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MATHEMATICAL MODEL OF ATTITUDE AND HEADING REFERENCE SYSTEM WITH BIAXIAL HORIZONTAL PLATFORM

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Abstract

Purpose: *Operation of attitude and heading reference systems in conditions of autonomy and high accuracy requires usage of gimballed platforms. The goal of the paper is detailed research of such systems kinematics and control moments. As result the full mathematical model of the precision attitude and heading reference system with the biaxial horizontal platform was derived.* ***Methods:*** *Obtaining of the mathematical model is based on the theory of gyros in general and corrected gyro compasses and theory of dynamically tuned gyros in particular. The basic laws of theoretical mechanics including concepts of Euler angles and directional cosines were taken into consideration.* ***Results:*** *The full mathematical model of the attitude and heading reference system is developed. The mathematical models of the vertical gyro and directional gyro as components of the researched system are given. The simulation results based on the developed models are presented.* ***Conclusions:*** *The mathematical model of the gimballed attitude and heading reference system including the vertical gyro and directional gyro is derived. The detailed expressions for control (correction) moments are obtained. The full analysis of the researched system kinematics was carried out. The obtained results can also be useful for design of inertial navigation systems of the wide class.*

Keywords: attitude and heading reference system; directional cosines; directional gyro; gimballed platforms; precision navigation systems; vertical gyro.

1. Introduction and Problem Statement

Nowadays the gimballed navigation systems are used when it is necessary to satisfy high accuracy in conditions of autonomous operation. The requirements to the high functional reliability and the ability to function in conditions of external disturbances are given to such systems too [1]. The gimballed systems can be components of guidance, navigation and control systems [2].

The researched attitude and heading reference system (AHRS) includes biaxial horizontal platform, which is stabilized by the vertical gyro signals. The principal axis of the gyro device is aligned by the direction of the local vertical based on accelerometer signals. The system uses the integral correction. In fact the biaxial horizontal platform with gyro devices represents the inertial vertical gyro.

The gimballed AHRS with biaxial horizontal platform has reduced dimensions [3]. Its possibility to rotate around the third axis (in the azimuth plane) is provided by means of a rotator, on which the directional gyro is mounted.

The rotator is stabilized relative to the given plane and can turn on the given angles providing alignment of the gyro. The directional gyro uses as an indicator of direction similar to the azimuth gyro. It can be used also as the gyro compass if stabilization is implemented relative to the meridian plane. The vertical gyro is mounted on the rotator too. Such construction provides calibration of the gyro device and determination of its corrections.

The researched AHRS uses dynamically tuned gyros (DTG) and accelerometers [4]. This system has some features. The first feature is division of control functions. In this case control by the position of the DTG principal axis is implemented by accelerometer’s signals. Stabilizing motors provide coincidence of the axis normal to the stabilized platform with the direction of DTG rotor. Division of functions provides relatively small moments of DTG torque sensors. The second feature of the researched system is usage of the biaxial platform.

In contrast to the traditional triaxial platform such construction provides azimuth motion of the platform together with the vehicle and constant orientation of the outer gimbal to the North. Therefore the system becomes sensitive to disturbances caused by changes of the heading and elliptic shape of the Earth. To compensate such disturbances it is necessary to use information about the linear speed, heading and its changes.

Advantages of the researched system are high accuracy and reduced dimensions in comparison with the triaxial platform.

Development of the model of such system requires usage of the trajectory reference frame. This significantly complicates control of the system.

2. Review of Last Publications

Features of the researched system are presented in [5]. Analysis of the kinematical schemes of gimballed corrected gyro compass and gimballed navigation systems with biaxial and triaxial gyrostabilized platforms is given in [4]. This paper keeps the basic idea proposed in [5] and used in [6]. The idea is to neglect the servo-systems errors and divide the system into the vertical and directional gyros. **The goal of the paper** is to consider the system kinematics and to derive expressions for control moments.

3. System kinematics

To describe the system kinematics it is necessary to introduce the following systems of coordinates:

1) the trajectory system of coordinates O′ζηξ (O′η is directed along the vehicle speed, O′ζ is perpendicular to the horizon plane, O′ξ lies in the horizon plane);

2) the body-axis system of coordinates $Ox_0y_0z_0$ (the axis Oy_0 is directed along the longitudinal axis of the vehicle; the axis Oz_0 is perpendicular to the horizon plane; the axis Ox_0 lies in the horizon plane);

3) the platform-axis system of coordinates $Ox_ny_nz_n$ (the axis Oy_n is directed along the external gimbal; the axis Oy_n is perpendicular to the horizon plane, the axis Ox_n is perpendicular to the axis Oy_0 and lies in the horizon plane);

4) the system of coordinates $Ox_ry_rz_r$, which is connected with Resal axes of DTG carrying out functions of the vertical gyro.

Mutual angular position of introduced systems of coordinates is given in Figures 1–5.

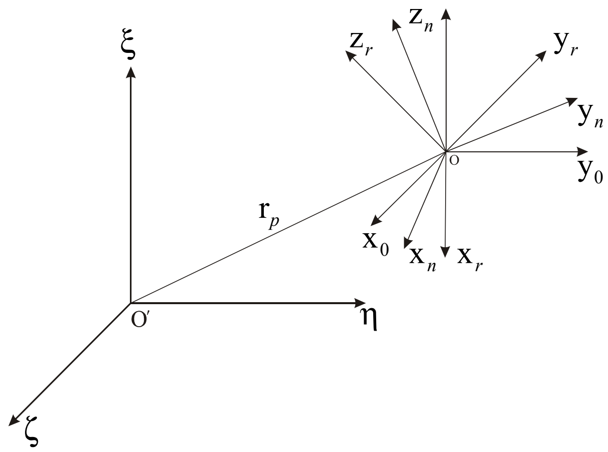


Fig. 1. Mutual position of introduced systems of coordinates: r_p is the radius-vector defining position of coordinate systems origins

