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METHOD FOR OPTIMAL RESOLUTION OF MULTI-AIRCRAFT CONFLICTS IN THREE-DIMENSIONAL SPACE

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Abstract

Purpose: *The risk of critical proximities of several aircraft and appearance of multi-aircraft conflicts increases under current conditions of high dynamics and density of air traffic. The actual problem is a development of methods for optimal multi-aircraft conflicts resolution that should provide the synthesis of conflict-free trajectories in three-dimensional space.* ***Methods:*** *The method for optimal resolution of multi-aircraft conflicts using heading, speed and altitude change maneuvers has been developed. Optimality criteria are flight regularity, flight economy and the complexity of maneuvering. Method provides the sequential synthesis of the Pareto-optimal set of combinations of conflict-free flight trajectories using multi-objective dynamic programming and selection of optimal combination using the convolution of optimality criteria. Within described method the following are defined: the procedure for determination of combinations of aircraft conflict-free states that define the combinations of Pareto-optimal trajectories; the limitations on discretization of conflict resolution process for ensuring the absence of unobservable separation violations.* ***Results:*** *The analysis of the proposed method is performed using computer simulation which results show that synthesized combination of conflict-free trajectories ensures the multi-aircraft conflict avoidance and complies with defined optimality criteria.* ***Discussion:*** *Proposed method can be used for development of new automated air traffic control systems, airborne collision avoidance systems, intelligent air traffic control simulators and for research activities.*

Keywords: aircraft; air traffic control; conflict resolution; dynamic programming; flight safety; multi-aircraft conflict; multi-objective optimization.

1. Introduction

The evolution of air traffic management (ATM) system is primarily aimed at increasing of its capacity in conditions of a constant growth of the flights intensity. For this a new air traffic control (ATC) procedures are introduced, separation minima are reduced and special airspace areas within aircraft are allowed to choose their own routes (Free Route Airspace) are allocated. However, high dynamics and density of current air traffic cause the increasing of separation minima infringements between several aircraft at the same time, i.e. multi-aircraft conflict situations in which prescribed separation minima were not maintained simultaneously between three or more aircraft.

The main requirements for ATM system are ensuring of flight safety and efficiency (regularity, economy). Potential conflict avoidance or actual conflict resolution is a complex cyclical process which consists of the sequence of interrelated actions: monitoring, estimation and prediction of aircraft trajectories; detection of potential conflicts; determination the potential conflict characteristics; finding of possible solutions for conflict resolution; decision-making and control of the selected solution realization. Therefore, the problem of conflict resolution should be considered as the multi-objective optimization problem with limitations and uncertainties in relative importance of optimality criteria and priorities.

2. Analysis of researches and publications

The results of analysis show that most of known methods for multi-aircraft conflicts resolution have a number of significant limitations.

The class of force fields methods contains various methods that use different properties of electric fields, vortex force fields and artificial gravitational fields [1-6]. Common disadvantages of these methods are that they use only heading change maneuvers for conflict resolution and the synthesized conflict-free trajectories are complex for civil aircraft. Application of force fields methods is more promising for avoidance of conflicts between unmanned aerial vehicles, which generally have a lower inertia of motion.

Another class consists of optimization methods that use optimality criteria to find the solution.

The method of multi-aircraft conflict resolution under Free Flight [7-9] is based on the cascade procedure of conflict-free trajectory planning. This method provides the resolution of horizontal plane conflicts using heading change maneuvers.

A well-known is the method of optimal cooperative conflict resolution between multiple aircraft that perform horizontal flight [10]. For conflict-free trajectories determination this method uses a single optimality criterion which defined as total time of aircraft maneuverings.

The method proposed in [11] provides the resolution of multi-aircraft conflicts in horizontal plane using heading changes with minimization of deviations from the planned trajectories. The main disadvantage is that the defined maneuvers are used only for conflict avoidance without returning to the planned flight trajectories.

In articles [12, 13] the method for two- and multi-aircraft conflicts resolution in three-dimensional space is proposed. This method provides a synthesis of heading or altitude change maneuvers considering the instantaneous kinetic energy of aircraft as optimality criterion.

The method described in [14] uses only speed change maneuvers for conflict avoidance.

The summarized disadvantages of discussed optimization methods are disusing of simultaneous combinations of heading, speed and altitude change maneuvers to avoid a conflict and using of single optimality criterion that do not allow to find the most effective solution.

Therefore it is necessary to develop new optimal methods for multi-aircraft conflict resolution in three-dimensional space that should provide the synthesis of conflict-free trajectories using heading, speed and altitude change maneuvers simultaneously according to different flight efficiency criteria.

The method developed in this article is the evolution of method proposed in articles [15, 16] taking into account an approach described in [17].

3. Problem statement

The conflict situation between $n \geq 3$ aircraft is considered. Conflict resolution is a controlled process and aircraft are the dynamic system $\mathbf{S}$. All aircraft change heading, airspeed and vertical speed to avoid the conflict.

The process of maneuvers synthesis is observed in the time interval $[t_0, t_k]$ where t_0 is the moment of a potential conflict detection, t_k is the planned time when all aircraft exit from an ATC area.

Controlled motion of each aircraft is described using the vector differential equation:

$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t), \mathbf{U}(t), t), \ \mathbf{X}(t_0) = \mathbf{X}_0,$$

where $\mathbf{X} = [x \quad y \quad h \quad V \quad V_h \quad \varphi]^{\mathrm{T}}$ – state vector; x, y – horizontal coordinates; h – altitude; V – true airspeed; V_h – vertical speed; φ – heading; $\mathbf{U} = [\gamma_a \quad V_a \quad V_{ha}]^{\mathrm{T}}$ – vector of controls; γ_a – assigned bank angle; V_a – assigned true airspeed; V_{ha} – assigned vertical speed.

