UDC 656.7.084:351.814.339:519.8(045)
10.18372/2306-1472.68.10907

Denys Vasyliev

METHOD OF MULTI-OBJECTIVE RESOLUTION OF TWO-AIRCRAFT CONFLICT IN THREE-DIMENSIONAL SPACE BASED ON DYNAMIC PROGRAMMING

Ukrainian State Air Traffic Services Enterprise (UkSATSE)
Airport, 08307, Boryspil, Kyiv region, Ukraine
National Aviation University
1, Kosmonavta Komarova Avenue, 03058, Kyiv, Ukraine
E-mail: dvasyliev@nau.edu.ua

Abstract

Purpose: *Current global trends of air traffic growth cause the increasing of number of aircraft conflicts. The actual problem is a development of new methods for conflict resolution that should provide the synthesis of conflict-free trajectories in three-dimensional space according to different flight efficiency criteria.* ***Methods:*** *The problem of multi-objective resolution of potential conflict between two aircraft in three-dimensional space is considered. The method of multi-objective resolution of conflict between two aircraft using heading, speed and altitude change maneuvers has been developed. Described method provides the synthesis of conflict-free flight trajectory according to criteria of flight regularity, flight economy and the complexity of maneuvering based on dynamic programming. The continuous-time and discrete-time equations of multi-objective dynamic programming for determining the set of Pareto-optimal estimations of conflict-flight trajectories are shown. The synthesis of Pareto-optimal trajectories is carried out using the forward procedure of discrete multi-objective dynamic programming. The simulation of flight trajectories is performed using the special model of controlled aircraft motion. The selection of optimal conflict-free trajectory from the set of Pareto-optimal trajectories is carried out using the convolution of optimality criteria. Within described method, the following procedures have been defined: for prediction of separation minima violations; for aircraft states and controls discretization; for interpolation of trajectories efficiency estimations according to defined optimality criteria.* ***Results:*** *The analysis of the proposed method is performed using computer simulation which results show that computed optimal conflict-free trajectory ensures the conflict avoidance and complies with defined optimality criteria.* ***Discussion:*** *The main advantages of the method are: heading, speed and altitude change maneuvers are used for conflict avoidance; the multi-objective optimization of conflict-free trajectories is applied; the using of dynamic programming enhances the computational efficiency. Proposed method can be used for development of advanced conflict resolution tools for automated air traffic control systems.*

Keywords: aircraft; air traffic control; conflict resolution; dynamic programming; flight safety; multi-objective optimization.

1. Introduction

Current global trends of air traffic growth cause the increasing of separation minima infringements, i.e. conflict situations in which prescribed separation minima were not maintained between aircraft. Therefore, the actual problem is improvement of air traffic control (ATC) methods and decision support systems for aircraft conflicts resolution.

Taking into account the current strict requirements to flight efficiency (regularity, economy etc.) a conflict-free trajectories synthesis should be considered as a multi-objective optimization problem.

2. Analysis of researches and publications

Most of the known methods of aircraft conflicts resolution do not provide a comprehensive solution of the problem and are not used in ATC. Particularly, the main disadvantage of force fields methods [1-4] is that the synthesized conflict-free trajectories are complex or even impossible for real aircraft. The optimization methods developed in [5-7] consider a single optimality criterion (objective) that narrows the range of possibilities for conflict resolution and does not allow finding the most effective solution in general. The optimization methods proposed in [8, 9] use only speed change for conflict avoidance. The common disadvantage of considered force fields methods and optimization methods proposed in [5, 8, 9] is that they are applied to conflict resolution only in the horizontal plane, when the largest number of conflicts occur between aircraft, at least one of which is climbing or descending [10, 11]. Also, all considered methods do not use a combination of heading, speed and altitude change maneuvers to avoid a conflict.

