

**ENVIRONMENT PROTECTION**

UDC 65.011.3:656.71:528.9 (045)  
DOI: 10.18372/2306-1472.70.11428

Inna Gosudarska

**ANALYSIS RESULTS OF DISTRIBUTION FUNCTIONS  
FOR PROBABILITY DENSITY ASSESSMENT OF ACCIDENT LOCATIONS  
IN THE VICINITY OF THE AIRPORT**

National Aviation University  
Kosmonavta Komarova avenue 1, 03680, Kyiv, Ukraine  
E-mail: gil\_@mail.ru

**Abstract**

*This paper contains the results of processing and statistical analysis of accident location data from the period 1973-2013 involving different types of large and light aircraft. The distribution of accident site locations which occurred during various phases of flight near-airport is presented. The analysis results of distribution functions are shown to estimate the probability density of accident locations during departures and arrivals. It is defined that the Weibull distribution and Gamma distribution are the most plausibly corresponded to describe the probability density of the accident site locations along the longitudinal and lateral coordinates relative to the runway centerline. The statistical characteristics of accident locations have obtained which took place in the vicinity of airports during take-off and landing phases of flight.*

**Keywords:** aircraft accident location; airport; environment; Gamma distribution; probability density function; runway; Weibull distribution.

**1 Introduction**

The countries as well as airlines and the International Society have done so much to prevent and reduce the damages for different environment objects due to using of civil aviation. However, today the existing hazards and risks are still high that create the adverse conditions in the vicinity of airports that have resulted the loss of ecological and material resources including death of people. The great majority of aircraft accidents occur during final approach and the initial climb phases of a flight that take place in or immediately adjacent to the airport. All this leads to additional technogenic impact and can be cause disastrous consequences for people in the vicinity of the airport.

**2 Analysis of the research and publications**

The current approaches for estimating the probability density of accident site locations in the vicinity of airports (the NLR method [1], the DOE method [2] and the ACRAM method [3]) were analysed. Each method includes certain disadvantages and has differences because of the different selection criteria used.

The statistical data of the accident locations involving different types of large and light aircraft were collected for improving the accident location model. The analysis of accident location data was performed for an 40-period (1973-2013) which happened within runways and in the vicinity of airports related to different phases of a flight:

- takeoff phase that includes the takeoff roll and the initial climb,
- landing phase that includes the landing approach and the landing roll.

Finding data with details of aircraft type and by accident site locations was a more difficult task. The data was compiled from a large amount of accident reports published by Federal Aviation Administration [4], the National Transportation Safety Board [5], UK Civil Aviation Authority [6], the Bureau d'Enquêtes et d'Analyses [7], UK Air Accidents Investigation Branch [8]. A vast amount of data on accident locations including information on aircraft type and airport has been collected from the ASN Aviation Safety database [9] and the Transportation Safety Board of Canada database [10]. The ADREP (the International Civil Aviation Organization) database [11] was the preferred source for data because allows the accurate location data to obtain. Data was gathered from reviews of national

organisation (the State Aviation Administration of Ukraine) [12] and international organisation (the Interstate Aviation Committee) [13] containing the complete record of the board’s investigation of each accident. So, a selection of a certain number of data includes the review different types of aircraft accidents (such as take-off overruns, undershoots, overshoots, lateral veer-offs, landing overruns and etc.) which associated with the various operational phases near-airport. A sufficient number of accident site location data has been found to perform the calculations and for improving the accident location model.

**3. Task**

The task is to establish trends of accident site locations in the vicinity of airports and determine the distances of accident locations relative to the runway ends by using data set from different sources. Moreover, the estimating the probability density of take-off and landing accident locations for large and light aircraft based on accurate location data.

**4. The distribution of take-off and landing accident locations in the vicinity of airports**

The function of the accident site location for large and light aircraft is estimated in the form of dependency probability densities [14]:

$$f(x, y) = f(x) \cdot f(y, x),$$

where  $f(x)$  is the function of the longitudinal location along the runway and its the extended centerline;

$f(y, x)$  is the function of the lateral distribution perpendicular to the runway centerline.

The accident location model for function of accident location has been investigated in form of two-dimensional probability density for longitudinal and lateral coordinates of accident sites during take-off and landing phases of flight. Results of processing of accident location data which happened within runways and areas around airports are represented in Figure 1.