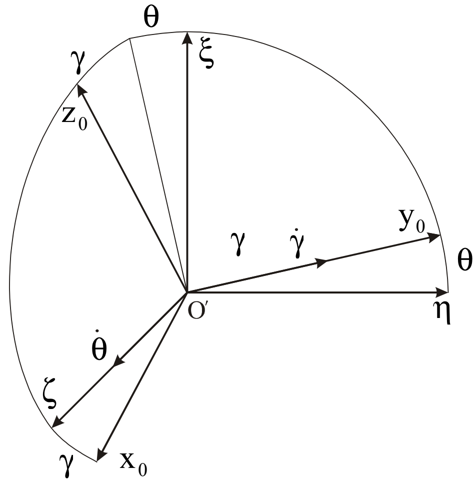


Fig. 2. Mutual location of the trajectory and body-axis systems of coordinates: θ, γ are angles of pitch and roll

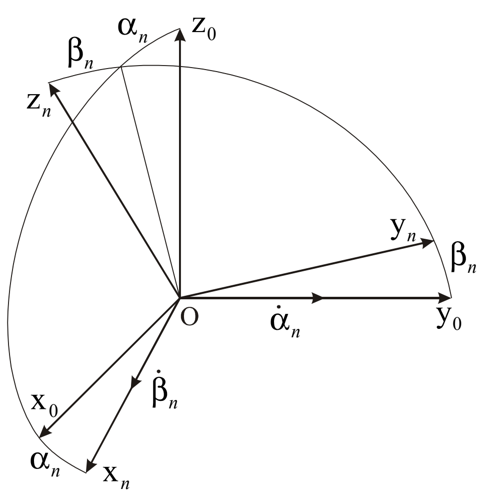


Fig. 3. Mutual location of the body-axis and platform-axis systems of coordinates (α_n, β_n are angles, which determine turn of a platform relative to the body-axis system)

z_0

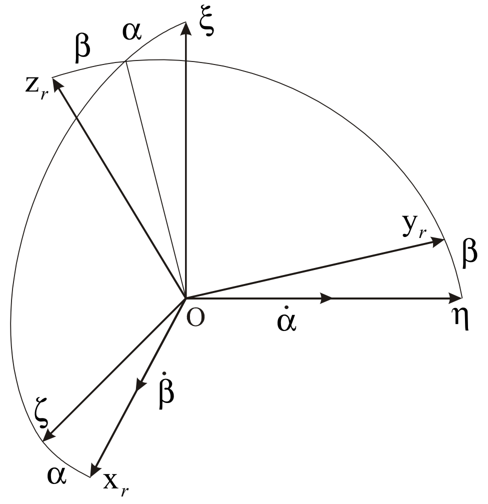


Fig. 4. Resal axes location relative to the trajectory system of coordinates (α, β determine the location of Resal axes relative to the horizon plane)

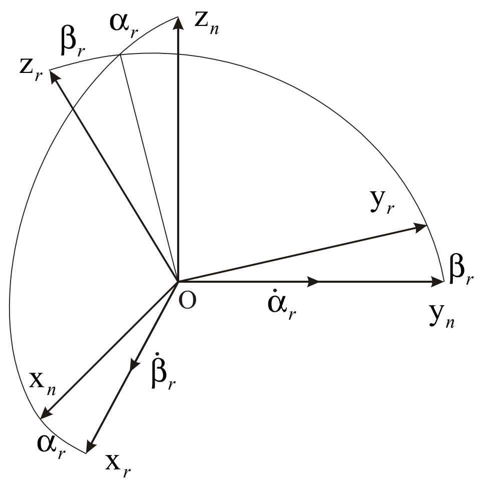


Fig. 5. Resal axes location relative to the platform (α_r, β_r determine the location of Resal axes relative to the platform)

In accordance with Figures 1–5 relations between introduced systems of coordinates can be described in the following way [5, 7]

$$\begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix}; \quad \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix};$$
$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \mathbf{A}_3 \begin{bmatrix} \zeta \\ \eta \\ \xi \end{bmatrix}; \quad \begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \mathbf{A}_4 \begin{bmatrix} x_n \\ y_n \\ z_n \end{bmatrix}; \tag{1}$$

$$\mathbf{A}_1 = \begin{bmatrix} \cos\gamma & \sin\theta\cdot\sin\gamma & -\cos\theta\cdot\sin\gamma \\ 0 & \cos\theta & \sin\theta \\ \sin\gamma & -\sin\theta\cdot\cos\gamma & \cos\theta\cdot\cos\gamma \end{bmatrix}; \tag{2}$$

$$\mathbf{A}_2 = \begin{bmatrix} \cos\alpha_n & 0 & -\sin\alpha_n \\ \sin\alpha_n\cdot\sin\beta_n & \cos\beta_n & \cos\alpha_n\cdot\sin\beta_n \\ \sin\alpha_n\cdot\cos\beta_n & -\sin\beta_n & \cos\alpha_n\cdot\cos\beta_n \end{bmatrix}; \tag{3}$$

$$\mathbf{A}_3 = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ \sin\alpha\cdot\sin\beta & \cos\beta & \cos\alpha\cdot\sin\beta \\ \sin\alpha\cdot\cos\beta & -\sin\beta & \cos\alpha\cdot\cos\beta \end{bmatrix}; \tag{4}$$

$$\mathbf{A}_4 = \begin{bmatrix} \cos\alpha_r & 0 & -\sin\alpha_r \\ \sin\alpha_r\cdot\sin\beta_r & \cos\beta_r & \cos\alpha_r\cdot\sin\beta_r \\ \sin\alpha_r\cdot\cos\beta_r & -\sin\beta_r & \cos\alpha_r\cdot\cos\beta_r \end{bmatrix}. \tag{5}$$

It should be noted that there is dependence

$$\mathbf{A}_3 = \mathbf{A}_4\cdot\mathbf{A}_2\cdot\mathbf{A}_1. \tag{6}$$

The expression (6) allows obtaining relations for determination of angles α_n, β_n.