Thus, it is necessary to develop the new methods of multi-objective conflict resolution that should provide the synthesis of conflict-free trajectories using heading, speed and altitude change maneuvers in three-dimensional space according to flight efficiency criteria. A promising approach to the synthesis of conflict-free flight trajectories using multi-objective dynamic programming was considered in article [12].

3. Problem statement

The problem of multi-objective resolution of potential conflict between two aircraft in three-dimensional space is considered.

Conflict resolution is a controlled process and aircraft are the dynamic system $\mathbf{S}$. One aircraft changes heading, airspeed and vertical speed to avoid the conflict, second aircraft flies according to planned trajectory.

The process of maneuvers synthesis is observed in the time interval $[t_0, t_k]$ where t_0 is the moment of detection of a potential conflict, t_k is the planned time of aircraft exit from an ATC area.

Controlled motion of the maneuvering aircraft is described using the vector differential equation:

$$\dot{\mathbf{X}}(t) = f(\mathbf{X}(t), \mathbf{U}(t), t),\ \mathbf{X}(t_0) = \mathbf{X}_0,$$

where $\mathbf{X} = [x \quad y \quad h \quad V \quad V_h \quad \varphi]^{\mathrm{T}}$ – state vector; x, y – horizontal coordinates; h – altitude; V – true airspeed; V_h – vertical speed; φ – heading; $\mathbf{U} = [\gamma_a \quad V_a \quad V_{ha}]^{\mathrm{T}}$ – vector of controls; γ_a – assigned bank angle; V_a – assigned true airspeed; V_{ha} – assigned vertical speed.

An absolute constraint is the flight safety ensured by separation minima maintenance. The state $\mathbf{X}(t)$ belongs to the set of conflict-free states $\mathbf{D}_{\mathbf{X}}(t)$ if the separation minima are not violated:

$$\mathbf{X}(t) \in \mathbf{D}_{\mathbf{X}}(t) | \mathbf{X}(t) \notin \Omega(t),$$

where $\Omega(t)$ – the space of a conflict:

The space of a conflict $\Omega(t)$ is a space of states where the separation minima are violated:

$$\Omega(t) = \{\mathbf{X}(t) | (d(\mathbf{X}(t), \mathbf{X}_{ref}(t)) < d_S) \wedge (\Delta h(\mathbf{X}(t), \mathbf{X}_{ref}(t)) < h_S)\},$$

where $d(\mathbf{X}(t), \mathbf{X}_{ref}(t))$ – horizontal distance between aircraft; $\mathbf{X}_{ref}(t)$ – state of the second aircraft; $\Delta h(\mathbf{X}(t), \mathbf{X}_{ref}(t))$ – vertical distance between; d_S – lateral (horizontal) separation minimum; h_S – vertical separation minimum. The initial state is conflict-free $\mathbf{X}(t_0) \notin \Omega(t_0)$.

Controls are limited according to the aircraft performances. Limitations are dependent on states and time:

$$\mathbf{U}(t) \in \mathbf{D}_{\mathrm{U}}(\mathbf{X}(t), t),$$

where $\mathbf{D}_{\mathrm{U}}(\mathbf{X}(t), t)$ – a set of possible controls $\mathbf{U}(t)$ in a state $\mathbf{X}(t)$.

Optimality criteria characterizing the efficiency of conflict resolution are flight regularity c_1, flight economy c_2 and the complexity of maneuvering c_3. The numerical estimations of trajectories according to defined optimality criteria are: $J_1(\mathbf{X}(t), \mathbf{U}(t), t)$ – deviation from planned flight time; $J_2(\mathbf{X}(t), \mathbf{U}(t), t)$ – deviation from planned altitude; $J_3(\mathbf{X}(t), \mathbf{U}(t), t)$ – fuel consumption; $J_4(\mathbf{X}(t), \mathbf{U}(t), t)$ – number of flight profile changes.