Figure 1 shows the distribution of accident locations which occurred during various phases of flight near-airport in two dimensions:

the longitudinal distribution of locations from the runway end,

the lateral distribution of locations from the runway centerline.

The database resulting from this research contains a total of 2030 aircraft accidents which happened in one of the flight phases: overshoots (5 %), undershoots (26 %), take-off overruns (9 %), landing overruns (26 %), lateral veer-offs (14 %) and other uncategorized (20 %). The processing of statistical data and necessary calculations was performed by a computer system STATISTICA 8.0 [15].

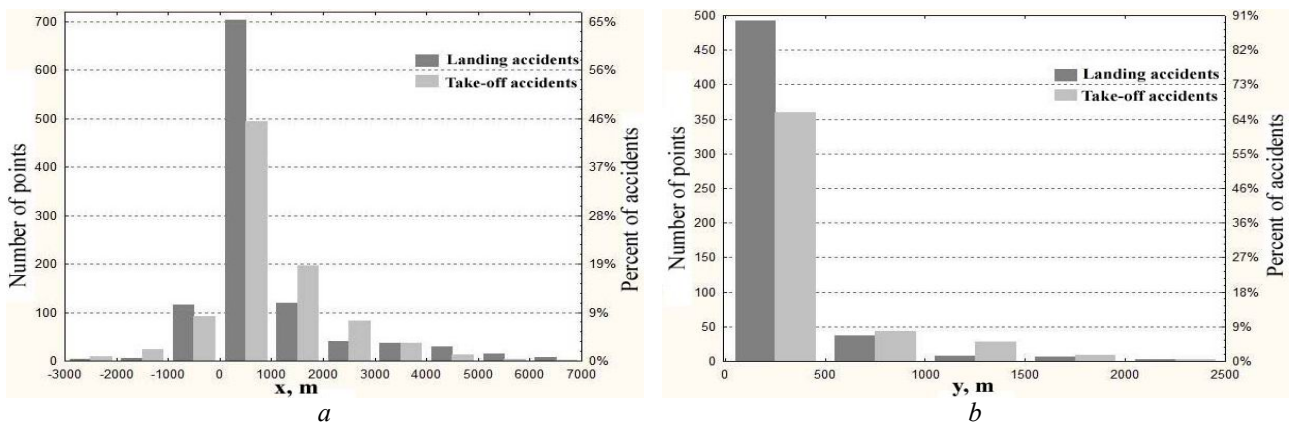


Fig. 1. The distribution of accident locations which happened during take-off and landing phases of flight: a – along runway centerline; b – perpendicular to the runway centerline

The statistical characteristics of the aircraft accident locations were obtained which occurred on the runways and in the vicinity of airports:

- landing accidents (the zero point on the axes is the landing end of the runway): arrival accident sites are concentrated within 1 km laterally from the runway centerline and extending outward to approximately 5-6 km of the runway end; the 65 %

of the points lie within an area 500 m wide and extending some 1 km from the runway end;

- take-off accidents (the zero point on the axes is the take-off end of the runway): departure accident sites are spread within 1.5 km laterally from the runway centerline and extending outward to approximately 4-5 km of the runway end; the 46 % of the points are concentrated within an area 500 m wide and extending some 1 km from the runway end.

The analysis of the statistical data related to the accident sites has allowed to define that the great majority of aircraft accidents take place on or immediately adjacent to the runway. It was found that landing accidents tend to be concentrated close to the runway centerline. Furthermore, take-off accidents tend to be concentrated near the runway end, but are not as located close to the runway centerline as are the landing accidents. Since the landing manoeuvres are performed at "approach zone", so the arrival accident sites tend to be most closely bunched around the landing threshold. A comparison of the two images in Figure 1 indicates that arrival accidents sites tend to be concentrated farther from the end of the runway than departure accident sites. The majority of departure accident sites in which the aircraft impacted with the ground are widely scattered relatively to the runway end.

The results of the investigation have showed that the probability of an aircraft accident during take-off and landing phases of flight in the proximity of the runway ends is higher than at larger distances from the runway end.