4. Vertical Gyro Model

For small turn angles it is possible to believe that the gyrostabilized platform motion coincides with motion of the vertical gyro's Resal axes. Accuracy of such supposition is defined by stabilization errors α_r, β_r. Angular motion of DTG, which carries out functions of the vertical gyro, can be described by the differential equations [8]

$$J\ddot{\alpha}_r + d\dot{\alpha}_r - H\dot{\beta}_r - \frac{H}{T}\beta_r + c\alpha_r = J\dot{\omega}_y + H_1\omega_x + $$
$$+ M_{1y} + M_{2y} + M_{3y} + M_{4y} + M_{dist\ y}$$
$$J\ddot{\beta}_r + d\dot{\beta}_r + H\dot{\alpha}_r + \frac{H}{T}\alpha_r + c\beta_r = J\dot{\omega}_x - H_1\omega_y + $$
$$+ M_{1x} + M_{2x} + M_{3x} + M_{4x} + M_{dist\ x} \tag{7}$$

where H is the kinetic moment; c is the residual rigidity of gimbals; d is the damping coefficient; T is the gyro time constant; $H_1 = H(1-S)$; S=10^{-3}; J is a sum of the equatorial moments of the rotor and gyro gimbals; ω_x, ω_y are projections of the platform angular rates; $\dot{\omega}_x, \dot{\omega}_y$ are projections of platform angular accelerations; $M_{iy} = 1,\ldots,4$, $M_{ix} = 1,\ldots,4$ and $M_{dist\ y}$, $M_{dist\ x}$ are control and disturbance moments acting along axes y, x respectively.

To determine projections of platform angular rates it is necessary to project angular rates of the trajectory reference frame onto platform-axis reference frame. For estimation of the vertical gyro errors it is convenient to represent projections ω_x, ω_y as functions of the angles α, β.

The angles α_r, β_r represent errors of platform stabilization and define position of the vertical gyro axes relative to the horizon plane. In fact they represent errors of vertical line determination. Respectively transformation for small angles α_r, β_r, and α, β using matrices (4) (5) becomes

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{A}_4 \cdot \mathbf{A}_3 \begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix}; \quad (8)$$

here $A_4 A_3 \approx \begin{bmatrix} 1 & 0 & -\alpha+\alpha_r \\ 0 & 1 & \beta\text{-}\beta_r \\ \alpha\text{-}\alpha_r & -\beta+\beta_r & 1 \end{bmatrix}$.

Using the expression (8) it is possible to write the platform angular rates in the following form

$$\begin{aligned} \omega_x &= \dot\beta - \dot\beta_r + \omega_\xi - \omega_\zeta(\alpha - \alpha_r); \\ \omega_y &= \dot\alpha - \dot\alpha_r + \omega_\eta + \omega_\zeta(\beta - \beta_r); \\ \omega_z &= (\dot\alpha - \dot\alpha_r)\mathrm{tg}\theta + \frac{\alpha - \alpha_r}{\cos^2\theta}\dot\theta + \omega_\zeta. \end{aligned} \quad (9)$$

Angular rates in the equations (9) are determined with accuracy to the second order of smallness. If the expression (9) is substituted in the equations (7), it is possible to obtain

$$\begin{aligned} &-H_1\dot\beta + H_1\omega_\zeta\alpha = -I\ddot\alpha_r + H\dot\beta_r - c\alpha_r - d\dot\alpha_r + \frac{H}{T}\beta_r + \\ &+ H_1(-\dot\beta_r + \omega_\xi + \omega_\zeta\alpha_r) + I\dot\omega_y + \\ &+ \sum_{i=1}^{4} M_{iy} + M_{\mathrm{d}ist\,y}; \\ &H_1\dot\alpha + H_1\omega_\zeta\beta = -I\ddot\beta_r - H\dot\alpha_r - c\beta_r - d\dot\beta_r - \frac{H}{T}\alpha_r - \\ &- H_1(-\dot\alpha_r + \omega_\eta - \omega_\zeta\beta_r) + I\dot\omega_x + \\ &+ \sum_{i=1}^{4} M_{ix} + M_{\mathrm{d}ist\,x}. \end{aligned} \quad (10)$$

If stabilization errors are believed to be constant, the relations (10) represent the vertical gyro model. Stabilization errors can be determined based on information about the gyro devices drifts, which can be obtained during operation.

Variables ω_ξ, ω_η, ω_ς in the equations (10) are projections of the angular rates of the trajectory reference frame. In accordance with [9, 10] they can be represented in the following form

$$\begin{aligned} \omega_\xi &= -\frac{V_\eta \sin K + V_\xi \cos K}{R_1}\sin K - \\ &- \frac{V_\eta \cos K - V_\xi \sin K}{R_M}\cos K - \Omega\cos\varphi\sin K; \\ \omega_\eta &= \frac{V_\eta \sin K + V_\xi \cos K}{R_1}\cos K - \\ &- \frac{V_\eta \cos K - V_\xi \sin K}{R_M}\sin K + \\ &+ \Omega\cos\varphi\cos K; \\ \omega_\zeta &= \frac{V_\eta \sin K + V_\xi \cos K}{R_1}\mathrm{tg}\varphi - \dot K + \Omega\sin\varphi, \end{aligned} \quad (11)$$

where V_ζ, V_η, V_ξ are lateral, longitudinal and vertical projections of the vehicle speed; K is the heading; R_1 is the main radius of curvature of the earth ellipsoid in the plane perpendicular to the meridian; R_M is the main radius of the earth curvature in the meridian plane; Ω is the angular rate of the diurnal rotation; $\dot K$ is rate of heading change.