The numerical estimations are defined at the time moment t_k as follows:

$$J_1 = \Lambda_1(\mathbf{X}(t_k), t_k),$$

$$J_2 = \Lambda_2(\mathbf{X}(t_k), t_k),$$

$$J_3 = \int_{t_0}^{t_k} \lambda_3(\mathbf{X}(t), \mathbf{U}(t), t)dt + \Lambda_3(\mathbf{X}(t_k), t_k),$$

$$J_4 = \int_{t_0}^{t_k} \lambda_4(\mathbf{X}(t), \mathbf{U}(t), t)dt + \Lambda_4(\mathbf{X}(t_k), t_k),$$

where Λ_1– estimation of deviation from planned flight time; Λ_2,– estimation of deviation from planned altitude; λ_3 – instantaneous fuel consumption; λ_4 – speed of flight profile changes; Λ_3 – estimation of fuel consumption for real exit from an ATC area relatively to actual position at the time moment t_k; Λ_4 – estimation of flight profile changes for real exit from an ATC area relatively to actual position at the time moment t_k.

The function λ_4 is defined as the sum of piecewise-defined functions:

$$\lambda_4(\mathbf{X}(t),\mathbf{U}(t),t)=\lambda_{41}(t)+\lambda_{42}(t)+\lambda_{43}(t),$$

$$\lambda_{41}(t)=\begin{cases}0, \gamma_з(t)=0,\\ 1, \gamma_з(t)\neq 0,\end{cases}$$

$$\lambda_{42}(t)=\begin{cases}0, V(t)=V_a(t),\\ 1, V(t)\neq V_a(t),\end{cases}$$

$$\lambda_{43}(t)=\begin{cases}0, V_h(t)=V_{h0},\\ 1, V_h(t)\neq V_{h0},\end{cases}$$

where V_{h0} – planned vertical speed.

Estimations constitute the vector $\mathbf{J}(\mathbf{X}(t),\mathbf{U}(t),t)=\{J_i(\mathbf{X}(t),\mathbf{U}(t),t)\},\ i=\overline{1,4}$.

As a result the problem of multi-objective conflict resolution is determined as follows:

$$\min_{\mathbf{U}(t)\in\mathbf{D}_\mathrm{U}(\mathbf{X}(t),t)}\mathbf{J}(\mathbf{X}(t),\mathbf{U}(t),t),\ \mathbf{X}(t)\in\mathbf{D}_\mathbf{X}(t),\ t\in[t_0,t_k].\quad(1)$$

The **aim** of this article is to develop the method of multi-objective resolution of two-aircraft conflicts in three-dimensional space based on dynamic programming.

4. Method of multi-objective conflict resolution

It is purposed to solve the problem (1) in two steps:

1) as the problem (1) is a problem of optimal control of dynamic system $\mathbf{S}$ the first step is a synthesis of a set of Pareto-optimal conflict-free trajectories using dynamic programming method;

2) second step is a selection of optimal conflict-free trajectory from a set of Pareto-optimal trajectories.

The basis of dynamic programming is the Bellman's principle of optimality [13]. In case of multi-objective optimization the optimality principle is formulated as follows – a segment from any point to the end of a Pareto-optimal trajectory is a Pareto-optimal trajectory that begins at this point [14].

Let $\mathbf{E}(\mathbf{X}(t),t)$ be a set of Pareto-optimal estimations of conflict-free trajectories for a state $\mathbf{X}(t)\in\mathbf{D}_\mathbf{X}(t)$:

$$\begin{gathered}\mathbf{E}(\mathbf{X}(t),t)=\{\mathbf{J}(\mathbf{X}(t),\mathbf{U}_e(t),t)\,|\,\neg\exists\,\mathbf{U}(t)\in\mathbf{D}_\mathrm{U}(\mathbf{X}(t),t):\\ \mathbf{J}(\mathbf{X}(t),\mathbf{U}(t),t)\le\mathbf{J}(\mathbf{X}(t),\mathbf{U}_e(t),t),\mathbf{U}_e(t)\neq\mathbf{U}(t)\},\end{gathered}\quad(2)$$

where

$$\mathbf{J}(\mathbf{X}(t),\mathbf{U}(t),t)=\left\{\Lambda_1,\ \Lambda_2,\ \int_t^{t_k}\lambda_3dt+\Lambda_3,\ \int_t^{t_k}\lambda_4dt+\Lambda_4\right\}.$$