### 5 Estimating the probability density of take-off and landing accident locations for large aircraft

The distribution functions were used: the Weibull function, the Gamma function, the Log-normal function, the Normal function, the Poisson function, the Chi-Square function and Binominal function.

These functions were used for statistical estimation the probability density of accident site locations. The range in values of the longitudinal and lateral coordinates ( $x, y$ ) of accident site locations was broken into intervals with interval of 1 km and 0.5 km which corresponded to the number data set. Then the number of points were calculated which concentrated at  $i$  interval and the statistics Chi-Square was estimated to compare the probability density of accident site locations. The smaller value for the Chi-Square statistic means that more likely hypothesis is true. The larger Chi-Square value the greater the difference between observed and expected data. The Kolmogorov-Smirnov test and Chi-Square test were used to check the hypothesis.

The following Table I presents the analysis results of distribution functions to estimate the probability density for longitudinal coordinate of take-off and landing accident locations due to large aircraft.

As seen in Table 1, the Gamma distribution is the most correspond to describe the probability density of the accident site locations along the longitudinal coordinates in phase of take-off due to large aircraft (Table 1). The value of the Chi-Square statistics is 9.119 which supports this hypothesis. Since a  $p$ -value of 0.823 means that the observed data are consistent with hypothesis of the Gamma distribution (Fig. 2).

Table 1

**Probability density assessment of longitudinal coordinate of accident site locations due to large aircraft**

Assessment, distance	Distribution						
	Binominal	Poisson	Chi-Square	Log-normal	Gamma	Weibull	Normal
<b>Take-off</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.455	-	0.512	0.067	0.025	0.043	0.086
Interval 1 km	0.303	-	0.343	0.062	0.023	0.026	0.079
Chi-Square test							
Interval 0.5 km	254.723	-	296.534	26.128	9.119	22.276	870.034
Interval 1 km	136.933	-	163.991	20.042	3.645	14.045	745.688
<b>Landing</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.536	0.679	0.536	0.056	0.044	0.039	0.188
Interval 1 km	0.389	0.371	0.325	0.032	0.028	0.021	0.188
Chi-Square test							
Interval 0.5 km	231.229	546.734	226.242	43.698	23.809	28.048	441.981
Interval 1 km	357.631	321.64	252.17	17.237	13.668	12.223	271.623

Based on result of researches it was found that the Gamma distribution is more closely correspond to describe the probability density of the accident site locations along the longitudinal coordinates in phase

of landing due to large aircraft (Table 1) and the Weibull distribution is for lateral coordinates. The calculated value of  $\chi^2 = 1.017$  (Table 2) and the significance level of 0.961 support this hypothesis.

Table 2

**Probability density assessment of lateral coordinate of accident site locations due to large aircraft**

Assessment, distance	Distribution						
	Binominal	Poisson	Chi-Square	Log-normal	Gamma	Weibull	Normal
<b>Take-off</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.257	0.254	0.213	0.053	0.024	0.039	0.143
Interval 1 km	0.137	0.137	0.137	0.033	0.013	0.021	0.143
Chi-Square test							
Interval 0.5 km	994.746	600.141	60.935	29.935	8.157	3.719	620.328
Interval 1 km	6.973	6.973	6.973	16.297	0.638	0.776	282.7
<b>Landing</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.148	-	-	0.099	0.018	0.101	0.293
Interval 1 km	0.052	-	-	0.026	0.009	0.021	0.192
Chi-Square test							
Interval 0.5 km	8.461	-	-	20.884	11.067	1.017	324.348
Interval 1 km	1.042	-	-	8.862	5.403	0.438	189.967