To determine apparent accelerations projections onto accelerometer sensitive axes it is convenient to neglect the instrumental errors of accelerometer alignment. And accelerometer sensitivity axes are believed to coincide with platform axes. In accordance with [9] the expression for apparent acceleration looks like

$$\mathbf{W}_{cw} = \dot{\mathbf{V}}_{cw} + \boldsymbol{\omega}_{tr} \times \mathbf{V}_{cw} + \boldsymbol{\Omega} \times \mathbf{V}_{cw} - \mathbf{g}, \quad (12)$$

where $\boldsymbol{\omega}_{tr}$ is the vector of the absolute angular rate of trajectory reference frame; **g** is the gravity acceleration; $\mathbf{V}_{cw}$ is the vector of the mass centre rate relative to the Erath; **Ω** is the vector of the diurnal rotation; φ is the geographical latitude. Projections of the vector equation (12) can be represented in the following form

$$\begin{bmatrix} W_\xi \\ W_\eta \\ W_\zeta \end{bmatrix} = \begin{bmatrix} \dot V_\xi \\ \dot V_\eta \\ \dot V_\zeta \end{bmatrix} + \begin{bmatrix} V_\zeta\omega_\eta - V_\eta\omega_\zeta \\ V_\xi\omega_\zeta - V_\zeta\omega_\xi \\ V_\eta\omega_\xi - V_\xi\omega_\eta \end{bmatrix} + \begin{bmatrix} V_\zeta\Omega\cos\varphi\cos K - V_\eta\Omega\sin\varphi \\ V_\xi\Omega\sin\varphi - V_\zeta\Omega\cos\varphi\sin K \\ V_\eta\Omega\cos\varphi\sin K - V_\xi\Omega\cos\varphi\cos K \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad (13)$$

The acceleration of the point, at which the platform is mounted at the vehicle, can be written as [5, 9]

$$\mathbf{W}_0 = \mathbf{W}_{cw} + \boldsymbol{\omega}_{abs} \times \mathbf{r}_{\mathrm{p}} + \boldsymbol{\omega}_{abs} \times (\boldsymbol{\omega}_{abs} \times \mathbf{r}_{\mathrm{p}}); \quad (14)$$

where $\boldsymbol{\omega}_{abs}$ is a vector of the absolute angular rate of the vehicle; $\mathbf{r}_p$ is the radius-vector of the point of platform setting relative to the centre of the vehicle mass.

Projections of the vector equation (14) onto the body-axis reference frame $Ox_0y_0z_0$ look like

$$\begin{bmatrix} W_{x_0} \\ W_{y_0} \\ W_{z_0} \end{bmatrix} = \mathbf{A}_1 \begin{bmatrix} W_\xi \\ W_\eta \\ W_\zeta \end{bmatrix} + \begin{bmatrix} \dot{\omega}_{x_0} z_p - \dot{\omega}_{z_0} y_p \\ \dot{\omega}_{z_0} x_p - \dot{\omega}_{x_0} z_p \\ \dot{\omega}_{x_0} y_p - \dot{\omega}_{y_0} x_p \end{bmatrix} + \\ + \begin{bmatrix} \omega_{y_0}(\omega_{x_0} y_p - \omega_{y_0} x_p) - \omega_{z_0}(\omega_{z_0} x_p - \omega_{x_0} z_p) \\ \omega_{z_0}(\omega_{y_0} z_p - \omega_{z_0} y_p) - \omega_{x_0}(\omega_{x_0} y_p - \omega_{y_0} x_p) \\ \omega_{x_0}(\omega_{z_0} x_p - \omega_{x_0} z_p) - \omega_{y_0}(\omega_{y_0} z_p - \omega_{z_0} y_p) \end{bmatrix}; \quad (15)$$

where $W_{x_0}, W_{y_0}, W_{z_0}$ are projections of the apparent acceleration onto the body-axis reference frame; $\omega_{x_0}, \omega_{y_0}, \omega_{z_0}$ are projections of the vehicle absolute angular rate; x_p, y_p, z_p are coordinates of point, at which the platform is set at the vehicle; $\mathbf{A}_1$ is the matrix of directional cosines between the trajectory and body-axis reference frames. This matrix is determined by the expression (2).

Projections of the absolute angular rate of the vehicle can be represented in the following form

$$\omega_{x_0} = (\omega_\xi + \dot{\theta})\cos\gamma + \omega_\eta \sin\gamma\sin\theta - \omega_\zeta \sin\gamma\cos\theta;$$
$$\omega_{y0} = \dot{\gamma} + \omega_\eta \cos\theta + \omega_\zeta \sin\theta;$$
$$\omega_{z_0} = (\omega_\xi + \dot{\theta})\sin\gamma - \omega_\eta \cos\gamma\sin\theta + \omega_\zeta \cos\gamma\cos\theta. \quad (16)$$

Based on the equations (15) it is possible to obtain the vector equation

$$\begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} W_{x_0} \\ W_{y_0} \\ W_{z_0} \end{bmatrix}, \quad (17)$$

where W_x, W_y, W_z are projections of the apparent acceleration onto the body-axis reference frame $Oxyz$; $\mathbf{A}_2$ is the matrix of the directional cosines between the body-axis and platform-axis reference frames. This matrix can be determined by the expression (3).

It should be noted that projections of the trajectory reference frame taking into consideration (1) are determined by relations

$$W_\xi = \dot{V}_\xi + V_\zeta\omega_\eta - V_\eta\omega_\zeta + V_\zeta\Omega\cos\varphi\cos K - V_\eta\Omega\sin\varphi;$$
$$W_\eta = \dot{V}_\eta + V_\xi\omega_\zeta - V_\zeta\omega_\xi + V_\xi\Omega\sin\varphi + V_\zeta\Omega\cos\varphi\sin K; \quad (18)$$
$$W_\zeta = \dot{V}_\zeta + V_\eta\omega_\xi - V_\xi\omega_\eta - V_\eta\Omega\cos\varphi\sin K - \\ - V_\xi\Omega\cos\varphi\cos K - g.$$

The equations (10) supplemented by the expressions (11), (15), (17), (18) represent the model of the inertial vertical. Such model allows researching of the AHRS using the biaxial gyrostabilized platform and the trajectory reference frame.

5. Control of Vertical Gyro

The important component of the vertical gyro model is description of control moments. These moments are formed in computing device and are applied to torque sensors of the vertical gyro. Such approach provides high accuracy of attitude and heading determination.

