i(t, x_i, u_i, v_i),\ \ x_i(t_{i-1}) = x_i^0, \quad (4)$$

$$i = \overline{1,r},$$

where $x_i = x_i(t)$ is n-measurement of state vector; $u_i = u_i(t)$ is m-measurement control vector; $v_i = v_i(t)$ – ℓ a measurement vector of turbulence; f_i – continuous and continuously differentiable on plurality variable t, x_i, u_i, v_i the vector function of generalized force; $t \in (t_i - t_{i-1})$.

The problem of terminal control consists in the vehicle translation from given initial state $x_1(t_0)$ to final (terminal) state $x_r(T)$, which is determined in the point of time $t = T$ by q - measurement ($q \le n$) vector equation:

$$S[x_r(T), T] = 0. \quad (5)$$

The quality of control procedure is estimated by the functional:

$$I = G[x_r(T), T] + \sum_{i=1}^{r} \int_{t_0}^{T} \Phi_i(t, x_i, u_i, v_i)dt, \quad (6)$$

where the given functions G and Φ_i have continuous partial differential coefficients on x_i, u_i, v_i. Restriction on state vectors and the control are taken into account during the selection of the functional type (6).

The conjugation boundary and starting conditions of sites of the process of deducing are set in the form of given border of requirements:

$$\varphi_i\left[x_i(T_i), x_{i+1}^0; u_i(T_i), u_{i+1}^0; T_i\right] = 0,$$

$$i = \overline{1,r}. \quad (7)$$

Program control $u = u(x,t)$, optimizing functional (6), implements optimum control on the open loop and guarantees execution of boundary requirements (5) in absence of activity of turbulences. Under actual condition the impact of an external environment $v_i(t)$ on the airship landing dynamics is considerable. With the purpose of neutralizing these turbulences the law of optimum by criterion (6) feedback control of the type is synthesized:

$$u = u(x, t). \quad (8)$$

Control (8), utilizing in each instant t the information on the current state $x(t)$, provides taking an airship from an arbitrary initial state into final (5) subjected to turbulences. The synthesis of feedback control of the type (8) can be implemented by the method of dynamic programming [5]. An essential deficiency of the given method is the problem of dimensionality, which consists in requiring very big memory of a computer even for problems of small dimensionality.

The numerical-analytical method of synthesis of the control algorithms of the airship landing with the use of the mathematical apparatus of differential transformations of functions and equations is considered below. As mentioned above, the method does not require numerical integration of differential equations and allows for analytical transformations, which considerably reduce the volume of computation during obtaining the numerical solution and, thus, allows to find solution for computing complexity of the given problem of synthesis.

Spectral model of an airship motion

The considered method of the control algorithms synthesis is based on the spectral model of control procedure [6]. The given model is obtained from equations of object motion by application of differential transformations to them (1). As a result the vector differential equation of the trajectory motion of an airship (4) in the images field is written as the following spectral model:

$$X_i(k+1, A_i, X_i^0) = \frac{T}{k+1} \underline{f_i} \times$$
$$\times \cdot [t, X_i(k, A_i, X_i^0), U_i(k, A_i)]; \quad (9)$$
$$X_i(0) = X_i^0(A_{i-1}, A_{i-2}, ..., A_i);$$
$$X_i(0) = X_i^0 = x_0; i = \overline{1,r}.$$

Spectral model (9) has a universal character and can be applied for problem of airship dynamics of different arranging. Let's point out, that as the differential transformations (1) are a precise operating method, the spectral model (9) has no methodical errors and potentially allows to receive a precise solution of the differential equation (4).

The method of control synthesis

The control algorithm synthesis with feedback can be executed by the method of closure of program control $u(t)$ for an arbitrary current state $x(t)$ [7]. At the first stage of synthesis we shall consider a undisturbed airship motion and we shall select inside each segment of its removing program control from a class of analytical functions $u_i(\tau, A_i)$, where

$A_i = (a_{i1}, a_{i2}, ..., a_{in})$ is the vector of free parameters, τ is the local time argument. Differential transformations (1) of function $u_i(\tau, A_i)$ are determined at $h = T_i$ and $\tau = 0$ its differential spectrum as:

$$\underline{u_i}(\tau A_i) = U_i(k, A_i) = \frac{T_i^k}{k!}\left[\frac{d^k u_i(t_{i-1} + \tau_i A_i)}{dt^k}\right]_{t=t_0} \quad (10)$$

