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TWO-LEVEL OPTIMAL CONTROL DESIGN

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The method of analytical design of the two-level optimal control algorithm is considered here together with using of the functional of the work generalized. The method offered may be used not only at the stage of preliminary synthesis of branching trajectories but for operative trajectories optimization of compound dynamic systems as well. The method of two-level tasks solution considered here may be extended to other objects of control.

Introduction

The methods of branching trajectories movement optimization of compound aerospace system require lots of essential computing resources for their realization as stated in [1].

Being offered in A.A. Krasovsky's work [2–4] and then advanced in the works by V.N. Bukov [5], V.S. Shendrick [6; 7] and others, the method of analytical design of control algorithms with using of the generalized functional work is much easier in computing relations that allows to use it not only at the stage of preliminary synthesis of branching trajectories but trajectories optimization of compound dynamic systems with the help of on board computers during normal system functioning.

Task setting

Task of automatic control considered below, wile be referred to as two-level one. The essence of this task is in the following. There is a dynamic system:

$$\dot{x} = f(x, u, w_1, t), \qquad (1)$$

where $x \in E^n, u \in E^{m_u}, w_1 \in E^{m_1}$, "u" and "w" are the vector controls that are to be transferred from the variety

$$(x(t_0), t_0) \in Q_0$$

to the variety

$$(x(t_f), t_f) \in Q_f,$$

so that to minimize the criterion

$$I = S(x(t_f), t_f) + \int_{t_0}^{t_f} L(x, u, w_1, w_2, t)dt \to \min_{u(\cdot)}, \qquad (2)$$

where w_2 is in general case the matrix control functions of "$m_2 \times m_2$" size, provided that $w_1 = w_1(\hat{x}, \hat{u}, t)$, $w_2 = w_2(\hat{x}, \hat{u}, t)$ are the known functions of tine and optimum values of phase coordinates and control, that is:

$$I = I(x(t_0), t_0; x(t_f), t_f; u(\cdot), x(\cdot))\Big|_{w_i(\hat{x}, \hat{u}, t)(i=1,2)}. \qquad (3)$$

The main idea

The task put will be referent to as the first level task. Its solution by the method based on the principle of minimum gives the equations [8–10]:

$$\partial H/\partial u\big|_{\wedge} = 0; \qquad (4)$$

$$\dot{\lambda} + \partial H/\partial x\big|_{\wedge} = 0; \qquad (5)$$

$$\dot{x} = f(\hat{x}, \hat{u}, w_1(\hat{x}, \hat{u}, t), t); \qquad (6)$$

$$(\hat{x}(t_0), t_0) \in Q_0, (\hat{x}(t_f), t_f) \in Q_f, \qquad (7)$$

where

$$H(\hat{x}, \hat{u}, \lambda, t) =$$

$$= \left[L(\hat{x}, \hat{u}, w_1, w_2, t) + \lambda^T f(\hat{x}, \hat{u}, w_1, t)\right]\Big|_{W_i(\hat{x}, \hat{u}, t)};$$

$i = (1, 2)$.

And method of dynamic programming the decision is described by the equations [10]

$$\frac{\partial}{\partial u}\left[L(\hat{x}, u, w_1, w_2 t) + \left(\frac{\partial V}{\partial t}\right)^T \Big|_{\wedge} f(\hat{x}, u, w_1, t)\right] = 0; \qquad (8)$$

$$\frac{\partial V}{\partial t} L(\hat{x}, \hat{u}, w_1, w_2 t) + \left(\frac{\partial V}{\partial t}\right)^T \Big|_{\wedge} f(\hat{x}, \hat{u}, w_1, t); \qquad (9)$$

$$V(\hat{x}(t_f), t_f) = S(\hat{x}(t_f), t_f); \qquad (10)$$

$$\dot{x} = f(\hat{x}, \hat{u}, w_1, t); \qquad (11)$$

$$(\hat{x}(t_0), t_0) \in Q_0, (\hat{x}(t_f), t_f) \in Q_f, \qquad (12)$$

where

$$V(x(t)t) =$$

$$= \left[\min_{u(\cdot)} \int_t^{t_f} L(x, u, w_1, w_2, t)dt + S(x(t_f), t_f)\right]_{W_i(\hat{x}, \hat{u}, t)(i=1,2)}.$$

The second level task consists of the equations. Its solution is in the choice of the equations $w(\hat{x}, \hat{u}, t)(i = 1,2)$ that would provide advantageous properties to the equations (4)–(7) or (8)–(12). Let's consider some approaches allowing to give substantial interpretation to the second level of a task.