The researched system is created by the scheme of the nondisturbed inertial vertical with the integral correction. The respective control moments can be described by the expressions

$$M_{1y} = k_i \int_0^t w_y, \quad M_{1x} = k_i \int_0^t w_x, \quad (19)$$

where $k_i = \frac{H_1}{R_M}$ is the coefficient of the integral correction; w_x, w_y are projections of apparent accelerations of the point, at which the platform is set. To organize the mode of the inertial vertical it is necessary to give signals to the integrators taking into consideration the corrections on translational and Coriolis accelerations caused by the Earth rotation and vehicle motion; noncoincidence of centre of mass of the vehicle and the point, at which the platform is set; and also vertical acceleration of the vehicle. To simplify these expressions it is necessary to believe that the system is set at the centre of the vehicle. Then expressions for the integral correction become

$$M_{1y} = k_i \int_0^t W_y - \Delta W_y, \quad M_{1x} = k_i \int_0^t W_x - \Delta W_x, \quad (20)$$

here W_x, W_y are readings of accelerometers; ΔW_x, ΔW_y are corrections.

Moments for compensation of trajectory reference frame angular motion can be determined in the following way [9, 10]

$$M_{2x} = H_1 \frac{V_\eta \sin K + V_\xi \cos K}{R_1} \cos K -$$

$$-\frac{V_\eta \cos K - V_\xi \sin K}{R_M}\sin K + \Omega\cos\varphi\cos K;$$

$$M_{2y} = H_1 + \frac{V_\eta \sin K + V_\xi \cos K}{R_1}\sin K + \frac{V_\eta \cos K - V_\xi \sin K}{R_M}\cos K + \Omega\cos\varphi\sin K \quad (21)$$

Control moments, which provide damping of the platform based on the external information v_{Ex}, v_{Ey}, can be determined by the expressions

$$M_{3x} = k_{3x}\int_0^t (v_{Ey} - v_y)/R_M dt;$$

$$M_{3y} = k_{3y}\int_0^t (v_{Ex} - v_x)/R_1 dt, \quad (22)$$

where k_{3x}, k_{3y} are transfer constants, v_{Ey}, v_{Ex} are linear speeds determined by the external aids; v_y, v_x are the vehicle linear speeds calculated based on accelerometers readings.

Control moments, which take into consideration gyro drifts, can be determined in the following way

$$M_{4y} = -I\ddot{\alpha}_r + H\dot{\beta}_r - c\alpha_r - d\dot{\alpha}_r + \frac{H}{T}\beta_r + H_1(-\dot{\beta}_r + \omega_\zeta\alpha_r);$$

$$M_{4x} = -I\ddot{\beta}_r - H\dot{\alpha}_r - c\beta_r - d\dot{\beta}_r - \frac{H}{T}\alpha_r - H_1(-\dot{\alpha}_r - \omega_\zeta\beta_r). \quad (23)$$

The expressions (19)–(23) represent control moments of the vertical gyro. The gyro drifts can be determined by results of tests. As a rule, the moments due to cross influence of the angular accelerations can not be taken into consideration in the researched systems. But they can be determined in the computing unit.

6. Directional Gyro Model

To create the mathematical description of the directional gyro it is necessary to use the following reference frames: platform-axis reference frame Oxyz; reference frame $Ox_1y_1z_1$ connected with the rotor of the directional gyro; reference frame $Ox_\kappa y_\kappa z_\kappa$ connected with Resal axes of DTG, which carries out functions of the directional gyro.

The mutual location of the platform axes and the rotator of the directional gyro is shown in Fig. 6.

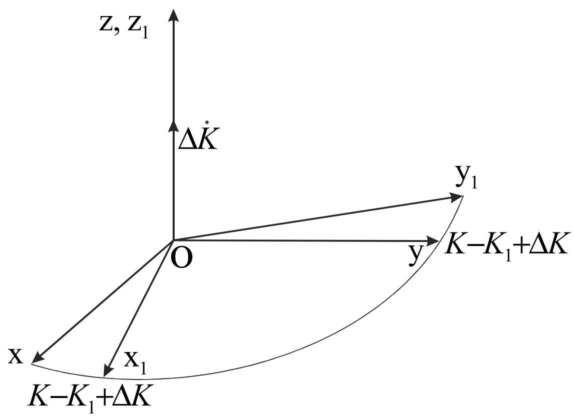


Fig. 6. Mutual location of the platform and the directional gyro

The following notations are used in Fig. 6: K is the heading; K_1 is the given orientation of the directional gyro principal axis; ΔK is error of heading determination. It should be noted that K_1 defines the angular orientation of the directional gyro relative to the meridian plane. Mutual location of the rotator axes and the Resal axes of the directional gyro is given in Fig. 7.

Angles α_K, β_K shown in Fig. 7 characterize an error of directional gyro rotor stabilization and an angle of deviation of the directional gyro principal axis from the simulated horizon plane.

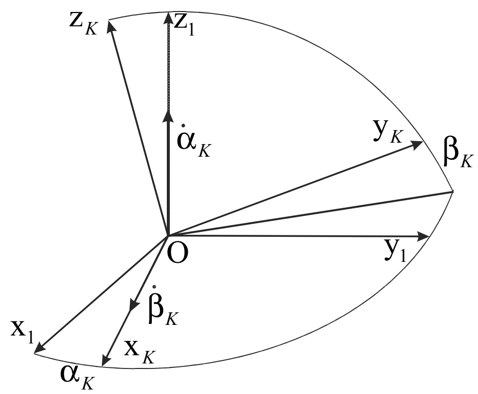


Fig. 7. Mutual location of the rotator axes and Resal axes of the directional gyro

Location of the directional gyro Resal axes can be also defined relative to the reference frame $O\zeta_1\eta_1\xi_1$ turned relative to the trajectory reference frame on an angle (K–K_1) as it is shown in Fig. 8. Here an angle ψ represents the horizontal component of angle defining deviation of the directional gyro principal axis from the given direction K_1.