An absolute constraint is the flight safety ensured by separation minima maintenance. The state $\mathbf{X}^m(t)$ of aircraft $m = \overline{1, n}$ belongs to the set of conflict-free states $\mathbf{D}_{\mathbf{X}}^m(t)$ if the separation minima with other aircraft are not violated:

$$\mathbf{X}^m(t) \in \mathbf{D}_{\mathbf{X}}^m(t) \big| \mathbf{X}^m(t) \notin \Omega^m(t),$$

where the space of a conflict $\Omega^m(t)$ is a space of states where the separation minima with other aircraft are violated:

$$\Omega^m(t) =$$
$$= \left\{\mathbf{X}^m(t) \big| \left(d\left(\mathbf{X}^m(t), \mathbf{X}^{ref}(t)\right) < d_S\right) \wedge \left(\Delta h\left(\mathbf{X}^m(t), \mathbf{X}^{ref}(t)\right) < h_S\right)\right\},$$
$$ref = \overline{1, n}, \ m \neq ref,$$

where $d\left(\mathbf{X}^m(t), \mathbf{X}^{ref}(t)\right)$ – horizontal distance between aircraft; $\mathbf{X}^{ref}(t)$ – state of aircraft ref; $\Delta h\left(\mathbf{X}^m(t), \mathbf{X}^{ref}(t)\right)$ – vertical distance between aircraft; d_S – lateral (horizontal) separation minimum; h_S – vertical separation minimum. The initial states of all aircraft are conflict-free $\mathbf{X}^m(t_0) \notin \Omega^m(t_0)$.

Controls are limited according to the aircraft performances. Let $\mathbf{D}_\mathrm{U}^m(\mathbf{X}^m(t),t)$ be a set of possible controls $\mathbf{U}^m(t)$ in a state $\mathbf{X}^m(t)$.

Optimality criteria characterizing the efficiency of conflict resolution are flight regularity c_1, flight economy c_2 and the complexity of maneuvering c_3.

The numerical estimations of trajectories for each aircraft according to defined optimality criteria are: J_1^m – deviation from planned flight time; J_2^m – deviation from planned altitude; J_3^m – fuel consumption; J_4^m – number of flight profile changes.

Estimations constitute the vector $\mathbf{J}^m = \{J_i^m\},\ i = \overline{1,4}$.

The numerical estimations of trajectories at time interval $[t_0, t_k]$ are defined as follows [15]:

$$J_1^m = \Lambda_1^m(\mathbf{X}^m(t_k), t_k),$$

$$J_2^m = \Lambda_2^m(\mathbf{X}^m(t_k), t_k),$$

$$J_3^m = \int_{t_0}^{t_k} \lambda_3^m(\mathbf{X}^m(t), \mathbf{U}^m(t), t)dt + \Lambda_3^m(\mathbf{X}^m(t_k), t_k),$$

$$J_4^m = \int_{t_0}^{t_k} \lambda_4^m(\mathbf{X}^m(t), \mathbf{U}^m(t), t)dt + \Lambda_4^m(\mathbf{X}^m(t_k), t_k),$$

where Λ_1^m – estimation of deviation from planned flight time; Λ_2^m,– estimation of deviation from planned altitude; λ_3^m – instantaneous fuel consumption; λ_4^m – speed of flight profile changes; Λ_3^m – estimation of fuel consumption for real exit from an ATC area relatively to actual position at the time moment t_k; Λ_4^m – estimation of flight profile changes for real exit from an ATC area relatively to actual position at the time moment t_k.

As a result the problem of optimal multi-aircraft conflict resolution is determined as follows:

$$\begin{cases} \min\limits_{\mathbf{U}^1(t)\in\mathbf{D}_\mathrm{U}^1(\mathbf{X}^1(t),t)} \mathbf{J}^1(\mathbf{X}^1(t), \mathbf{U}^1(t), t),\ \mathbf{X}^1(t) \in \mathbf{D}_\mathbf{X}^1(t), \\ \min\limits_{\mathbf{U}^2(t)\in\mathbf{D}_\mathrm{U}^2(\mathbf{X}^2(t),t)} \mathbf{J}^2(\mathbf{X}^2(t), \mathbf{U}^2(t), t),\ \mathbf{X}^2(t) \in \mathbf{D}_\mathbf{X}^2(t), \\ \dots \\ \min\limits_{\mathbf{U}^n(t)\in\mathbf{D}_\mathrm{U}^n(\mathbf{X}^n(t),t)} \mathbf{J}^n(\mathbf{X}^n(t), \mathbf{U}^n(t), t),\ \mathbf{X}^n(t) \in \mathbf{D}_\mathbf{X}^n(t), \end{cases} \quad (1)$$

$$t \in [t_0, t_k]$$

The **aim** of this article is to develop the method for optimal resolution of multi-aircraft conflicts in three-dimensional space based on multi-objective dynamic programming.

4. Synthesis of conflict-free trajectories using multi-objective dynamic programming

The problem (1) is solved by synthesizing the set of Pareto-optimal combinations of the conflict-free trajectories of all aircraft and choosing of the optimal combination.