Let eff be the operator of determination of Pareto-optimal estimations:

$$\mathbf{E}(\mathbf{X}(t),t)=\mathrm{eff}\{\mathbf{J}(\mathbf{X}(t),\mathbf{U}(t),t)\},$$

where $\mathbf{U}(t)\in\mathbf{D}_\mathrm{U}(\mathbf{X}(t),t)$.

Let $\mathbf{X}(t+\tau)\in\mathbf{D}_\mathbf{X}(t+\tau)$ be a state on arbitrary Pareto-optimal conflict-free trajectory, where τ is a small value. Then the Pareto-optimal estimation can be written as follows:

$$\mathbf{E}(\mathbf{X}(t+\tau),t+\tau)=\mathrm{eff}\{\mathbf{J}(\mathbf{X}(t+\tau),\mathbf{U}(t+\tau),t+\tau)\},\quad(3)$$

where

$$\mathbf{U}(t+\tau)\in\mathbf{D}_\mathrm{U}(\mathbf{X}(t+\tau),t+\tau),$$

$$\mathbf{J}(\mathbf{X}(t+\tau),\mathbf{U}(t+\tau),t+\tau)=$$

$$=\left\{\Lambda_1,\Lambda_2,\ \int_{t+\tau}^{t_k}\lambda_3dt+\Lambda_3,\ \int_{t+\tau}^{t_k}\lambda_4dt+\Lambda_4\right\}.$$

The application of the optimality principle to expressions (2) and (3) defines the equation of multi-objective dynamic programming:

$$\mathbf{E}(\mathbf{X}(t),t)=$$

$$=\mathrm{eff}\bigcup_{\mathbf{U}(t)\in\mathbf{D}_\mathrm{U}(\mathbf{X}(t),t)}\left(\left\{0,0,\int_t^{t+\tau}\lambda_3dt,\int_t^{t+\tau}\lambda_4dt\right\}\oplus\mathbf{E}(\mathbf{X}(t+\tau),t+\tau)\right),\quad(4)$$

where $\mathbf{X}(t)\in\mathbf{D}_\mathbf{X}(t)$, $\mathbf{X}(t+\tau)\in\mathbf{D}_\mathbf{X}(t+\tau)$, $\oplus$ – direct sum.

The set of Pareto-optimal conflict-free flight trajectories $\mathbf{P}$ is determined by the set of estimations $\mathbf{E}(\mathbf{X}_0,t_0)$ at the moment of conflict detection t_0.

The synthesis of the set of Pareto-optimal conflict-free flight trajectories is performed using discrete multi-objective dynamic programming.

The dynamic system $\mathbf{S}$ is discretized in time (the conflict resolution process is decomposed into k stages) and in state space. Discretization step Δt is defined taking into account values of possible controls. The main requirement is the stabilization of assigned airspeed during time interval Δt. It is assumed that aircraft maneuvers for conflict avoidance during stages $j=\overline{1,k-1}$ and returns to the planned flight trajectory during last stage k.

The number of stages is defined using following expression:

$$k=\left[\frac{t_k+(t_{los}+t_{end})/2-2t_0}{2\Delta t}\right],$$

where t_{los} – time of potential conflict start; t_{end} – time of potential conflict end; $[\cdot]$ – rounding operator.

Time interval $[t_{j-1},t_j],\ t_j=t_{j-1}+\Delta t$ corresponds to each stage j, except for the last one. The time interval of the last stage $j=k$ is different because of the different time of reaching the fixed final state when transiting from the states at the previous stage $(k-1)$.