Based on recursion expression (9) and differential spectra of control (4), the differential spectra $X_i(k, A_i, X_i^0)$ of a state vector $x_i(t)$ is formed. Let's take advantage of the property of the differential transformations [4], according to which the algebraic total of all a builder (discretes) differential spectra of any analytical function in point $t = t_v$, is equal to zero discrete of a differential spectrum of function in point $t_{v+1} = t_v + h$ or value of the original of function in the same point:

$$\sum_{k=0}^{\infty} X_v(k) = X_{v+1}(0) = x(t_v + h).$$

From the obtained relation at $t_v = t_{i-1}$ and $h = T_i$, we determine a state vector at the end of each landing phase of an airship:

$$x_i(k, A_i, x_i^0) = \sum_{k=0}^{\infty} X_i(k, A_i, X_i^0),\ i = \overline{1, r}. \quad (11)$$

Then the equation of the final state (5) in view of the expression for conjugation of boundary and initial sites of landing process (7), and also the expressions for a state vector at the end of each landing phase (11) is conversed as followed:

$$S[A_1, A_2, ..., A_r] = 0. \quad (12)$$

The given boundary condition in the implicit shape define q a builder of vectors of free parameters $A_i, i = \overline{1, r}$ as functions from T_i and x_i^0.

Remaining builders of vectors of free parameters are determined from the requirements of optimality of functional (6). The differential transformations (1) of functional (6) in view of differential spectra (9) and (10) allow presenting functional (6) as the function of vectors of free parameters A_i:

$$I[A_1, A_2, ..., A_r] = G[A_1, A_2, ..., A_r] + \sum_{i=1}^{r} T_i \sum_{r=0}^{\infty} \frac{\underline{\Phi_i}\left[t, X_i(k, A_i, X_i^0), U_i(k, A_i)\right]}{k+1}.$$

The necessary requirements of an optimality of the given function enable to receive combined equations for determining remaining (n-q)r of unknown builders of free parameters vectors $A_1, A_2, ..., A_r$:

$$\frac{\partial I(A_1, A_2, ..., A_r)}{\partial a_{ij}} = 0;\ i = \overline{1, r};\ j = \overline{q+1, n}. \quad (13)$$

The obtained system of the nonlinear equations (12) and (13) in the implicit shape defines builders of a vector of free parameters $A = (A_1, A_2, ..., A_r)$ as functions from a vector of an arbitrary initial state $x_0 = x_i(t_0)$.

As a result of execution of the first stage of synthesis of the control algorithms in the implicit form, the nonlinear communication of optimum program control $u[t, A(T, x_0)]$ with a vector of the initial state $x_0 = x_i(t_0)$ is established. This control cannot be applied over all the time slice T of airship landing in case disturbations effect on it. The control $u[t, A(T, x_0)]$ can be utilized only for control in the initial instant t_0.

Thus, the differential transformations (1) allow to receive in the analytic form combined equations (12) and (13) for arbitrary values of the initial state x_0, time instant t_0 and interval T.

At the second stage of synthesis is considered the disturbed airship motion at the landing stage, which permanently declines from the optimum program trajectory. In this case control $u[t, A(T, x_0)]$ is calculated from combined equations (12) and (13) for current values of time t and state x (t). The solution of combined equations (12) and (13) for each current instant t and state x (t) during airship landing subjected to turbulence, continuously sets control u (t, x), linking current state x (t) with boundary (terminal) requirements (5).

In the closed circuit of control only the current value of control $u[t, A(T, x_0)]$ will be utilized which in the following instant is calculated using equations (12) and (13). It provides "pliable" adaptation of the landing path of an airship to the action of unknown turbulence factors.

Let's point out distinctive features of the offered method. The initial mathematical model (4)–(7) of an airship motion refers to multipoint nonlinear boundary-value problems. The solution of such problems by the known numerical methods demands considerable volume of computation, which in real time causes difficulties. The offered method, utilizing the mathematical apparatus of differential transformations, allows for receiving the system of spectral models (9), connected of boundary conditions, measured to given points. The models look like the system of recursion expressions which do not contain time argument, and allow yielding calculations in the analytical way. The initial multipoint nonlinear boundary-value problem (4)–(7) as a result of the applied approach is put to the system of final equations, whose continuous solution allows for implementing feedback control in real time.