Let $\mathbf{E}(\mathbf{X}^m(t), t)$ be a set of Pareto-optimal estimations of conflict-free trajectories for a state $\mathbf{X}^m(t) \in \mathbf{D}_\mathbf{X}^m(t)$:

$$\mathbf{E}(\mathbf{X}^m(t),t) = \{\mathbf{J}^m(\mathbf{X}^m(t), \mathbf{U}_e^m(t), t) | \neg\exists \mathbf{U}^m(t) \in \mathbf{D}_\mathrm{U}^m(\mathbf{X}^m(t),t):$$
$$\mathbf{J}^m(\mathbf{X}^m(t), \mathbf{U}^m(t), t) \le \mathbf{J}(\mathbf{X}^m(t), \mathbf{U}_e^m(t), t), \mathbf{U}_e^m(t) \ne \mathbf{U}^m(t)\},$$

$$\mathbf{J}^m(\mathbf{X}^m(t), \mathbf{U}^m(t), t) = \left\{\Lambda_1^m, \Lambda_2^m, \int_t^{t_k} \lambda_3^m dt + \Lambda_3^m, \int_t^{t_k} \lambda_4^m dt + \Lambda_4^m\right\}.$$

Based on Bellman's principle of optimality [18] the system of multi-objective dynamic programming equations for determination of the set $\mathbf{E}_\mathbf{Z}(t)$ of Pareto-optimal estimations of combinations of conflict-free trajectories is written as follows:

$$\begin{cases} \mathbf{E}(\mathbf{X}^1(t),t) = \mathrm{eff}\,\Im^1,\ \mathbf{X}^1(t) \in \mathbf{D}_\mathbf{X}^1(t),\ \mathbf{X}^1(t+\tau) \in \mathbf{D}_\mathbf{X}^1(t+\tau), \\ \mathbf{E}(\mathbf{X}^2(t),t) = \mathrm{eff}\,\Im^2,\ \mathbf{X}^2(t) \in \mathbf{D}_\mathbf{X}^2(t),\ \mathbf{X}^2(t+\tau) \in \mathbf{D}_\mathbf{X}^2(t+\tau), \\ \dots \\ \mathbf{E}(\mathbf{X}^n(t),t) = \mathrm{eff}\,\Im^n,\ \mathbf{X}^n(t) \in \mathbf{D}_\mathbf{X}^n(t),\ \mathbf{X}^n(t+\tau) \in \mathbf{D}_\mathbf{X}^n(t+\tau), \end{cases}$$

$$\Im^m = \bigcup_{\mathbf{U}^m(t)\in\mathbf{D}_\mathrm{U}^m(\mathbf{X}^m(t),t)} \left(\left\{0, 0, \int_t^{t+\tau} \lambda_3^m dt, \int_t^{t+\tau} \lambda_4^m dt\right\} \oplus \mathbf{E}(\mathbf{X}^m(t+\tau), t+\tau)\right),$$
$$m = \overline{1,n},$$

with boundary condition

$$\mathbf{E}(\mathbf{X}^m(t_k), t_k) = \{\Lambda_1^m, \Lambda_2^m, \Lambda_3^m, \Lambda_4^m\},$$

where eff – the operator of determination of Pareto-optimal estimations; τ – a small value; $\oplus$ – direct sum.

The set $\mathbf{P}_\mathbf{Z}$ of Pareto-optimal combinations of conflict-free flight trajectories is determined by the set of estimations $\mathbf{E}_\mathbf{Z}(t_0)$ at the moment of conflict detection t_0.

Finding all possible solutions of the equations system is difficult computational problem. It is proposed to limit the number of combinations of conflict-free trajectories and apply the following method of their synthesis using discrete multi-objective dynamic programming.

The dynamic system $\mathbf{S}$ is discretized in time (the conflict resolution process is decomposed into k stages) and in state space. It is assumed that all aircraft maneuver for conflict avoidance during stages $j=\overline{1,k-1}$ and return to the planned flight trajectories during last stage k. Time interval $[t_{j-1},t_j],\ t_j=t_{j-1}+\Delta t$ corresponds to each stage j, except for the last one. The time interval of the last stage $j=k$ is different because of the different time that each aircraft needs to reach the fixed final state when transiting from the states at the previous stage.

The flight of each aircraft is considered separately, taking into account the overall limitation on maintaining the separation minima.

In general, it is considered that each aircraft m can transit into the state $\mathbf{X}^m(j)$ from several states $\mathbf{X}^m(j-1)$ at the previous stage:

$$\mathbf{X}^m(j)=f\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right).$$

States $\mathbf{X}^m(j)$ form the set $\mathbf{D}^m(j)$ of possible states .The final state $\mathbf{X}_k^m=\mathbf{X}^m(k)$ is specified only by the horizontal coordinates of the point at which an aircraft exits an ATC area at planned time. It is expected that aircraft may transit into the final state from all the states of the previous stage.

To increase the computational efficiency of the method, it is proposed to monitor the separation minima violations in defined states at each stage.

The set of conditionally conflict-free states $\widehat{\mathbf{D}}_{\mathbf{X}}^m(j)$ of aircraft m includes the states $\mathbf{X}^m(j)\in\mathbf{D}^m(j)$ in which the violations of separation minima with other aircraft may be present for some of their states $\mathbf{X}^{ref}(j)$, or the violations of separation minima are absent:

$$\begin{gathered}\mathbf{X}^m(j)\in\widehat{\mathbf{D}}_{\mathbf{X}}^m(j):\exists\mathbf{X}^{ref}(j),\left((d\ge d_s)\wedge(\Delta h\ge h_s)\right)\vee\\ \vee\left((d\ge d_s)\wedge(\Delta h<h_s)\right)\vee\left((d<d_s)\wedge(\Delta h\ge h_s)\right),\\ \mathbf{X}^{ref}(j)\in\mathbf{D}^{ref}(j),\ ref=\overline{1,n},\ m\ne ref,\end{gathered}$$

where d, Δh – horizontal distance and vertical interval between aircraft in states $\mathbf{X}^m(j)$ and $\mathbf{X}^{ref}(j)$ respectively.

It is necessary to check the maintaining of separation minima in limited time periods in which unobservable violations of separation cannot occur.