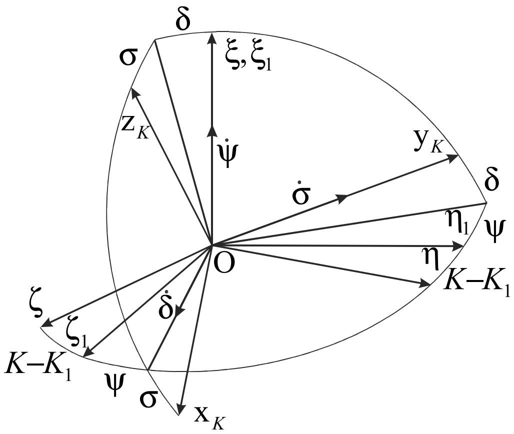


Fig. 8. Mutual location of the directional gyro directional axis relative to the reference frame $O\zeta_1\eta_1\xi_1$

By comparing Figures 6–8 and taking into consideration the matrix $\mathbf{A}_2$ it is possible to write [5]

$$\Delta K = \psi - (\alpha - \alpha_r)\text{tg}\theta - \alpha_K .$$

Equations of the directional gyro can be represented in the following form [8]

$$\begin{aligned} &J\ddot{\alpha}_k + H\dot{\beta}_k + c\alpha_k + d\dot{\alpha}_k + \frac{H}{T}\beta_k = -H_1\omega_{x_1} + M_{z1} + \\ &+ M_{z2} + M_{z3} + M_{z4} + M_{dist\,z}; \\ &J\ddot{\beta}_k + H\dot{\alpha}_k - c\beta_k - d\dot{\beta}_k + \frac{H}{T}\alpha_K = H_1\omega_{z_1} + M_{x1} + \\ &+ M_{x2} + M_{x3} + M_{x4} + M_{dist\,x}. \end{aligned} \tag{24}$$

Expressions for determination of angular rate projections of the directional gyro rotor look like

$$\begin{cases} \omega_{x_1} = \omega_x \cos(K_1 + \Delta K) + \omega_y \sin(K_1 + \Delta K); \\ \omega_{z_1} = \omega_z + \dot{K} + \Delta\dot{K} \end{cases} \tag{25}$$

where $\omega_x, \omega_y, \omega_z$ are defined by relations (9), where index r is changed by index k.

After substitution of (25) in (24) equations of the directional gyro become

$$\begin{aligned} &J\ddot{\alpha}_k + H\dot{\beta}_k + c\alpha_k + d\dot{\alpha}_k + \frac{H}{T}\beta_k = -H_1\{[\dot{\beta} - \dot{\beta}_k + \\ &+ \omega_\xi - \omega_\zeta(\alpha - \alpha_k)]\cos(K_1 + \Delta K) + \\ &+ [\dot{\alpha} - \dot{\alpha}_k + \omega_\eta - \omega_\zeta(\beta - \beta_k)]\sin(K_1 + \Delta K)\} + \\ &+ M_{z1} + M_{z2} + M_{z3} + M_{z4} + M_{dist\,z}; \end{aligned}$$

$$\begin{aligned} &J\ddot{\beta}_k + H\dot{\alpha}_k - c\beta_k - d\dot{\beta}_k + \frac{H}{T}\alpha_k = H_1\{[(\dot{\alpha} - \dot{\alpha}_k)\text{tg}\theta + \\ &+ \frac{\alpha - \alpha_k}{\cos^2\theta}\dot{\theta} + \omega_\zeta] + \dot{K} + \Delta\dot{K}\} + \\ &+ M_{x1} + M_{x2} + M_{x3} + M_{x4} + M_{dist\,x}. \end{aligned} \tag{26}$$

After some transformations the relations (26) can be represented in the following form

$$\begin{aligned} &- H_1[(\dot{\beta} + \omega_\xi - \omega_\zeta\alpha)\cos(K_1 + \Delta K) + \\ &+ (\dot{\alpha} + \omega_\eta + \omega_\zeta\beta)\sin(K_1 + \Delta K)] = \\ &= J\ddot{\alpha}_k + H\dot{\beta}_k + c\alpha_k + d\dot{\alpha}_k + \frac{H}{T}\beta_k - H_1[(-\dot{\beta}_k + \\ &+ \omega_\zeta\alpha_k)\cos(K_1 + \Delta K) - \\ &- (\dot{\alpha}_k - \omega_\zeta + \beta_k)\sin(K_1 + \Delta K)] + M_{z1} + M_{z2} + \\ &+ M_{z3} + M_{z4} + M_{dist\,z}; \end{aligned}$$

.

$$\begin{aligned} &H_1(\dot{\alpha}\text{tg}\theta + \frac{\alpha}{\cos^2\theta}\dot{\theta} + \omega_\zeta + \dot{K} + \Delta\dot{K}) = J\ddot{\beta}_k + H\dot{\alpha}_k - \\ &- c\beta_k - d\dot{\beta}_k + \frac{H}{T}\alpha_k + \\ &+ H_1(-\dot{\alpha}_k\text{tg}\theta - \frac{\alpha_k}{\cos^2\theta}\dot{\theta} +) + M_{x1} + M_{x2} + \\ &+ M_{x3} + M_{x4} + M_{dist\,x}. \end{aligned} \tag{27}$$

The equations (27) represent the directional gyro model.

7. Control of Directional Gyro

The system in the mode of the gyro compass functions as the corrected gyro with appropriate control moments. It is typical for the researched device to use correction by the signal of the angle transmitter, which it is mounted on the inner gimbal of the directional gyro. The appropriate control moments can be described by the expressions

$$M_{x1} = k_{x1}\beta; \; M_{z1} = k_{z1}\beta, \tag{28}$$

where k_{x1}, k_{z1} are transfer constants.

Control moments (28) based on accelerometer signals can be represented in the following form

$$M_{x2} = k_{x2}\frac{A_y}{g}; \; M_{z2} = k_{z2}\frac{A_y}{g}, \tag{29}$$

where k_{x2}, k_{z2} are transfer constants; $A_y = W_y - \Delta W_y$, here W_y is the accelerometer output signal; ΔW_y is a correction taking into consideration influence of translational and Coriolis accelerations, and the vertical acceleration.