Discrete dynamic system $\mathbf{S}$ is determined as follows:

$$\mathbf{S}=\{\mathbf{D_X},\mathbf{X}_0,\mathbf{X}_k,\mathbf{D_U}(\mathbf{X}),\mathbf{D_U^S}(\mathbf{X}),f(\mathbf{X},\mathbf{U}),\Delta J_i(\mathbf{X},\mathbf{U}),\mathbf{T}_{ref}\},$$

where $\mathbf{D_X}$ – a set of conflict-free states of aircraft which performs maneuvers; $\mathbf{X}_0$, $\mathbf{X}_k$ – the initial and the final state of aircraft which performs maneuvers; $\mathbf{D_U}(\mathbf{X})$ – a set of possible controls $\mathbf{U}$ in the state $\mathbf{X}$; $\mathbf{D_U^S}(\mathbf{X})$ – a set of conflict-free controls $\mathbf{U}$ in the state $\mathbf{X}$; $f(\mathbf{X},\mathbf{U})$ – a transition function from the state $\mathbf{X}$ under the action of control $\mathbf{U}$; $\Delta J_i(\mathbf{X},\mathbf{U})$ – efficiency estimation of transition from the state $\mathbf{X}$ under the action of control $\mathbf{U}$, $i=\overline{1,4}$; $\mathbf{T}_{ref}$ – a discretized planned trajectory of an aircraft that does not maneuver (second aircraft).

In general, it is considered that maneuvering aircraft can transit into the state $\mathbf{X}(j)$ at the stage j from several states $\mathbf{X}(j-1)$ at the previous stage $(j-1)$:

$$\mathbf{X}(j)=f(\mathbf{X}(j-1),\mathbf{U}(j-1)).$$

The final state $\mathbf{X}_k=\mathbf{X}(k)$ is specified only by the horizontal coordinates of the point at which an aircraft exits an ATC area (control point). Aircraft may transit into the final state from all the states of the previous stage.

The procedures for each stage $j=\overline{1,k}$ are:

– determination of sets of possible controls $\mathbf{D_U}(\mathbf{X}(j-1))$;

– prediction of separation minima violations when transiting from states $\mathbf{X}(j-1)$ under the action of controls $\mathbf{U}(j-1)\in\mathbf{D_U}(\mathbf{X}(j-1))$ and determination of corresponding sets of conflict-free controls $\mathbf{D_U^S}(\mathbf{X}(j-1))\in\mathbf{D_U}(\mathbf{X}(j-1))$;

– simulation of aircraft flight trajectories, determination of a set of conflict-free states $\mathbf{D_X}(j)$ and values $\Delta J_i(\mathbf{X}(j-1),\mathbf{U}(j-1))$ when transiting from states $\mathbf{X}(j-1)$ under the action of conflict-free controls $\mathbf{U}(j-1)\in\mathbf{D_U^S}(\mathbf{X}(j-1))$;

– determination of sets of Pareto-optimal estimations of conflict-free trajectories $\mathbf{E}(\mathbf{X}(j))$ when transiting into states $\mathbf{X}(j)\in\mathbf{D_X}(j)$.

Prediction of separation minima violations is performed using geometrical method at time interval $[t_{j-1},t_j]$. The prediction is based on the determination of time interval when the lateral separation minimum d_S and vertical separation minimum Δh_S are simultaneously violated.

Time moments of lateral violation start t_{ds} and lateral violation end t_{de} are defined by solving the equation:

$$d_s=\sqrt{(f_x-(x_{ref}+V_{x\,ref}t))^2+(f_y-(y_{ref}+V_{y\,ref}t))^2},\quad(5)$$

$$f_x=x+V_xt+\frac{V_xat^2}{2V}+\frac{V_ygt^2}{2V}tg(\gamma_a),$$

$$f_y=y+V_yt+\frac{V_yat^2}{2V}-\frac{V_xgt^2}{2V}tg(\gamma_a),$$

$$t\in[t_{j-1},t_j],$$

where x, y, V_x, V_y – coordinates and speed components of the maneuvering aircraft at the time moment t_{j-1}; x_{ref}, y_{ref}, $V_{x\,ref}$, $V_{y\,ref}$ – coordinates and speed components of the second aircraft at time moment t_{j-1}; a, γ_a – acceleration and assigned bank of the maneuvering aircraft at the time moment t_{j-1}. Equation (5) is solved using numerical methods.