The procedures for each stage $j=\overline{1,k}$ are:

– determination of sets of possible controls $\mathbf{D}_{\mathbf{U}}^m\left(\mathbf{X}^m(j-1)\right)$;

– simulation of aircraft flight trajectories and determination of sets of conditionally conflict-free states $\widehat{\mathbf{D}}_{\mathbf{X}}^m(j)$;

– determination of efficiency estimations $\Delta J_i^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)$ when transiting from states $\mathbf{X}^m(j-1)$ under the action of controls $\mathbf{U}^m(j-1)$.

The trajectories simulation is performed using the kinematics-energy model of the controlled aircraft motion proposed in article [19], which takes into account the dynamic properties of motion, aircraft performance characteristics stored in the EUROCONTROL Base of Aircraft Data (BADA), and allows to calculate the fuel consumption.

The efficiency estimations ΔJ_i^m for aircraft m are defied using expressions [15]:

$$\Delta J_1^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)=\begin{cases}0,\ j\ne k,\\ \left|t_k-t_f^m\right|,\ j=k,\end{cases}\quad(2)$$

$$\Delta J_2^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)=\begin{cases}0,\ j\ne k,\\ \left|h_k^m-h_f^m\right|,\ j=k,\end{cases}\quad(3)$$

$$\Delta J_3^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)=Q^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right),\quad(4)$$

$$\begin{gathered}\Delta J_4^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)=\lambda_{41}^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)+\\ +\lambda_{42}^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right)+\lambda_{43}^m\left(\mathbf{X}^m(j-1),\mathbf{U}^m(j-1)\right),\end{gathered}\quad(5)$$

$$\lambda_{41}^m=\begin{cases}1,\ \left|\varphi^m(j)-\varphi^m(j-1)\right|>\Delta\varphi,\\ 0,\end{cases}$$

$$\lambda_{42}^m=\begin{cases}1,\ V_a^m(j-1)\ne V^m(j-1),\\ 0,\end{cases}$$

$$\lambda_{43}^m=\begin{cases}1,\ V_{ha}^m(j-1)\ne V_{h0}^m,\\ 0,\end{cases}$$

where t_f^m – actual time of reaching the final state $\mathbf{X}_k^m$; h_k^m, h_f^m – planned and actual altitude of the control point overflight; Q^m – fuel consumption;

$\Delta\varphi$ – parameter that takes into account the small heading changes; V_{h0}^{m} – planned vertical speed.

To determine a set $\mathbf{E}(\mathbf{X}^m(j))$ of Pareto-optimal estimations of conflict-free trajectories when transiting into the state $\mathbf{X}^m(j)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(j)$ from states $\mathbf{X}^m(j-1)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(j-1)$ the following equation of multi-objective dynamic programming is used [15, 16]:

$$\mathbf{E}(\mathbf{X}^m(j))=\bigcup_{\mathbf{X}^m(j-1)\in\Pi(\mathbf{X}^m(j))}\left(\mathbf{E}(\mathbf{X}^m(j-1))\oplus\Delta\mathbf{J}^m\right),$$
$$\Delta\mathbf{J}^m=\{\Delta J_i^m(\mathbf{X}^m(j-1),\mathbf{U}'^m(j-1))\}, \qquad (6)$$
$$\mathbf{X}^m(j)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(j),\ \mathbf{X}^m(j-1)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(j-1),$$

where $\Pi(\mathbf{X}^m(j))$ – a set of states of aircraft m at the stage $(j-1)$ due to which the transition into the state $\mathbf{X}^m(j)$ is possible; $\mathbf{U}'^m(j-1)$ – are controls which allow an aircraft m to transit from the state $\mathbf{X}^m(j-1)\in \Pi(\mathbf{X}^m(j))$ into the state $\mathbf{X}^m(j)$.

At each stage, except the last one, sets of Pareto-optimal estimations of conflict-free trajectories $\mathbf{E}(\mathbf{X}^m(j))$ are determined using equation (6), corresponding sets $\mathbf{D}_{\mathbf{U}}^{\mathbf{E}}(\mathbf{X}^m(j))$ of Pareto-optimal controls and sets $\Pi_{\mathbf{E}}(\mathbf{X}^m(j))\in \Pi(\mathbf{X}^m(j))$ of Pareto-optimal states are determined.

Then the conflict-free combinations $\mathbf{I}(j)$ of states are determined. All conflict-free combinations at the stage j create the set $\mathbf{D}_{\mathbf{I}}(j)$. Let denote a conflict-free combination as follows:

$$\mathbf{I}(j)=\{\mathbf{X}^1(j),\mathbf{X}^2(j),\dots,\mathbf{X}^n(j)\}=\{\mathbf{X}^m(j)\},$$
$$\mathbf{X}^m(j)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(j).$$

Each combination contains the states which meet following conditions [17].

Condition 1. The violations of separation minima between all aircraft are absent in the states of combination $\mathbf{I}(j)$:

$$\forall a,b:((A\ge d_S)\wedge(B\ge h_S))\vee((A\ge d_S)\wedge(B<h_S))\vee$$
$$\vee((A<d_S)\wedge(B\ge h_S)),$$
$$A=d(\mathbf{X}^a(j),\mathbf{X}^b(j)),$$
$$B=\Delta h(\mathbf{X}^a(j),\mathbf{X}^b(j)),\mathbf{X}(j)\in \mathbf{I}(j),$$
$$a=\overline{1,n},\ b=\overline{1,n},\ a\ne b.$$

Condition 2. All aircraft have transited into states $\mathbf{X}^m(j)\in \mathbf{I}(j)$ under the action of Pareto-optimal controls $\mathbf{U}^m(j-1)\in \mathbf{D}_{\mathbf{U}}^{\mathbf{E}}(\mathbf{X}^m(j))$ from the states $\mathbf{X}^m(j-1)\in \Pi_{\mathbf{E}}(\mathbf{X}^m(j))$, which combinations where conflict-free at the previous stage:

$$\{\mathbf{X}^m(j-1)\}\in \mathbf{D}_{\mathbf{I}}(j-1),$$
$$\mathbf{X}^m(j-1)\in \Pi_{\mathbf{E}}(\mathbf{X}^m(j)),\ \mathbf{X}(j)\in \mathbf{I}(j).$$