Control moments caused by the angular motion of the trajectory reference frame and the rotator are determined by the expressions

$$M_{x3} = H_1(\omega_\zeta + \dot{K} + \Delta\dot{K});$$
$$M_{z3} = -H_1[\omega_\xi \cos(K_1 + \Delta K) + \omega_\eta \sin(K_1 + \Delta K).] \quad (30)$$

or after substitution of expressions of trajectory reference frame angular rates [5, 11]

$$M_{x3} = H_1\left(V_1 \mathrm{tg}\varphi - \dot{K} + \Omega\sin\varphi + \dot{K} + \Delta\dot{K}\right);$$
$$M_{z3} = -H_1\left(-V_1 \sin K - V_2 \cos K - \Omega\cos\varphi\sin K\right)\times$$
$$\times\cos(K_1 + \Delta K) - H_1(V_1 \cos K - V_2 \sin K +$$
$$+\Omega\cos\varphi\cos K)\sin(K_1 + \Delta K), \quad (31)$$

here

$$V_1 = \frac{V_\eta \sin K + V_\xi \cos K}{R_1};$$
$$V_2 = \frac{V_\eta \cos K - V_\xi \sin K}{R_M}.$$

The moments taking into consideration gyro drifts are described by the expressions

$$M_{z4} = J\ddot{\alpha}_k + H\dot{\beta}_k + c\alpha_k + d\dot{\alpha}_k + \frac{H}{T}\beta_k -$$
$$- H_1\{[-\dot{\beta}_k + \omega_\zeta\alpha_k)]\cos(K_1 + \Delta K) +$$
$$+[-\dot{\alpha}_k - \omega_\zeta\beta_k]\sin(K_1 + \Delta K)\}; \quad (32)$$
$$M_{x4} = J\ddot{\beta}_k + H\dot{\alpha}_k - c\beta_k - d\dot{\beta}_k +$$
$$+\frac{H}{T}\alpha_k + H_1(-\dot{\alpha}_k \mathrm{tg}\theta - \frac{\alpha_k}{\cos^2\theta}\dot{\theta}+).$$

Equations (27) supplemented by expressions (29)–(32) represent the model of the directional gyro using biaxial gyrostabilized platform.

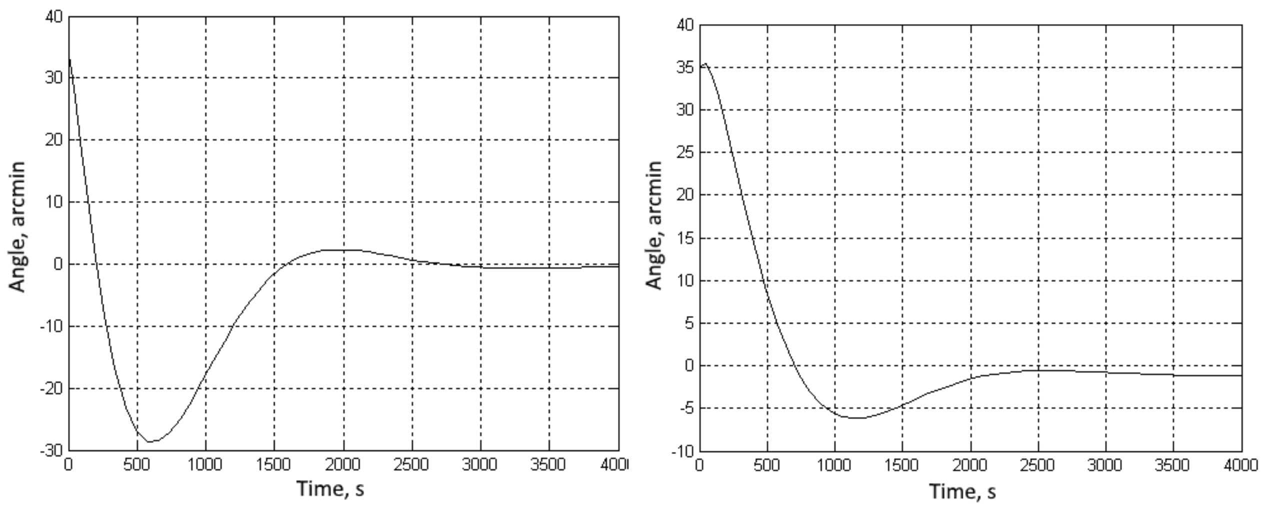
In the mode of the azimuth gyro it is necessary to believe

$$M_{x1} = 0, M_{x2} = 0, M_{z3} = 0.$$

8. Simulation Results and Their Discussion

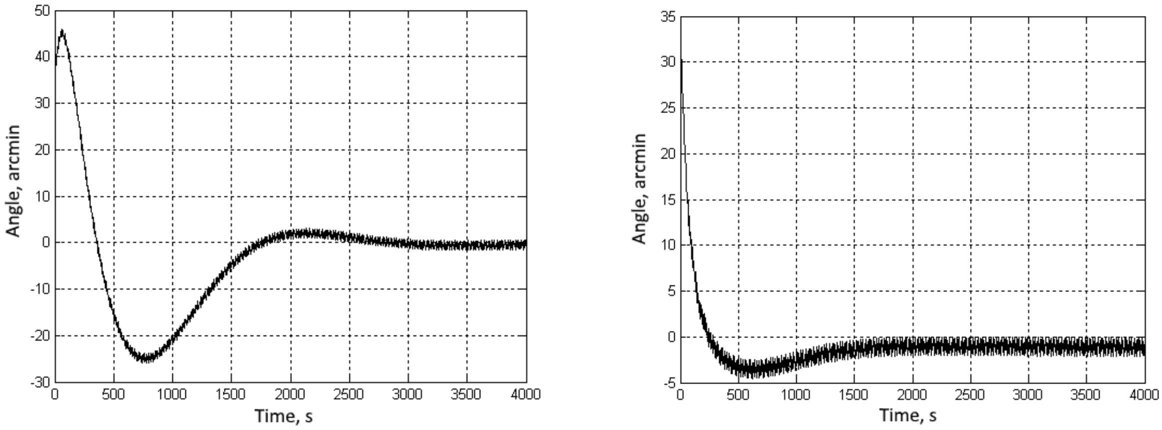
Simulation results are given in Figures 9–11. They represent the transient processes of high-precision attitude determination. It should be noted that simulation was carried out for the AHRS operated on marine vehicles in conditions of autonomy [12].