Time moments of vertical violation start t_{hs} and vertical violation end t_{he} are defined by solving the equation:

$$\Delta h_s = \left(h + V_h t\right) - \left(h_{ref} + V_{h\,ref} t\right),\ t \in [t_{j-1}, t_j],\ (6)$$

where h, V_h – altitude and vertical speed of the maneuvering aircraft at the time moment t_{j-1}; h_{ref}, $V_{h\,ref}$ – altitude and vertical speed of the second aircraft at the time moment t_{j-1}. The solution of equation (6) is:

$$t_h = \frac{\pm \Delta h_S - \left(h - h_{ref}\right)}{V_{ha} - V_{h\,ref}}.$$

In case the time intervals of lateral and vertical violations are intersected, the control $\mathbf{U}(j-1)$ is considered to be a conflict one:

$$[t_{j-1}, t_j] \cap [t_{ds}, t_{de}] \cap [t_{hs}, t_{he}] \neq \varnothing \Rightarrow$$
$$\Rightarrow \mathbf{U}(j-1) \notin \mathbf{D}_{\mathbf{U}}^{S}(\mathbf{X}(j-1)).$$

The simulation of trajectories is performed using the kinematics-energy model of the controlled aircraft motion proposed in article [15]. This model takes into account the dynamic properties of motion, aircraft performance characteristics stored in the EUROCONTROL Base of Aircraft Data (BADA), and allows to calculate the fuel consumption.

The efficiency estimations ΔJ_i are defied using following expressions:

$$\Delta J_1(\mathbf{X}(j-1), \mathbf{U}(j-1)) = \begin{cases} 0,\ j \neq k, \\ \left|t_k - t_f\right|,\ j = k, \end{cases} \quad (7)$$

$$\Delta J_2(\mathbf{X}(j-1), \mathbf{U}(j-1)) = \begin{cases} 0,\ j \neq k, \\ \left|h_k - h_f\right|,\ j = k, \end{cases} \quad (8)$$

$$\Delta J_3(\mathbf{X}(j-1), \mathbf{U}(j-1)) = Q(\mathbf{X}(j-1), \mathbf{U}(j-1)),\quad (9)$$

$$\begin{aligned} \Delta J_4(\mathbf{X}(j-1), \mathbf{U}(j-1)) = \lambda_{41}(\mathbf{X}(j-1), \mathbf{U}(j-1)) + \\ + \lambda_{42}(\mathbf{X}(j-1), \mathbf{U}(j-1)) + \lambda_{43}(\mathbf{X}(j-1), \mathbf{U}(j-1)), \end{aligned} \quad (10)$$

$$\lambda_{41} = \begin{cases} 1,\ \left|\varphi(j) - \varphi(j-1)\right| > \Delta\varphi, \\ 0, \end{cases}$$

$$\lambda_{42} = \begin{cases} 1,\ V_a(j-1) \neq V(j-1), \\ 0, \end{cases}$$

$$\lambda_{43} = \begin{cases} 1,\ V_{ha}(j-1) \neq V_{h0}, \\ 0, \end{cases}$$

where t_f – actual time of reaching the final state $\mathbf{X}_k$; h_k, h_f – planned and actual altitude of the control point overflight; Q – fuel consumption; $\Delta\varphi$ – parameter that takes into account the small heading changes; V_{h0} – planned vertical speed.