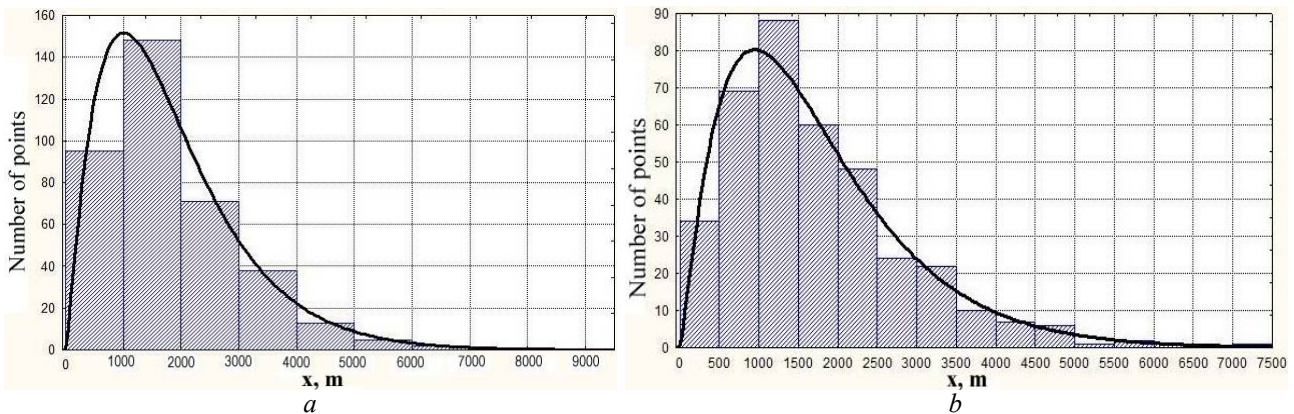


Fig. 2. The probability density of the Gamma distribution of longitudinal location for take-off accidents due to large aircraft: a – interval 1 km, b – interval 0,5 km

**6. Estimating the probability density of take-off and landing accident locations for light aircraft**

The analysis results of distribution functions to estimate the probability density for longitudinal coordinate of take-off and landing accident locations due to light aircraft are shown in Table 3.

From the obtained results it was deduced that the Gamma distribution is more closely correspond to describe the probability density of the accident site locations along the longitudinal coordinates due to light aircraft during take-off phase of flight.

Table 3

**Probability density assessment of longitudinal coordinate of accident site locations due to light aircraft**

Assessment, distance	Distribution						
	Binominal	Poisson	Chi-Square	Log-normal	Gamma	Weibull	Normal
1	2	3	4	5	6	7	8
<b>Take-off</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.543	0.298	0.247	0.038	0.036	0.034	0.101
Interval 1 km	0.264	0.2	0.2	0.038	0.009	0.027	0.101

Table 3 (continued)

1	2	3	4	5	6	7	8
Chi-Square test							
Interval 0.5 km	248.402	832.332	121.98	20.175	6.283	7.351	182.768
Interval 1 km	29.336	16.8	16.8	19.12	1.848	0.077	104.349
<b>Landing</b>							
Kolmogorov-Smirnov distance							
Interval 0.5 km	0.575	-	0.575	0.057	0.056	0.052	0.218
Interval 1 km	0.204	-	0.154	0.049	0.027	0.024	0.181
Chi-Square test							
Interval 0.5 km	322.472	-	302.783	80.849	41.78	29.082	875.661
Interval 1 km	30.738	-	11.766	52.799	13.814	17.619	434.371

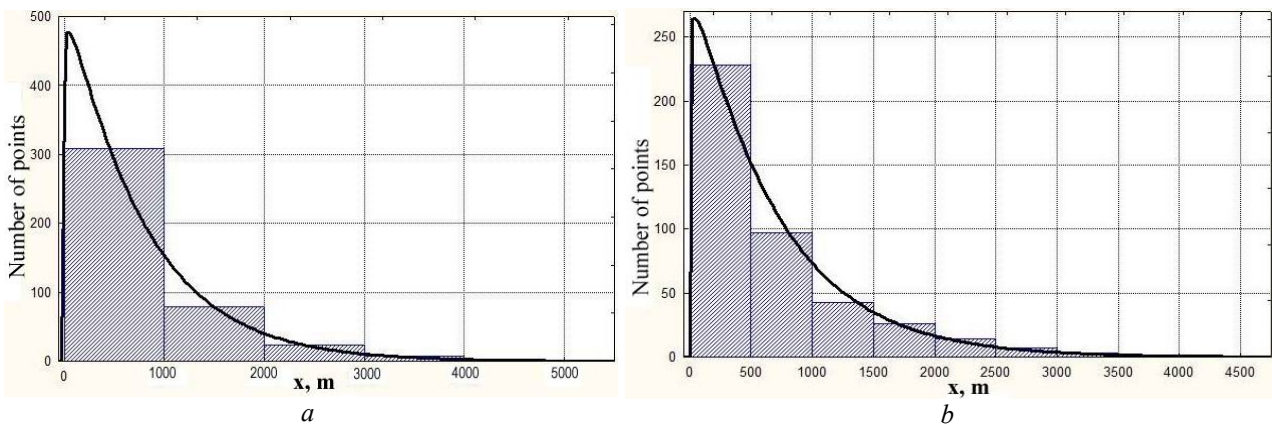


Fig. 3. The probability density of the Gamma distribution of longitudinal location for take-off accidents due to light aircraft: *a* – interval 1 km, *b* – interval 0,5 km