Thus, the offered numerical – analytical method gives the problem of synthesis of the made control laws to the solution of the system of nonlinear equations without numerical integration and differentiation of the equations of the airship motion trajectory. The basic advantage of the designed method consists in that it is established in the implicit form (12), (13) nonlinear communication of control $u[t, A(T, x)]$ with the vector of current state x (t). It allows for forming control on the feedback from the parameters of a trajectory airship motion during its landing.

Modeling

The configuration of modeled aerostatic vehicle, basically, corresponded to an airship of vertical take-off and landing of «Zeppelin NT» type. It was accepted, that the vehicle is fulfilled by the semi-rigid scheme and has "λ"-like tail unit. Envelope volume is 8 225 м^3, length – 75 m, width – 19,5 m (including a tail unit). Three reversible mid-flight engines with declined propellers up to 120° and rated power of 200 h.p. each supposed to be mounted on the airship. Two of them are mounted on each body side in the area of the center of application of the aerostatic lift and work synchronously at creating driving forces. The third engine is mounted in the rear part of the body. Apart from the direct assignment it also supplies the tail propeller declined in the horizontal plane up to 90^0. The given propeller is intended for turn realization and parrying of transversal forces, as well as improvement of directional controllability of the vehicle.

The propellers control will be utilized alongside with aerodynamic control.

The longitudinal airship motion at the landing stage is considered. An airship motion as the solid body were accepted motion equations in the vertical plane obtained from common nonlinear motion equations [8] as the initial mathematical model. In the model exterior forces and moments acting on the vehicle on behalf of gravitational and aerodynamic forces, aerostatic lift and control actions were taken into account.

The aerodynamic airship performances are calculated using Johnes and De Laurier method [9]. The action on an airship of aerodynamic forces and moments depending on acceleration was taken into account according to [10].

The propellers were simulated as power sources able to create the negative thrust up to 50 % of the rated one. The maximum velocity of rotational displacement of propellers is accepted of equal to 5°/second. The pilot simulation as device of a control loop has not been made.

Making use of the equations of longitudinal airship motion together with differential transformations (1) there was obtained in the images field the spectral model of a vehicle motion at the landing stage, which then was applied to the control algorithms synthesis by the elevator and diversion of propellers [11].

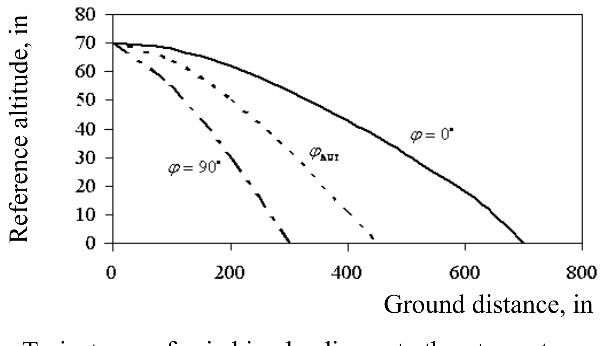
The solution of the problem of control algorithms synthesis was followed by simulation of a controllable airship motion at the landing stage. Sequentially setting integer values of argument $k = 0,1,2\ \ldots$ the builders (discretes) of differential spectra of a variable trajectory airship motion were calculated.

For restitution of time processes of parameters changes of airship dynamics by differential spectra was accepted computationally method of time restitution processes in the form of Taylor series [4]. According to this method, for obtaining values of object motion parameters in an instant $t_i = t_{i-1} + h$ it is enough only to sum algebraically discretes of a differential spectrum, which are calculated for an instant t_{i-1}.

Airship landing was modeled with simultaneous thrust vector and elevator control. On account of precise landing criteria absence at control algorithm synthesis by thrust vector it was supposed, that close to zero vertical speed was achieved at zero height over the set distance from the beginning of landing process.

As entrance signals at control algorithm synthesis of a thrust vector, flight altitude changing, speed of flight altitude changing (vertical descent rate), pitch changing and speeds of pitch changing have been accepted.

Below in figure the received airship landing trajectories are shown.