1. Assume that – the dynamic system (1) has controls of various efficiency one part of which can be brought to the vector u, and another part – to the w_1.

Solving a task at the second level, we shall search for such controls w_1, which would provide the most favorable conditions to use the first level controls u.

So, for example the controls determining the object configuration can be considered as "w_1". The object is appeared to be adjusted with the help to w_1 for it's control with the "u" to be the most effective.

2. The control of w_2 influences the equations calculating the control "u" and this control is given some additional properties that, in the end, influence the character of system movement (1). Using the concept of control w_2 one is able to solve, on the one hand, the choice problem of weight factors of the functional (2), when considering these factors as control w_2 and, on the other hand it is possible to control the equations complexity, by which the control u is calculated. In particular, the latter makes sense in the case of inexact information about control objects parameters determining optimum control.

Besides, such an approach can be considered as one of the ways of realization of a reduction of mathematical models of optimum processes (6) or [11–15].

3. When the tasks are put incorrectly it makes sense to use control w_2 as the stabilizing addition to functional, that will help organize steady procedure of the approximate finding of optimum control [15].

The way of search $w(\hat{x},\hat{u},t)(i=1,2)$ essentially depends on initial setting of functional and object model. Let's consider some results received when casing the second approach in the case, when

$$f(x,u.w_1,t)=\varphi(x,t)+\psi(x,u,w_1,t); \quad (13)$$

$$L(x,u,w_1,w_2,t)=Q(x,t)+\Omega(x,u,w_1,w_2,t). \quad (14)$$

Hamiltonian becomes such:

$$H(x,u,w_1,w_2,\lambda,t)=Q(x,t)+\Omega(x,u,w_1,w_2,t)+$$
$$+\lambda^T\varphi(x,t)+\lambda^T\psi(x,u,w_1,t). \quad (15)$$

We investigate it on a minimum on u:

$$\left(\frac{\partial H}{\partial u}\right)^T=\left(\frac{\partial \Omega}{\partial u}\right)^T+\lambda^T\frac{\partial\psi}{\partial u}=0. \quad (16)$$

From a parity (16) we receive, that

$$\lambda^T\psi=-\left(\frac{\partial \Omega}{\partial u}\right)^T\left(\frac{\partial \psi}{\partial u}\right)^{-1}\psi.$$

After substitution $\lambda^T\psi$ in (15), we find expression for minimized hamiltonian

$$H(x,u,w_1,w_2,\lambda,t)=Q(x,t)+\Omega(x,u,w_1,w_2,t)+$$
$$+\lambda^T\varphi(x,t)-\left(\frac{\partial \Omega}{\partial u}\right)^T\left(\frac{\partial \psi}{\partial u}\right)^T\psi=Q(x,t)+\lambda^T\varphi(x,t), \quad (17)$$

where the control w_1 and w_2 are chosen so that the equality would be carried out:

$$\Omega(x,u,w_1,w_2,t)-\left(\frac{\partial \Omega}{\partial u}\right)^T\left(\frac{\partial \psi}{\partial u}\right)^{-1}\psi=0. \quad (18)$$