The chi-square calculated value is 6.283 which supports this hypothesis (Table 3). Since a *p*-value of 0.392 indicates that the observed data have not rejected the hypothesis of the Gamma distribution (Figure 3).

It was defined that the Gamma distribution and Weibull distribution are the most plausibly corresponded to describe the probability density of the accident site locations along the longitudinal coordinates due to light aircraft during landing phase of flight (Table 3).

The analysis results of distribution functions for probability density assessment of lateral coordinate for take-off and landing accident locations due to light aircraft are approximately the same.

**6. Results**

The longitudinal and lateral distribution of take-off and landing accident locations due to large and light aircraft are modelled using the Weibull function (Figure 4 and Figure 5).

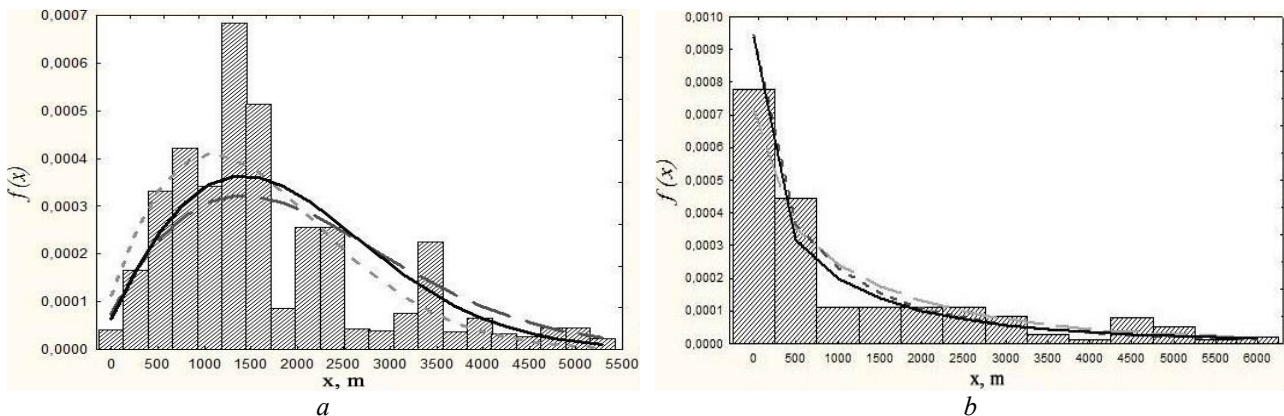


Fig. 4. Weibull distribution to describe the longitudinal coordinates of accident locations due to large aircraft: *a* – take-off, *b* – landing