Trajectory of airship landing at thrust vector control and fixed deflection of an elevator

Variants horizontal direction of thrust vector ($\varphi = 0^\circ$), thrust vector downwards deflection at 90° ($\varphi = 90^\circ$), synthesized control of thrust vector (φ_{AUT}) were considered. The similar trajectories of landing were simulated and with application of conventional numerical methods (standard method of Runge – Kutta of the fourth order) for integration of input equations of an airship motion.

The matching of the obtained results has shown, that the application of simulation on the basis of differential transformations at dynamics examination of mobile objects allows essentially (by ~ 2–3 times) for reducing computing expenditures. Thus, there is an opportunity of realization of analytical examination of the problem. The obtained numerical simulation results wear illustrative character and testify to the possibility of differential transformations to use the problems of control algorithms synthesis and simulation of flight airship dynamics.

Conclusion

The approach to airship motion simulation and control algorithms synthesis based on differential transformations of initial mathematical model of the vehicle motion is offered. The given approach is formalized as the relevant mathematical model that allows for eliminating from viewing time functions and replaces them by their differential spectra in the images field. The offered method does not demand a numerical integration of differential equations of object motion and essentially reduces computing expenditures at model operation in comparison with the conventional numerical methods.

The opportunity and operational effectiveness of the given approach is shown at modeling of airship motion such as «Zeppelin NT» at the landing stage.

Literature

1. *Nagabhushan B.L., Faiss G.D.* Thrust vector of a V/STOL airship // Journal of aircraft. – 1984. – Vol. 21, №6. – P. 408–413.

2. *Nagabhushan B.L., Tomlinson N.P.* Thrust-vectored takeoff, landing and ground handling of an airship //Journal of aircraft. – Vol. 23, №3. – 1985. – P. 250–256.

3. *Gusynin V.P., Baranov V.L., Gusynin A.V.* Synthesis of control of the ascent into orbit of multimode aerospace systems // Proc. of AAS/AIAA Astrodynamics specialists conf. – Quebec City, Quebec. – Canada. –Jul. 30 – Aug. 02, 2001. – P. 50–60.

4. *Пухов Г.Е.* Дифференциальные спектры и модели. – К.: Наук. думка, 1990. – 180 с.

5. *Сейдж Э.П., Уайт Ч.С.* Оптимальное управление системами. – М.: Радио и связь, 1982. – 350 с.

6. *Диференціальні* перетворення в задачах керування рухом літальних апаратів: Навч. посіб./ В.Л. Баранов, В.П. Гусинін, А.В. Гусинін та ін. – К.: НАУ. – 2003. – 158 с.

7. *Уруский О.С., Баранов В.Л.* Синтез замкнутых законов терминального управления на основе дифференциальных преобразований // Электронное моделирование. – 1996. – 18, №3. – С. 3–8.

8. *Jones S.P., Krausman J.A.* Nonlinear Dynamic simulation of tethered aerostat//Journal of aircraft. – 1982. –Vol. 19. – P. 679–686.

9. *Jones S.P., DeLaurier J.D.* Aerodynamic estimation techniques for aerostat and airship//Journal of aircraft. – 1983. – Vol. 20. – P. 120–126.

10. *Jones R.* Acceleration derivatives in the gase of a body moving in an ideal fluid // ARC R&M 748. – Apr., 1921. – P. 45–54.

11. *Гусынин В.П., Гусынин А.В.* Системоаналоговое моделирование процесса управления дирижаблем с отклоняемым вектором тяги // Тр. конф. «Dynamical systems modeling and stability investigation». – К. , 2001. – С. 164.

The editors received the article on 13 April 2005.

А.В. Гусинін

Застосування диференціальних перетворень до задач моделювання руху та синтезу алгоритмів керування аеростатичними літальними апаратами

Запропоновано підхід до моделювання руху та синтезу алгоритмів керування дирижаблем, які основані на диференціальних перетвореннях вихідної математичної моделі руху апарата. Можливість та ефективність застосування цього підходу продемонстровано при моделюванні руху дирижабля типу “Zeppelin NT” на етапі посадки.

А.В. Гусынин

Применение дифференциальных преобразований к задачам моделирования движения и синтеза алгоритмов управления аэростатическими летательными аппаратами

Предложен подход к моделированию движения и синтеза алгоритмов управления дирижаблем, основанный на дифференциальных преобразованиях исходной математической модели движения аппарата. Возможность и эффективность применения данного подхода продемонстрировано при моделировании движения дирижабля типа “Zeppelin NT” на этапе посадки.