Fig. 9 presents attitude determination by means of ideal situation, when the external disturbances are absent. Influence of irregular sea wave is shown in Fig. 10.



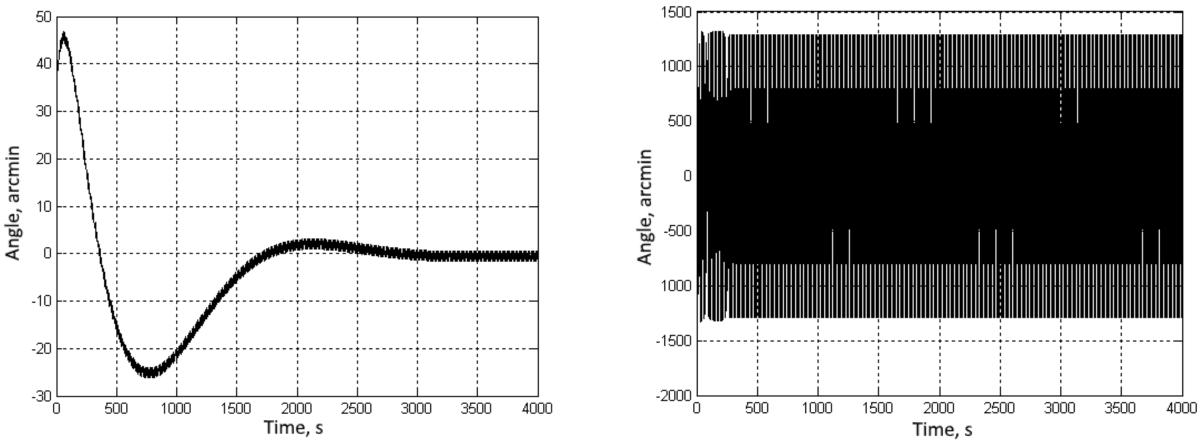
a b

Fig. 9. Transient processes of angles α (a) and β (b) during nondisturbed operation



a b

Fig. 10. Transient processes of angles α (a) and β (b) during disturbed operation (amplitude of irregular sea wave is 12 degrees, period of irregular sea wave is 12,57 s)



a b

Fig. 11. Control with presence (a) and absence (b) of the integral correction

Advantages of the integral correction for AHRS operated in difficult conditions of external disturbances are verified by the simulation results shown in Fig. 11.

Simulation results prove the possibility to achieve high accuracy of attitude and heading determination.

9. Conclusions

The full mathematical model of the gimballed AHRS including the vertical gyro and directional gyro are derived.

The detailed expressions for control moment both for the vertical gyro and for the directional gyro are obtained.

The necessary complex transformations taking into consideration the trajectory reference frame as the initial navigation reference frame are carried out.

The full analysis of the system kinematics is represented. This allows obtaining expressions for errors of attitude and heading determination.

The simulation of attitude and heading processes determination for system operated on the marine vehicles is carried out. Represented results of simulation include situation with nondisturbed motion, and influence of irregular sea waves.

The presented results prove efficiency of integral correction in conditions of the external accelerations influence.

The obtained results can be also useful for design of inertial navigation systems of the wide class.

The obtained expressions of control moments can be useful both for gimballed and for strapdown inertial navigation systems of the wide class.

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Математична модель системи визначення просторової орієнтації з використанням двоосної горизонтальної платформи.

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Мета: Функціонування системи визначення просторового положення в умовах автономності та високої точності потребує використання платформ у кардановому підвісі. Метою статті є детальне дослідження кінематики та моментів управління такої системи. У результаті досліджень було отримано модель високоточної системи визначення просторової орієнтації з використанням двохосної горизонтальної платформи. **Методи:** Створення математичної моделі здійснювалося на підставі теорії гіроскопів у цілому та теорії коректованих гірокомпасів та динамічно настроюваних гіроскопів зокрема. Було взято до уваги концепцію кутів Ейлера та спрямовуючих косинусів. **Результати:** Представлено повний опис системи визначення просторового положення. Наведено математичні моделі таких складових досліджуваної системи як гіровертикаль та гіроскоп напрямку. Представлено результати моделювання з використанням розробленої моделі. **Висновки:** Отримано математичну модель платформної системи визначення просторової орієнтації, включаючи моделі гіровертикалі та гіроскопа напрямку. Наведено детальні вирази для отримання моментів управління (корекції). Виконано повний аналіз кінематики досліджуваної системи. Отримані результати можуть бути корисними під час проектування інерціальних навігаційних систем широкого класу.

Ключові слова: високоточні навігаційні системи; гіровертикаль; гіроскоп напрямку; платформи у кардановому підвісі; система визначення просторової орієнтації; спрямовуючі косинуси.

О.А. Сущенко

Математическая модель системы определения пространственной ориентации с использованием двухосной горизонтальной платформы.

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Цель: Функционирование системы определения пространственного положения в условиях автономности и высокой точности требует использования платформ в кардановом подвесе. Целью статьи является подробное исследование кинематики и моментов управления такой системы. В результате исследований была получена модель высокоточной системы определения пространственной ориентации с использованием двухосной горизонтальной платформы. **Методы:** Разработка математической модели основывалась на теории гироскопов в целом и теории корректируемых гирокомпасов и динамически настраиваемых гироскопов в частности. Были приняты во внимание концепции углов Эйлера и направляющих косинусов. **Результаты:** Представлено полное описание системы определения пространственного положения. Приводятся математические модели таких составляющих исследуемой системы как гировертикаль и гироскоп направления. Представлены результаты моделирования с использованием разработанной модели. **Выводы:** Получена математическая модель платформенной системы определения пространственной ориентации включая модели гировертикали и гироскопа направления. Приведены подробные выражения для моментов управления (коррекции). Выполнен полный анализ кинематики исследуемой системы. Полученные результаты могут быть полезными при проектировании инерциальных навигационных систем широкого класса.

Ключевые слова: высокоточные навигационные системы; гировертикаль; гироскоп направления; направляющие косинусы; платформы в кардановом подвесе; система определения пространственной ориентации.

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