UDC 621.3.01:518
DOI: 10.18372/2306-1472.70.11422

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ESTIMATE OF ACCURACY OF APPROXIMATE SOLUTIONS OF NON-LINEAR BOUNDARY VALUE PROBLEMS BY THE MULTI-STEP DIFFERENTIAL TRANSFORM METHOD

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Abstract

Purpose: *The present paper is aimed at estimating of accuracy and justification the application effectiveness of the multi-step differential transform method for solving non-linear boundary value problems.* ***Methods:*** *This article reviews the multi-step differential transform method for solving non-linear boundary value problem.* ***Results:*** *The upper bound of estimate of accuracy of approximate solutions of non-linear boundary value problems by the multi-step differential transform method for the case of accounting of restricted quantity of discretes of differential spectra is offered. We present results of numerical solution of a non-linear boundary value problem and shown the efficiency of application of the multi-step differential transform method compared with traditional differential transform method.* ***Discussion:*** *It is shown, that upper bound of error estimate of the multi-step differential transform method compared with traditional differential transform method is decreased in p^s time, where s is the quantity of accounted discretes, p is the quantity of intervals, over which the given time interval is divided. The multi-step differential transform method gives the principal possibility to get more exact value of random analytic function $x(t)$ on the end of interval at restricted quantity of discretes of differential spectrum compared with the differential transform method application.*

Keywords: approximate solution; differential transform method; estimate of accuracy; multi-step differential transform method; simulation; upper and lower bounds of error estimate.

1. Introduction

Non-linear boundary value problems occur frequently in modeling of different problems in various areas of science and engineering, including optimal control, flight dynamics, nuclear physics, quantum mechanics and others. In general, boundary value problems are described by non-linear differential equations, don't have analytical solutions and are solved by various numerical and numerical-analytical methods. However, application of majority of these methods is associated with overcoming of the variety of mathematical and computational difficulties.

One way to overcome given shortcoming is the application of operation method of differential-taylor transformations (differential transform method, DTM), whose basic definition and the fundamental theorems are given in [1-5]. It can be applied directly to solve non-linear differential equations without preliminary linearization, eliminates dependence of variables from time argument, admits the possibility to obtain solution in analytic form and considerably reduces the computing volume [6,7]. But along with the evident advantages, the DTM has some drawbacks. Restoring the differential equation solution as a formal Taylor series, generally can be impossible through the small radius of convergence, which could be essentially less than transient time.

For extending the search region solution of non-linear boundary value problems, different modifications of DTM methods have been proposed including the multi-step differential transform method (MsDTM) [8-13]. The sense of last method consists in dividing of entire interval into sub-intervals, searching over each subinterval the solution by the DTM based method and obtaining the general solution of equation as sum of solutions over sub-intervals. In [14] was proposed the modification of the MsDTM by using of approximation of non-linear terms of differential equations by Adomian polynomials permitting to obtain the solution of non-linear differential equations in more wide range with acceptable accuracy.

The solution accuracy of non-linear boundary value problems, obtained using different methods of DTM, essentially dependent from the quantity of accounted discretes used for restoring Taylor series solution. The bigger quantity of discretes would be accounted, the more exact the approximate solution would be obtained. In practice at dynamic process simulation the maximum quantity of accounted discretes always are restricted through the great volume of needed calculation that complicates the obtaining of task solution in the real and speeding up time. This results to increase the simulation error of dynamical processes in the field of originals and necessity of estimate of accuracy of solution obtained.

2. Research tasks

The present paper is aimed at estimating of accuracy and justification the application effectiveness of the MsDTM for solving non-linear boundary value problems.

3. Estimate of error of approximate solution

Consider the following non-linear boundary value problem

$$\frac{\mathrm{dx}}{\mathrm{dt}} = f\left(t, \mathrm{x}, \mathrm{x}', ..., \mathrm{x}^{(\mathrm{m})}\right) = 0, \tag{1}$$

subject to the initial conditions

$$x^{(r)}(t_0) = c_r, \ r = 0,1,...,m-1 .$$

The problem solution (1) will consider over the interval $t_0 \le t \le T$, where the length of interval $L = T - t_0$ is selected inside the radius R of convergence of Taylor series, i.e. $0 \le L \prec R$. Assume that analytic function x(t) is continuously differentiated in any point $t \in [t_0, T]$, has derivatives of m^{th} - order, which are limited in total for any whole $m \ge 1$ so that,

$$\left|x^{(m)}(t)\right| \le C < +\infty, \ \ t \in [t_0, T]. \tag{2}$$

Let us apply the basic features of DTM to the function x(t)

$$X(k) = \frac{H^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_0} \Leftrightarrow \mathrm{x(t)} = \sum_{\mathrm{k}=0}^{\infty}\left(\frac{\mathrm{t}}{\mathrm{H}}\right)^k X(k)$$

. (3)

where $x(t)$ is the original function, which represents the continuous and bounded together with all its derivatives the function of real argument t; $X(k)$ is the discrete function of integer argument $k = 0,1,2,...$, which is termed as the differential image of original $x(t)$ (the differential spectrum); H is the scale stationary value having dimensionality of argument t and often equals the time interval L, over which we want to find the function $x(t)$; $\Leftrightarrow$ is the correspondence symbol between the original $x(t)$ and its differential image $X(k)$.