Condition 3. Depending on the actual separation between two aircraft in states $\mathbf{X}^a(j)$ and $\mathbf{X}^b(j)$ the one of following conditions must be met:

– if $(A\ge d_S)\wedge(B\ge h_S)$ aircraft have transited into the states $\mathbf{X}^a(j)$, $\mathbf{X}^b(j)$ from any states, which meet the condition 2;

– if $(A\ge d_S)\wedge(B<h_S)$ aircraft have transited into the states $\mathbf{X}^a(j)$, $\mathbf{X}^b(j)$ from states, which meet the condition 2 and in which the horizontal and vertical separation or only horizontal separation between this aircraft is ensured;

– if $(A\ge d_S)\wedge(B<h_S)$ aircraft have transited into the states $\mathbf{X}^a(j)$, $\mathbf{X}^b(j)$ from states, which meet the condition 2 and in which the horizontal and vertical separation or only vertical separation between this aircraft is ensured.

Each combination $\mathbf{I}(j)$ determines the combinations of Pareto-optimal trajectories, which transfer the aircraft to the states of this combination. In general, each combination $\mathbf{I}(j)$ corresponds to several combinations of Pareto-optimal trajectories.

At the last stage k for each aircraft m the set $\mathbf{z}^m$ of full trajectories $\mathbf{T}^m=\{\mathbf{X}_0^m,\mathbf{X}^m(1),\dots\mathbf{X}_k^m\}$, which transfer the aircraft to the final state $\mathbf{X}_k^m$ from the states $\mathbf{X}^m(k-1)\in \widehat{\mathbf{D}}_{\mathbf{X}}^m(k-1)$, is determined. For this the special backward procedure is applied [15].

The combinations $\mathbf{Z}$ of full conflict-free trajectories of all aircraft are determined. Combination $\mathbf{Z}$ is a combination of trajectories, which transfer the all aircraft from states of combination $\mathbf{I}(k-1)$ to the final states $\mathbf{X}_k^m$:

$$\mathbf{Z}=\{\mathbf{T}^1,\mathbf{T}^2,\dots,\mathbf{T}^n\}=\{\mathbf{T}^m\},$$
$$\mathbf{T}^m\in \mathbf{z}^m:\mathbf{X}^m(k-1)\in \mathbf{T}^m,\mathbf{X}^m(k-1)\in \mathbf{I}(k-1).$$

Each possible combination $\mathbf{Z}$ of trajectories $\mathbf{T}^m$ is characterized by extended vector optimality criterion $\mathbf{J}_{\Sigma}(\mathbf{Z})$:

$$\mathbf{J}_\Sigma(\mathbf{Z})=\{\mathbf{J}^m(\mathbf{T}^m)\},\ \mathbf{J}^m(\mathbf{T}^m)\in\mathbf{E}(\mathbf{X}_k^m),\ m=\overline{1,n}.$$

Then the set $\mathbf{P}_\mathbf{Z}$ of Pareto-optimal combinations of conflict-free trajectories is determined:

$$\mathbf{P}_\mathbf{Z}=\{\mathbf{Z}_p \mid \neg\exists\ \mathbf{Z}:\mathbf{J}_\Sigma(\mathbf{Z})\le\mathbf{J}_\Sigma(\mathbf{Z}_p),\mathbf{Z}_p\ne\mathbf{Z}\}.$$

Selection of optimal $\mathbf{Z}^*$ combination of conflict-free trajectories from the set of Pareto-optimal $\mathbf{P}_\mathbf{Z}$ is performed with use of linear convolutions of optimality criteria by solving the optimization problem [20]:

$$\mathbf{Z}^*=\arg\min_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\max_{\mathbf{W}\in\mathbf{D}_w}\sum_{i=1}^{3}w_i\bar{c}_i(\mathbf{Z}),\ \mathbf{Z}\in\mathbf{P}_\mathbf{Z},\qquad(7)$$

where $\bar{c}_1$, $\bar{c}_2$, $\bar{c}_3$ – normalized estimations of combinations $\mathbf{Z}$ efficiency by flight regularity c_1, flight economy c_2 and the complexity of maneuvering c_3 criteria respectively with the domain of allowable values $\mathbf{D}_{\bar{c}}=\{\bar{c}\mid\bar{c}\in[0,1]\}$; w_i – the weighting coefficients reflecting the relative importance of criteria and forming a vector $\mathbf{W}=\{w_i\},\ i=\overline{1,3}$ with the domain of allowable values $\mathbf{D}_w$ and minimal value w_0:

$$\mathbf{D}_w=\left\{\mathbf{W}\left|\sum_{i=1}^{3}w_i=1;\ w_i\ge w_{i+1},\ i=\overline{1,2};\ w_3\ge w_0>0\right.\right\}.$$

The estimation $\bar{c}_1$ is the normalized linear convolution of the normalized total deviation from the planned flight time and the normalized total deviation from planned altitude of all aircraft:

$$\bar{c}_1(\mathbf{Z})=\frac{c_\Sigma(\mathbf{Z})-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}c_\Sigma(\mathbf{Z})}{\max\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}c_\Sigma(\mathbf{Z})-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}c_\Sigma(\mathbf{Z})},$$

$$c_\Sigma(\mathbf{Z})=0{,}5\frac{\sum\limits_{m=1}^{n}J_1^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_1^m(\mathbf{T}^m)}{\max\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_1^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_1^m(\mathbf{T}^m)}+,$$

$$+0{,}5\frac{\sum\limits_{m=1}^{n}J_2^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_2^m(\mathbf{T}^m)}{\max\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_2^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_2^m(\mathbf{T}^m)},\ \mathbf{T}^m\in\mathbf{Z}.$$