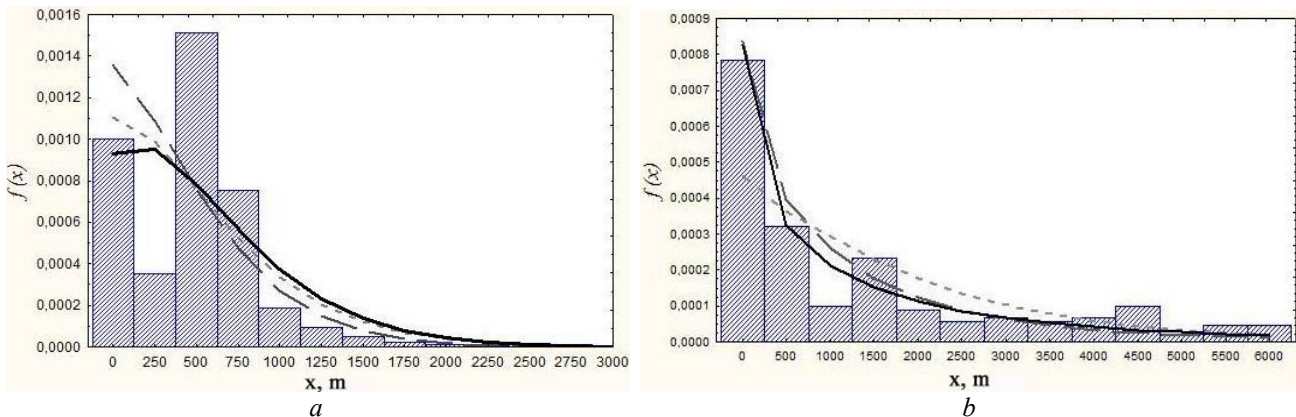


Fig. 5. Weibull distribution to describe the longitudinal coordinates of accident locations due to light aircraft: a – take-off, b – landing

The longitudinal distribution of accident locations due to large and light aircraft along the runway and its extended centerline are modelled using the Gamma function.

As a result, it was concluded that the Gamma distribution (1) and Weibull distribution (2) are the most plausibly corresponded to describe the probability density of the accident site locations along the longitudinal and lateral coordinates of aircraft accidents due to large and light aircraft during take-off and landing phases of flight:

$$f(x, \alpha, \beta) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)\right] \quad 1$$

$$f(x, \alpha, \beta) = \frac{\alpha}{\beta^\alpha} x^{\alpha-1} \exp\left[-\left(\frac{x}{\beta}\right)^\alpha\right] \quad 2$$

### 7. Conclusions

The accident location data involving different types of large and light aircraft has been collected. The analysis of worldwide aircraft accidents related to take-off and landing phases of flight has been performed for the period 1973 to 2013 which occurred within runways and in the vicinity of airports. The processing of data points has been performed by the computer system STATISTICA 8.0 that include the following accident types: the lateral veer-offs, the overshoots, the undershoots, the take-off overruns and landing overruns. Based on result of researches, it is found that a vast amount of aircraft accident sites tend to be concentrated close to the runway ends and relatively near the extended centerline.

From an analysis of the obtained results it was determine that accident probability density decreases

with increasing distance from the runway. The 60 % of the arrival accidents points are plotted within a narrow strip, approximately 500 m wide and extending some 1 km from the runway end. Also, almost 45 % of the departure accident points lie within an area 500 m wide and extending some 1 km from the runway end.

The analysis of distribution functions has been performed to estimate the probability density of accident locations due to large and light aircraft during take-off and landing phases of flight. The results of investigation have allowed to deduce that the Gamma distribution and Weibull distribution are the most plausibly corresponded to describe the probability density of the accident site locations along the longitudinal and lateral coordinates of aircraft accidents due to large and light aircraft during take-off and landing phases of flight.

### References

- [1] Pikaar A., Weijts J., M. de Jong *An enhanced method for the calculation of third party risk around large airports with application to Schiphol*. Netherlands, NLR, 2000, 104 p.
- [2] *Accident analysis for aircraft crash into hazardous facilities* (Standard DOE-STD-3014-2006). U.S. Department of Energy, 215 p.
- [3] Kimura C.Y., Glaser R.E., Mensing R.W., Haley T.A., Barto A.B. *Data development technical support document for the aircraft crash risk analysis methodology* (Standard UCRL-ID-124837). U.S. Department of Energy, 1996, 268 p.
- [4] *Accident and Incident Dates*. Federal Aviation Administration. Available at: [http://www.faa.gov/data\\_research/accident\\_incident](http://www.faa.gov/data_research/accident_incident).
- [5] *Aviation Accident Reports*. U. S. National Transportation Safety Board. Available at:

[http://www.nts.gov/investigations/reports\\_aviation.html](http://www.nts.gov/investigations/reports_aviation.html).

[6] *Aviation Safety Reviews*. UK Civil Aviation Authority. Available at: <http://www.caa.co.uk/application.aspx>.

[7] *Annual reports*. Bureau d'Enquêtes et d'Analyses. Available at: <http://www.bea.aero/en/rapport.annuel/index.php>.

[8] *Annual Safety Reports*. UK Air Accidents Investigation Branch. Available at: [http://www.aaiib.gov.uk/publications/annual\\_safety\\_reports.cfm](http://www.aaiib.gov.uk/publications/annual_safety_reports.cfm).