The expression to the left of symbol $\Leftrightarrow$ in (3) defines the differential direct transform, permitting by the original $x(t)$ to find the image $X(k)$, and on the right - the differential inverse transform, which recovers the original $x(t)$ by the images $X(k)$.

Following the ideology of the MsDTM, the entire time interval $[\mathrm{t}_0, T]$ is divided into p given sub-intervals, $T_q = t_q - t_{q-1}$, $q = \overline{1,\mathrm{p}}$, $\sum_{q=1}^{\mathrm{p}} T_q = T - t_0 = L$, of equal step-size $h = L/p$. Restrict the quantity of discretes of differential spectrum $X(k)$ some given number $s \succ 0$ so, that integer argument $k = 0,1,2,...$ is changed within bounds $k = 0,1,2,...,s$. Applying the DTM over the first sub-interval $[\mathrm{t}_0, t_1]$, we will obtain the approximate solution of equation (1) with taking into account the finite quantity of discretes s in the form:

$$x_1(t_0) \approx \sum_{k=0}^{s} X_1(k)(t - \mathrm{t}_0)^k, \ t \in [t_0, t_1].$$

Taking into account the initial condition $x_1^{(r)}(t_0) = c_r$ and the expression (3) we will find for the first sub-interval all values of differential spectrum $\mathrm{X}_1(k)$, $k = 0,1,2,...,s$. For $q \ge 2$ and at each following sub-interval $[t_{q-1}, t_q]$ we will use the initial condition $x_q^{(r)}(t_{q-1}) = x_{q-1}^{(r)}(t_{q-1})$. Then the expression (1) for the q^{th} sub-interval will be following:

$$X_q(k) = \frac{H^k}{k!}\left[\frac{d^k x_{q-1}(t)}{dt^k}\right]_{t=t_{q-1}}, k \geq 0.$$

By applying given approach to each sub-interval will obtain the sequence of approximate solutions $x_q(t),\ q = 0,1,...,p$ for the solutions $x(t)$ of equation (1):

$$x_q(t) \approx \sum_{k=0}^{s} X_q(k)(t - t_{q-1})^k = y_q(t),\ t \in [t_{q-1}, t_q]. \quad (4)$$

Finally, the MsDTM assumes the following solution of equation (1):

$$x(t) = \begin{cases} x_1(t) \approx y_1(t),\ t \in [t_0, t_1], \\ x_2(t) \approx y_2(t),\ t \in [t_1, t_2], \\ ..... \\ x_p(t) \approx y_p(t), t \in [t_{p-1}, t_p] \end{cases} \quad (5)$$

It is easily observed that if $p = 1$, then the MsDTM reduces to the classical DTM.

The quantity of accounted discretes s for restoring of the solution (5) as Taylor series is one of the most essential factors that effect on the solution accuracy obtained. Restriction of given quantity of discretes leads to error of result obtained. The upper bound of error estimate $|\varepsilon_0| = |x(t) - y(t)|$ of the DTM (3) is given by the expression [15,16]:

$$|\varepsilon_0| \leq \frac{L^{s+1}}{(s+1)!} \sup_{0<t_l<L} \left|x^{(s+1)}(t_l)\right|. \quad (6)$$

Taking into account the constraint (2), the expression (6) can be written as:

$$|\varepsilon_0| \leq C\frac{L^{s+1}}{(s+1)!} = |\hat{\varepsilon}_0|. \quad (7)$$

In [17] as a criterion of preliminary estimation of upper error bound of obtained solution $G(\mathrm{n,m})$ at the restriction of quantity of discretes used for restoring solution as a power series is considered the expression that links solutions, obtained for n and $\mathrm{n+m}$ discretes:

$$\mathrm{G(n,m)} = \sqrt{\frac{1}{\mathrm{s}}\sum_{i=0}^{N}\left|1 - \frac{x(t_i, n)}{x(t_i, n+m)}\right|^2},\ t_i \in [0, L]. \quad (8)$$

Use the given criterion is enough awkward, because besides the dual solution calculation it demands the additional m discretes $X(\mathrm{n}+1), X(n+2),..., X(n+m)$ calculation too. For non-linear differential equations with complex nonlinearities, the expression (8) in some particular cases can give the wrong solution, when discretes $X(\mathrm{n})$ and $X(n+m)$ have the same sign. In these cases, are necessary to choose $\mathrm{m} > 0$, so that $X(\mathrm{n})$ and $X(n+m)$ have the different signs.