Taking into account (13), (14), (17), (18), the result of the solution of the second level task set by the equations (4)–(7) or (8)–(12), will be written down respectively:

$$\left.\left(\frac{\partial \Omega}{\partial u}\right)^T\right|_{\wedge}+\lambda^T\left.\frac{\partial\psi}{\partial u}\right|_{\wedge}=0; \quad (19)$$

$$\dot{\lambda}+\left.\frac{\partial Q}{\partial u}\right|_{\wedge}+\lambda^T\left.\frac{\partial\varphi}{\partial u}\right|_{\wedge}=0; \quad (20)$$

$$\dot{\hat{x}}=\varphi(\hat{x},t)+\psi(\hat{x},\hat{u},w_1,t)=0; \quad (21)$$

$$(\hat{x}(t_0),t_0)\in Q_0,(\hat{x}(t_f),t_f)\in Q_f; \quad (22)$$

$$\Omega(\hat{x},\hat{u},w_1,w_2,t)-\left.\left(\frac{\partial \Omega}{\partial u}\right)^T\right|_{\wedge}\left.\left(\frac{\partial \psi}{\partial u}\right)^{-1}\right|_{\wedge}\psi|_{\wedge}=0; \quad (23)$$

$$\left.\left(\frac{\partial \Omega}{\partial u}\right)^T\right|_{\wedge}+\left.\left(\frac{\partial V}{\partial x}\right)^T\right|_{\wedge}\left.\left(\frac{\partial \psi}{\partial u}\right)^T\right|_{\wedge}-\left.\left(\frac{\partial V}{\partial t}\right)\right|_{\wedge}=$$
$$=Q(\hat{x},t)+\left.\left(\frac{\partial V}{\partial x}\right)^T\right|_{\wedge}\varphi(\hat{x},t); \quad (24)$$

$$V(\hat{x}(t_f),t_f)=S(\hat{x}(t_f),t_f); \quad (25)$$

$$\dot{\hat{x}}=\varphi(\hat{x},t)+\psi(\hat{x},\hat{u},w_1,t)=0; \quad (26)$$

$$(\hat{x}(t_0),t_0)\in Q_0,(\hat{x}(t_f),t_f)\in Q_f; \quad (27)$$

$$\Omega(\hat{x},\hat{u},w_1,w_2,t)-\left.\left(\frac{\partial \Omega}{\partial u}\right)^T\right|_{\wedge}\left.\left(\frac{\partial \psi}{\partial u}\right)^{-1}\right|_{\wedge}\psi|_{\wedge}=0, \quad (28)$$

where $\lambda=\left.\left(\frac{\partial V}{\partial x}\right)\right|_{\wedge}$ is specified.

Note, that the equations (19)–(23) or (18)–(28) are the solutions of the two-level task as a whole.

It should be noted that the further constructive results depend on the kind of functions: $\Omega(x,u,w_1,w_2,t)$ and $\psi(x,u,w_1,t)$. So for example A.A. Krasovsky was the first to study /issue/ the two-level assuming that [2; 3]:

$$\Omega(x,u,w_1,w_2,t)=p(u,t)+w_2(t);$$

$$\psi(x,u,w_1,t)=\mu(x,t)u.$$

Depending on the various types of objects of control, the functions $\Omega(x,u,w_1,w_2,t)$ and $\psi(x,u,w_1,t)$. can differ greatly from (20), (21). However, method of the solution of the two-level tasks problems given in the article applied to extended on these cases as well.

Conclusion

The solution method of the two-level optimum tasks considered in this article allows to simplify the analytical design of control algorithms with the functional of the work generalized.

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Конструювання дворівневого оптимального керування

Розглянуто метод аналітичного конструювання алгоритмів дворівневого оптимального керування з використанням функціонала узагальненої роботи. Показано, що запропонований метод можна застосовувати не тільки на етапі попереднього синтезу розгалужених траєкторій, але і для оптимізації траєкторій складених динамічних систем. Розглянутий метод розв’язання дворівневих задач може бути розповсюджений на інші об’єкти керування.

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Конструирование двухуровневого оптимального управления

Рассмотрен метод аналитического конструирования алгоритмов двухуровневого оптимального управления с использованием функционала обобщенной работы. Показано, что предложенный метод можно применять не только на этапе предварительного синтеза ветвящихся траекторий, но и для оперативной оптимизации траекторий составных динамических систем. Рассматриваемый метод решения двухуровневых задач может быть распространен на другие объекты управления.