[9] *ASN Aviation Statistics*. Available at: <http://aviation-safety.net/statistics>.

[10] *Aviation Statistics*. Transportation Safety Board of Canada. Available at: <http://www.tsb.gc.ca/eng/stats/aviation/index.asp>.

[11] *ADREP ICAO*. Request 198/80 (USSR). Available at: <http://www.icao.int>.

[12] The State Aviation Administration of Ukraine,

Annual Safety Reports until 2013 “Analysis of aviation safety level and identification of potential accidents associated with the Ukrainian-registered aircraft”, available at: <http://www.avia.gov.ua/documents/Bezpeka-aviatsii/Bezpeka-polotiv/>.

[13] The Interstate Aviation Committee, Aviation Accident Reports until 2013 “Civil aviation flight safety level of the contracting states of the Agreement on civil aviation and use of airspace”, available at: <http://mak-iac.org/en/rassledovaniya/bezopasnost-poletov/>.

[14] Davies P.A., Quinn D.J. *Public Safety Zones: Cork, Dublin and Shannon Airports: Annex A*. Ireland, 2003, pp. A24–A28.

[15] *Statistical analysis system of data*. StatSoft STATISTICA 8.0. (CD-ROM).

Received 6 June 2016.

#### **І.І. Государська**

#### **Результати аналізу функцій розподілу для оцінки густини ймовірності розміщення місць авіаційних подій в околиці аеропорту**

Національний авіаційний університет, просп. Космонавта Комарова, 1, Київ, Україна, 03680.

E-mail: [gil\\_@mail.ru](mailto:gil_@mail.ru)

Стаття містить результати обробки та статичного аналізу даних про місця розміщення авіаційних подій важких та легких повітряних суден, які сталися протягом 1973–2013 рр. Представлено розподіл розміщення місць авіаційних подій під час виконання різних етапів польоту поблизу аеропорту. Показано результати аналізу функцій розподілу для оцінки густини ймовірності розміщення місць авіаційних подій під час зльоту і посадки. Розподіли Вейбула і Гама визначені як такі, що з найбільшою достовірністю відповідають опису густини ймовірності розміщення місць авіаційних подій уздовж поздовжніх і поперечних координат відносно вісі злітно-посадкової смуги. Отримані статистичні характеристики місць, де трапляються авіаційні події під час зльоту і посадки повітряних суден в околиці аеропортів.

**Ключові слова:** аеропорт; довкілля; злітно-посадкова смуга; розміщення місця авіаційної події; розподіл Вейбула; розподіл Гама; функція густини ймовірності.

#### **И.И. Государская**

#### **Результаты анализа функций распределения для оценки плотности вероятности размещения мест авиационных происшествий в окрестности аэропорта**

Национальный авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03680.

E-mail: [gil\\_@mail.ru](mailto:gil_@mail.ru)

Статья содержит результаты обработки и статического анализа данных про месторасположения авиационных происшествий тяжелых и легких воздушных судов, которые произошли на протяжении 1973–2013 гг. Представлено распределение размещения мест авиационных происшествий при выполнении различных этапов полета вблизи аэропорта. Показаны результаты анализа функций распределения для оценки плотности вероятности размещения мест авиационных происшествий во время взлета и посадки. Определено, что распределения Вейбулла и Гамма с наибольшей достоверностью соответствуют описанию плотности вероятности размещения мест авиационных происшествий вдоль продольных и поперечных координат относительно оси взлетно-посадочной полосы. Получены статистические характеристики мест, где совершаются авиационные происшествия во время взлета и посадки воздушных судов в окрестности аэропортов.

**Ключевые слова:** размещение места авиационного происшествия; аэропорт; взлетно-посадочная полоса; окружающая среда; распределение Вейбулла; распределение Гамма; функция плотности вероятности.

**Gosudarska Inna** (1979). Candidate of Engineering.

Department of Safety of Human Activities, National Aviation University, Kyiv, Ukraine.

Education: National Pedagogical Dragomanov University, Kyiv, Ukraine (2002).

National Aviation University, Kyiv, Ukraine (2004, 2008).

Research area: methods of third party risk assessments in the vicinity of the airports, ensuring ecological safety of airports.

Publications: 27.

E-mail: gil\_@mail.ru