For the MsDTM, the expression (6) for the upper bound of error estimate $|\varepsilon_q| = |\mathrm{x_q(t) - y_q}(t)|$ over q^{th} sub-interval with taking into account s discretes can be written as:

$$|\varepsilon_{\mathrm{q}}| \leq \frac{(L/\mathrm{p})^{\mathrm{s+1}}}{(s+1)!} \sup_{\xi\in[\mathrm{t_q}, t_{q-1}]} \left|x_q^{(s+1)}(\xi)\right|,\ q = 1,...p. \quad (9)$$

On the ground of the constraint (2) can make the conclusion, that over q^{th} sub-interval

$$C_q = \sup_{\xi\in[t_q, t_{q-1}]} \left|x_q^{(s+1)}(\xi)\right| \leq C < +\infty. \quad (10)$$

Really, if the $(s+1)^{\text{th}}$ derivative of function $x_q(t)$ achieves the maximum value over the interval $[t_0, t_q]$ then $C_q = C$, otherwise $C_q < C$.

The error estimate (9) with taking into account the constraint (10) can be written as

$$|\varepsilon_{\mathrm{q}}| \leq C\frac{(L/p)^{\mathrm{s+1}}}{(s+1)!}. \quad (11)$$

From the expression (11) follows that obtained error at dividing the entire interval into equal p sub-intervals, is the same over sub-intervals and depends only from the quantity of accounted discretes s. Convert the given estimate to the relative error estimate. Let us select as a comparison base the error (7) for the DTM. Then for the relative error on q^{th} sub-interval obtain:

$$\left|\frac{\varepsilon_{\mathrm{q}}}{\hat{\varepsilon}_0}\right| \leq \frac{(L/p)^{\mathrm{s+1}}}{L^{s+1}},\quad q = 1, ..., p. \quad (12)$$

Consider the full relative error of the MsDTM $\varepsilon_s = \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_p$ over the entire interval in relation to the error ε_0 of the DTM:

$$\left|\frac{\varepsilon_s}{\hat{\varepsilon}_0}\right| = \left|\frac{\varepsilon_1}{\hat{\varepsilon}_0}\right| + \left|\frac{\varepsilon_2}{\hat{\varepsilon}_0}\right| + ... + \left|\frac{\varepsilon_p}{\hat{\varepsilon}_0}\right|. \quad (13)$$

From expression (12) follows, that components of relative errors are changed in the bounds:

$$0 \le \left| \frac{\varepsilon_{s_i}}{\hat{\varepsilon}_0} \right| \le 1, \quad i = 1,2,..., p. \qquad (14)$$

Bigger deviation of approximate solution from exact solution, usually, falls on the end of time interval. At that, it will be maximum in the case of the same signs ε_{s_i}. Then, taking into account (12), the expression (13) will be following:

$$\left| \frac{\varepsilon_s}{\hat{\varepsilon}_0} \right| \le \left| \frac{1}{p^{s+1}} \right| + \left| \frac{1}{p^{s+1}} \right| + ... + \left| \frac{1}{p^{s+1}} \right| = \frac{1}{p^s} = p^{-s}. \qquad (15)$$

This means that the upper bound of error estimate of the MsDTM in p^s time less than the upper bound of error estimate of the DTM at dividing of given interval into p sub-intervals of equal step-size, i.e.:

$$|\varepsilon_s| \le p^{-s} |\hat{\varepsilon}_0|, \qquad (16)$$

where s is the quantity of accounted discretes of differential spectrum $X(k)$ above the zeroth discrete $X(0)$, i.e. the quantity s is equal the number of the last accounted discretes of differential spectrum $X(k)$. The analysis of obtained expression shown, that with increasing of quantity of accounted discretes s, the upper bound of summary error is reduced on the exponential rule and at $s \to \infty$ reduced to the zeroth lower bound. Therefore, the range of changing of summary error at dynamical processes simulation using MsDTM is defined by constraints:

$$0 \le |\varepsilon_s| \le p^{-s} \cdot |\hat{\varepsilon}_0|, \qquad (17)$$

where $|\hat{\varepsilon}_0|$ is defined by expression (7).