The normalized estimations $\bar{c}_2$ and $\bar{c}_3$ are defined as follows:

$$c_i(\mathbf{Z})=\frac{\sum\limits_{m=1}^{n}J_{i+1}^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_{i+1}^m(\mathbf{T}^m)}{\max\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_{i+1}^m(\mathbf{T}^m)-\min\limits_{\mathbf{Z}\in\mathbf{P}_\mathbf{Z}}\sum\limits_{m=1}^{n}J_{i+1}^m(\mathbf{T}^m)},$$

$$\mathbf{T}^m\in\mathbf{Z},\ i=\overline{2,3}.$$

5. Discretization aspects

The conflict resolution process is decomposed into k stages. Discretization step Δt is defined taking into account values of possible controls. The first requirement is the stabilization of assigned airspeed during time interval Δt.

To ensure separation minima between aircraft while they transiting between states it is necessary to check the violations at fixed time moments in limited time periods during which an unobservable violations of separation cannot occur.

When the changes of heading, airspeed speed and vertical speed are used to avoid a conflict with crossing angle of initial tracks $\le 90^\circ$ the discretization step Δt is determined according to inequalities system:

$$\begin{cases}\Delta t<d_s/V_{\max},\\ \Delta t<\Delta h_s/V_{h\max},\end{cases}$$

where V_{max} – maximum ground speed of all aircraft; $V_{h\max}$ – maximum vertical speed of all aircraft.

The number of stages is defined using following expression:

$$k=\left[\frac{\sum\limits_{m=1}^{n}(t_m-t_0)}{2\Delta t\cdot n}+\frac{\sum\limits_{p=1}^{q}(t_{conf}^p-t_0)}{2\Delta t\cdot q}\right],$$

where t_m – planned time of exit from ATC area for aircraft m; t_{conf}^p – center of the time interval when the conflict between a pair of aircraft $p=\overline{1,q}$ is existed; q – number of aircraft pairs; $[\cdot]$ – rounding operator.

The discretization of states and controls is performed using following procedure based on approach that was described in [14].

Generally, it is assumed that each aircraft can make a left/right turn with bank angle γ or to do not change the heading; to increase/decrease the airspeed on ΔV or to do not change it; to increase/decrease the vertical speed on ΔV_h or to do not change it. Let $\mathbf{D}_{\mathbf{U}0}(\mathbf{X}(j-1))$ be the basic set that contains all possible 27 combinations of controls. When applying controls $\mathbf{U}(j-1)\in \mathbf{D}_{\mathbf{U}0}(\mathbf{X}(j-1))$, an aircraft transits into different states $\mathbf{X}'(j)$ with efficiency estimations $\Delta J_i'(\mathbf{X}(j-1),\mathbf{U}(j-1))$ that are defined using expressions (2)-(5). The rule for formation of new states $\mathbf{X}(j)$ which combine states $\mathbf{X}'(j)$ is defined.

At the stage $j=1$ all states $\mathbf{X}'(1)$ are new $\mathbf{X}(1)=\mathbf{X}'(1)$ and efficiency estimations of transition to new states $\mathbf{X}(1)$ are equal to $\Delta J_i'(\mathbf{X}_0,\mathbf{U}(0))$.

At the stages $j=\overline{2,k-1}$ the new states $\mathbf{X}(j)$ are formed as a combination of states $\mathbf{X}'(j)$ in which aircraft altitudes are equal, horizontal coordinates and headings have proximate values (proximity criteria are adjustable). It means that an aircraft can transit into the state $\mathbf{X}(j)$ under the action of several controls $\mathbf{U}'(j-1)$. As a result the set $\Pi(\mathbf{X}(j))$ of states at the stage $(j-1)$ from which an aircraft can transit into the state $\mathbf{X}(j)$ is defined.

Coordinates and heading of an aircraft in new state $\mathbf{X}(j)$ are determined as arithmetic mean of these parameters for the states $\mathbf{X}'(j)$ which are combined in this new state $\mathbf{X}(j)$.

Estimations $\Delta J_i(\mathbf{X}(j-1),\mathbf{U}'(j-1))$ when transiting into new states $\mathbf{X}(j)$ from the states of the set $\Pi(\mathbf{X}(j))$ is determined using nearest-neighbor interpolation of values $\Delta J_i'(\mathbf{X}(j-1),\mathbf{U}(j-1))$ for states $\mathbf{X}'(j)$ which are combined:

$$\Delta J_i(\mathbf{X}(j-1),\mathbf{U}'(j-1))=\Delta J_i'(\mathbf{X}(j-1),\mathbf{U}(j-1)),$$
$$\mathbf{X}(j-1)\in \Pi(\mathbf{X}(j)),\ \mathbf{U}'(j-1)=\mathbf{U}(j-1).$$

6. Computer simulation

The conflict situation between three aircraft Boeing 737-800 was simulated. The lateral separation minimum is equal to $d_s=18{,}5$ km (10 nautical miles) and the vertical separation minimum is equal to $\Delta h_S=300$ m (1000 feet). The initial parameters of aircraft flight are represented in Table 1. The characteristics of predicted multi-aircraft conflict are represented in Table 2.

It was assumed that to avoid the conflict all aircraft should make manoeuvres. The process was discretized in time on $k=5$ stages. The discretization step for the first 4 stages is equal to $\Delta t=59$ s.

It was assumed that aircraft №1 and №2 can change heading and airspeed; aircraft №3 can change heading and vertical speed. The bank angle during turns is equal to 20°, turning time is limited to 15 s. During transition between stages the absolute value of airspeed change is equal to 5 m/s and the absolute value of vertical speed change is equal to 4 m/s.

The minimal value of weighting coefficients in the optimization problem (7) is equal to $w_0=0{,}1$.

The optimal combination of conflict-free trajectories is shown in Fig. 1. The dependences of horizontal and vertical distance between aircraft from time are shown in Fig. 2 and Fig. 3. The assigned airspeeds and vertical speed for conflict resolution are represented in Table 3. The efficiency parameters of optimal combination of trajectories are represented in Table 4.

Table 1

The initial parameters of aircraft flight

Parameter	Aircraft		
	№1	№2	№3
Heading, degrees	0	80	220
Airspeed, m/s	225	200	220
Vertical speed, m/s	0	0	-6
Initial coordinates $(x_0;y_0)$, km	(40; 0)	(0; 30)	(55,7; 60,6)
Initial altitude, m	10050	10050	11350
Assigned altitude, m	10050	10050	9150
Distance to the control point, km	80	75	80
Planned time of control point overflight, s	356	375	364

Table 2

The characteristics of predicted multi-aircraft conflict

Parameter	Pair of aircraft		
	№1, №2	№1, №3	№2, №3
Time interval of separation violation, s	[118, 245]	[167, 191]	[167, 207]
Predicted minimum horizontal distance between aircraft, m	6215	9395	3397
Predicted minimum vertical distance between aircraft, m	0	154	58

Table 3

The assigned airspeeds and vertical speed for conflict resolution

Stage	1	2	3	4	5
Airspeed of aircraft №1, m/s	225	225	225	230	230
Airspeed of aircraft №2, m/s	200	200	205	210	210
Airspeed of aircraft №3, m/s	220	220	220	220	220
Vertical speed of aircraft №3, m/s	-2	-2	-10	-10	-10

Table 4

The efficiency parameters of optimal combination of conflict-free flight trajectories

Parameter	Aircraft		
	№1	№2	№3
Deviation from the planned flight time, s	4,5	9	19,4
Deviation from the assigned altitude at control point, m	–	–	20
Additional fuel consumption, %	-2,15	5,6	10,3
Number of flight profile changes	4	7	5

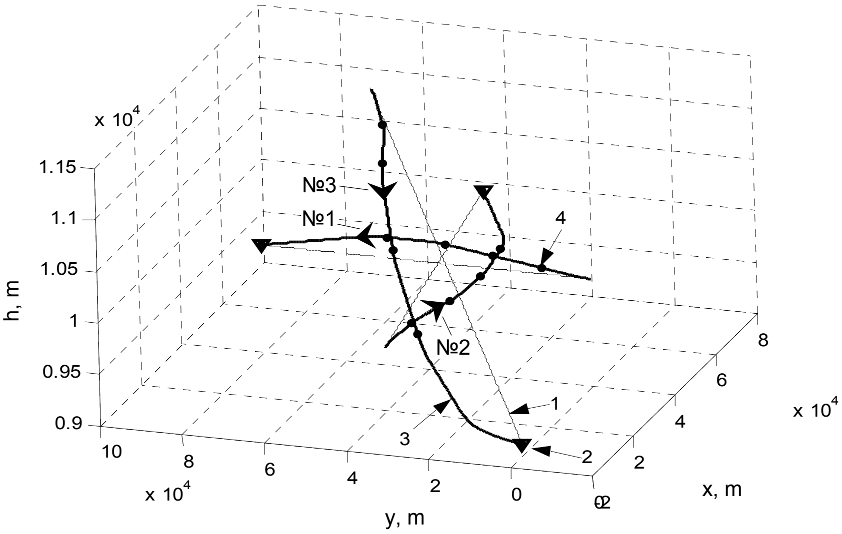

Fig. 1. The aircraft trajectories in three-dimensional space:
1 – planed trajectories; 2 – control points on the routes; 3 – optimal conflict-free trajectories; 4 – states at the stages.

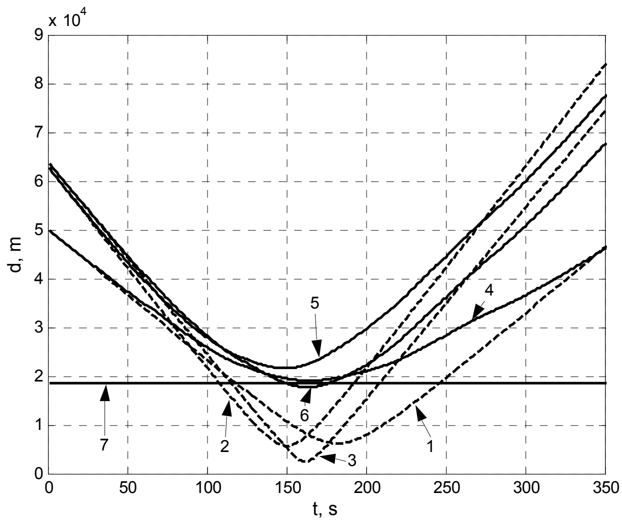

Fig. 2. The dependence of horizontal distances between aircraft from time:
1, 2, 3 – between aircraft №1 and №2, №1 and №3, №2 and №3 respectively during flight by planed trajectories; 4, 5, 6 – between aircraft №1 and №2, №1 and №3, №2 and №3 respectively during conflict resolution; 7 – separation minimum.