From the expression (17) can make the conclusion, that the MsDTM at the restricted quantity of discretes s of differential spectrum $X(k)$ gives the possibility to get more exact solution of boundary value problem (1) in the point $t = T$ at condition execution (2), than the DTM.

The DTM (3) gives the exact value of analytical function $x(t)$ in the point $t = T$ only in particular case, when solution $x(t)$ is approximated by polynomial of $n \le s$ order. In other cases, the error of the DTM (3) is equal $|\varepsilon_0| > 0$ for restricted quantity of discretes $s < +\infty$ of differential spectrum $X(k)$. The error $|\varepsilon_0| > 0$ couldn't be reduced to zero by dividing the interval $[t_0, T]$ over any finite sub-interval quantity $p < +\infty$, as the zeroth error value of the DTM (3) gives in the general case over the random non-null time interval only at taking into account infinite amount of Taylor series terms or discretes of differential spectrum $X(k)$ [16].

The constraint (17) shows, that with increasing of the quantity of accounted discretes s of differential spectrum, the application effectiveness of the MsDTM compared with the DTM is increasing on the law of exponential function. Therefore, for the high-accuracy calculation is appropriate to apply the MsDTM instead of the DTM.

Enhance the solution accuracy of non-linear boundary value problems and also to expand the admissible solution interval, the restriction on which is defined by the radius of convergence of Taylor series is possible on the basis of application of shifted differential transformations. In contrast to traditional, shifted transformations are obtained by transferring of the center of expansion of original to a Taylor series from the initial point $t = 0$ to the shifted point $t = t_q$. The best from the standpoint of reducing the solution error is the arrangement of the center of expansion of the original in a Taylor series in the middle of the given interval [16]. In fact, it means that given interval is divided into two sub-intervals of same length and obtaining the solution over each sub-interval using two models in the area of shifted transformations: the direct model (from shifted point to the end of interval) and the inverse model (from shifted point to the start of interval). At that has been obtained that compared with traditional DTM the upper bound of error estimate of shifted differential transformations is decreasing in 2^s time, where s is the quantity of accounted discretes of differential spectrum. This is agreed with above result obtained for the MsDTM, the use of which allows to decrease the error in p^s time, where p is the quantity of sub-intervals into which given interval is divided.

4. The effectiveness of multi-step differential transform method

The effectiveness of the MsDTM can be illustrated by following example.

Let us consider the following boundary value problem, which is described by non-linear ordinary differential equation with the quadratic source term [14]:

$$\frac{dx(t)}{dt} = 2x(t) - x^2(t) + 1, \; x(0) = 0, \quad t \in [0;1.2]. \qquad (18)$$

The exact solution is given by:

$$x(t) = 1 + \sqrt{2}\tanh\left(\sqrt{2}t + \frac{1}{2}\log\left(\frac{\sqrt{2}-1}{\sqrt{2}+1}\right)\right). \quad (19)$$

Let us divide the given interval $[0;1,2]$ into 10 sub-intervals of the equal step-size $h = 1,2/10$.

By applying the DTM, we write the equation (18) in the spectral form for each sub-interval:

$$\begin{aligned} &(k+1)X_q(k+1) = 2X_q(k) - \widetilde{A}_{kq} + \sigma(k), \\ &X_1(0) = x_1(0) = x(0) = 0,\ t_0 = 0,\ q = 1, \\ &x_q(t_{q-1}) = x_{q-1}(t_{q-1}),\ q = 2,...,10, \\ &\sigma(k) = \begin{cases} 1, k = 0, \\ 0, k \neq 0. \end{cases} \end{aligned} \quad (20)$$

Accordingly, with procedure [14], for non-linear part of equation (18) $f(x) = x^2$ we calculate over each sub-interval the components A_{kq} of Adomian polynomials and thereon the corresponding components $\widetilde{A}_{kq}$ for replacement by them of components of differential images of non-linear part of equation:

$$\widetilde{A}_{0q} = X_q^2(0),\ \widetilde{A}_{1q} = 2X_q(0)X_q(1),$$
$$\widetilde{A}_{2q} = X_q^2(1) + 2X_q(0)X_q(2),$$
$$\widetilde{A}_{3q} = 2X_q(0)X_q(3) + 2X_q(1)X_q(2),$$
$$\widetilde{A}_{4q} = 2X_q(0)X_q(4) + 2X_q(1)X_q(3) + X_q^2(2),$$
$$\widetilde{A}_{5q} = 2X_q(0)X_q(5) + 2(X_q(2)X_q(3) + X_q(1)X_q(4))$$

Substituting values $\widetilde{A}_{kq}$ in (19) and taking into account (7), we find the approximate solution of equation (18) over each subinterval. Summating given solutions obtain the general solution of equation (18) on the given interval.