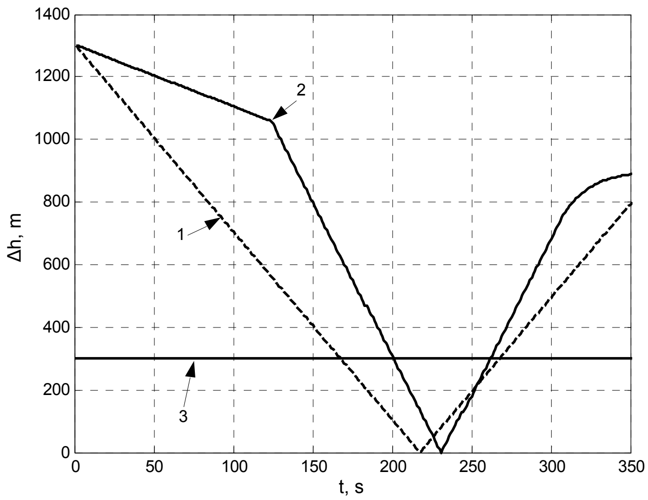

Fig. 3. The dependence of vertical distances between aircraft №1 and №2, №2 and №3 from time: 1 – during flight by planed trajectories; 2 – during conflict resolution; 3 – separation minimum.

7. Conclusions

The method for optimal resolution of multi-aircraft conflicts in three-dimensional space has been developed. Method provides the synthesis of optimal combination of conflict-free flight trajectories of aircraft that use heading, airspeed and vertical speed change maneuvers according to flight regularity, economy and the complexity of maneuvering criteria. The synthesis of Pareto-optimal combinations of conflict-free trajectories is carried out using the multi-objective dynamic programming and special procedure for determination of combinations of aircraft conflict-free states. The selection of optimal combination of conflict-free trajectories from the set of Pareto-optimal is carried out using the convolution of optimality criteria.

The correctness, adequacy and efficiency of proposed method were proved by computer simulation.

Developed method can be used for development of new automated ATC systems, airborne collision avoidance systems (ACAS), intelligent ATC simulators and for research activities.

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Метод оптимального розв'язання групових конфліктних ситуацій між повітряними суднами у тривимірному просторі

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Мета: В сучасних умовах високої динамічності і щільності повітряного руху значно зростає небезпека критичного зближення відразу декількох повітряних суден і виникнення групових конфліктних ситуацій. Актуальною проблемою є розробка методів оптимального розв'язання групових конфліктних ситуацій, які повинні забезпечувати синтез безконфліктних траєкторій у тривимірному просторі. **Методи:** Розроблено метод оптимального розв'язання групових конфліктних ситуацій із застосуванням маневрування зміною курсу, швидкості та висоти польоту. Критеріями оптимальності визначені регулярність, економічність польотів і складність маневрування. Метод полягає у послідовному синтезі множини Парето-оптимальних комбінацій безконфліктних траєкторій сукупності повітряних суден з використанням дискретного багатокритеріального динамічного програмування та визначенні оптимальної комбінації із застосуванням із застосуванням згортки критеріїв оптимальності. В рамках методу визначено: процедуру формування безконфліктних комбінацій станів сукупності повітряних суден, які визначають комбінації Парето-оптимальних траєкторій; обмеження, які накладаються при дискретизації процесу розв'язання конфлікту для забезпечення відсутності неспостережуваних порушень ешелонування. **Результати:** Дослідження запропонованого методу виконано шляхом комп'ютерного моделювання, результати якого показали, що синтезована комбінація безконфліктних траєкторія забезпечує усунення групової конфліктної ситуації та відповідає встановленим критеріям оптимальності. **Обговорення:** Запропонований метод може бути використаний при розробці нових автоматизованих систем управління повітряним рухом, бортових систем попередження зіткнень, інтелектуальних тренажерів керування повітряним рухом та для проведення наукових досліджень.

Ключові слова: багатокритеріальна оптимізація; безпека польотів; групова конфліктна ситуація між повітряними суднами; динамічне програмування; повітряне судно; розв'язання конфліктної ситуації; управління повітряним рухом

Д.В. Васильев. Метод оптимального разрешения групповых конфликтных ситуаций между воздушными судами в трехмерном пространстве

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Цель: В современных условиях высокой динамичности и плотности воздушного движения значительно возрастает опасность критического сближения сразу нескольких воздушных судов и возникновения групповых конфликтных ситуаций. Актуальной проблемой является разработка методов оптимального разрешения групповых конфликтных ситуаций, которые должны обеспечивать синтез бесконфликтных траекторий в трехмерном пространстве. **Методы:** Разработан метод оптимального разрешения групповых конфликтных ситуаций с применением маневрирования по изменению курса, скорости и высоты полета. Критериями оптимальности определены регулярность, экономичность полетов и сложность маневрирования. Метод заключается в последовательном синтезе множества Парето-оптимальных комбинаций бесконфликтных траекторий совокупности воздушных судов с использованием дискретного многокритериального динамического программирования и определении оптимальной комбинации с использованием свертки критериев оптимальности. В рамках метода определены: процедура формирования бесконфликтных комбинаций состояний совокупности воздушных судов, определяющих комбинации Парето-оптимальных траекторий; ограничения, которые накладываются при дискретизации процесса разрешения конфликта для обеспечения отсутствия ненаблюдаемых нарушений эшелонирования. **Результаты:** Исследование предложенного метода выполнено путем компьютерного моделирования, результаты которого показали, что синтезированная комбинация бесконфликтная траектория обеспечивает устранение групповой конфликтной ситуации и соответствует установленным критериям оптимальности. **Обсуждение:** Предложенный метод может быть использован при разработке новых автоматизированных систем управления воздушным движением, бортовых систем предупреждения столкновений, интеллектуальных тренажеров управления воздушным движением и для проведения научных исследований.

Ключевые слова: безопасность полетов; воздушное судно; групповая конфликтная ситуация между воздушными судами; динамическое программирование; многокритериальная оптимизация; разрешение конфликтной ситуации; управление воздушным движением

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