Let us find the value of function $x(t)$ in the end point $t = 2,0$. Result obtained shown the following. The exact solution (19) of function $x(t)$ of equation (18) in the point $t = 2,0$ is equal 2,35777. Application of the DTM doesn't allow to obtain the solution over given interval due to exceeding the value of given interval the radius of convergence of Taylor series by whom the approximate solution is approximated.

The found solution by MsDTM with taking into account first 6 discretes of the differential spectrum and dividing given interval into two sub-intervals (analogue of application of shifted differential transformations) is 2,08641 ($\varepsilon = 1{,}15 \cdot 10^{-1}$), while at dividing interval into 10 sub-intervals, the approximate solution has the value 2,3577717 ($\varepsilon = 3{,}19 \cdot 10^{-8}$). The given example is illustrated the effectiveness of the MsDTM application for solving non-linear boundary value problems.

5. Conclusions

The upper bound of error estimate for approximate solution of non-linear boundary value problem by the MsDTM has been offered. It is shown, that upper bound of error estimate of the MsDTM compared with traditional DTM is decreased in p^s time, where s is the quantity of accounted discretes, p is the quantity of intervals, over which the given time interval is divided. The MsDTM gives the principal possibility to get more exact value of random analytic function $x(t)$ on the end of interval at restricted quantity of discretes of differential spectrum compared with the DTM application.

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Received 20 December 2017

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Оцінка точності наближеного розв’язку нелінійних крайових задач багатоетапним методом диференціальних перетворень

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Мета: Метою цієї статті є оцінка точності та обґрунтування ефективності застосування багатоетапного метода диференціальних перетворень для розв’язку нелінійних крайових задач. **Методи:** В статті розглянуто багатоетапний метод диференціальних перетворень до розв’язку нелінійної крайової задачі. **Результати:** Запропоновано оцінку зверху точності наближеного розв’язку нелінійних крайових задач багатоетапним методом диференціальних перетворень для випадку урахування обмеженої кількості дискрет диференціальних спектрів. Представлені результати численного розв’язку нелінійної крайової задачі та показана ефективність застосування багатоетапного метода диференціальних перетворень порівняно з основними диференціальними перетвореннями. **Обговорення:** Показано, що оцінка зверху наближеного розв’язку нелінійної крайової задачі багатоетапним методом диференціальних перетворень порівняно з основними диференціальними перетвореннями знижується в p^s раз, де s – кількість дискрет, що враховується, p – кількість підінтервалів, на які розбивається заданий часовий інтервал. Отримано, що застосування метода багатоетапних диференціальних перетворень дає принципову можливість отримати точне значення довільної аналітичної функції $x(t)$ на кінці інтервалу при обмеженій кількості дискрет диференціального спектру.

Ключові слова: багатоетапний метод диференціальних перетворень; верхня та нижня межі оцінки похибки; метод диференціальних перетворень; оцінка точності; моделювання; наближений розв’язок.

В. П. Гусынин[1], А. В. Гусынин[2], Е. М. Тачинина[3]. **Оценка точности приближенного решения нелинейных краевых задач многоэтапным методом дифференциальных преобразований**

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Цель: Целью данной статьи является оценка точности и обоснование эффективности применения многоэтапного метода дифференциальных преобразований для решения нелинейных краевых задач. **Методы:** В статье рассмотрен многоэтапный метод дифференциальных преобразований к решению нелинейной краевой задачи. **Результаты:** Предложена оценка сверху точности решения нелинейных краевых задач многоэтапным методом дифференциальных преобразований для случая учета ограниченного количества дискрет дифференциальных спектров. Представлены результаты численного решения нелинейной краевой задачи и показана эффективность применения многоэтапного метода дифференциальных преобразований в сравнении с основными дифференциальными преобразованиями. **Обсуждение:** Показано, что оценка сверху приближенного решения нелинейной краевой задачи многоэтапным методом дифференциальных преобразований по сравнению с основными дифференциальными преобразованиями снижается в p^s раз, где s - количество учитываемых дискрет, p - количество подинтервалов, на которое разбивается заданный временной интервал. Получено, что применение метода многоэтапных ДТ-преобразований дает принципиальную возможность получить точное значение произвольной аналитической функции $x(t)$ на конце интервала при ограниченном количестве дискрет дифференциального спектра.

Ключевые слова: верхняя и нижняя граница оценки погрешности; метод дифференциальных преобразований; многоэтапный метод дифференциальных преобразований; моделирование; оценка точности; приближенное решение.

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