The numerical estimations J_i of an arbitrary trajectory $\mathbf{T} = \{\mathbf{X}_0, \mathbf{X}(1), \ldots \mathbf{X}(m)\}$ are defined as follows:

$$J_i(\mathbf{T}) = \sum_{j=1}^{m} \Delta J_i(\mathbf{X}(j-1), \mathbf{U}(j-1)).$$

The using of discrete dynamic programming requires the ability of aircraft to transit into the state $\mathbf{X}(j) \in \mathbf{D}_{\mathbf{X}}(j)$ at j stage from several states $\mathbf{X}(j-1) \in \mathbf{D}_{\mathbf{X}}(j-1)$ at the previous stage $(j-1)$

The determination of fixed states and controls which allow to transit from several states $\mathbf{X}(j-1)$ to one state $\mathbf{X}(j)$ is a difficult problem. It is proposed to combine the sequential determination of conflict-free states and relative Pareto-effective controls using interpolation when solving the problem of dynamic programming.

It is assumed that maneuvering aircraft can change heading, airspeed and vertical speed at all the stages except the last one. Let $\mathbf{D}_{\mathbf{U}0}(\mathbf{X}(j-1))$ be the basic set of controls that contains all possible combinations of controls for changes of heading, airspeed and vertical speed. When applying controls from the set $\mathbf{D}_{\mathbf{U}}^{S}(\mathbf{X}(j-1)) \in \mathbf{D}_{\mathbf{U}0}(\mathbf{X}(j-1))$, an aircraft transits into different states at stage j:

$$\mathbf{X}'(j) = f(\mathbf{X}(j-1), \mathbf{U}(j-1)),$$
$$\mathbf{X}'(j) \in \mathbf{D}_{\mathbf{X}}(j),\ \mathbf{U}(j-1) \in \mathbf{D}_{\mathbf{U}}^{S}(\mathbf{X}(j-1)),$$

with efficiency estimations $\Delta J_i'(\mathbf{X}(j-1), \mathbf{U}(j-1))$ that are defined using expressions (7)-(10).

It is proposed to introduce the rule for formation of new states $\mathbf{X}(j)$ which combine states $\mathbf{X}'(j)$. The proximity of coordinates and heading as well as the equality of airspeeds are the backgrounds for states combining. Thus, it is considered that an aircraft can transit into the state $\mathbf{X}(j)$ under the action of several controls $\mathbf{U}'(j-1)$. As a result the set $\Pi(\mathbf{X}(j))$ of states at the stage $(j-1)$ from which an aircraft can transit into the state $\mathbf{X}(j)$ is defined.

Coordinates and heading of an aircraft in new state $\mathbf{X}(j)$ are determined as arithmetic mean of these parameters for the states $\mathbf{X}'(j)$ which are combined in this new state $\mathbf{X}(j)$.

Estimations $\Delta J_i(\mathbf{X}(j-1), \mathbf{U}'(j-1))$ when transiting into new states $\mathbf{X}(j)$ from the states of the set the $\Pi(\mathbf{X}(j))$ is determined using nearest-neighbor interpolation of values $\Delta J_i'(\mathbf{X}(j-1), \mathbf{U}(j-1))$ for states $\mathbf{X}'(j)$ which are combined:

$$\Delta J_i(\mathbf{X}(j-1), \mathbf{U}'(j-1)) = \Delta J_i'(\mathbf{X}(j-1), \mathbf{U}(j-1)),$$
$$\mathbf{X}(j-1) \in \Pi(\mathbf{X}(j)),\ \mathbf{U}'(j-1) = \mathbf{U}(j-1).$$

At the last stage an aircraft flies direct to the control point with constant speed and maintains the assigned flight level.

For the synthesis of Pareto-optimal conflict-free trajectories the forward procedure of discrete multi-objective dynamic programming is used. To determine the set of Pareto-optimal estimations $\mathbf{E}(\mathbf{X}(j))$ of conflict-free trajectories the equation of multi-objective dynamic programming (4) is transformed into recursive form [12]:

$$\mathbf{E}(\mathbf{X}(j)) = \\ = \operatorname{eff} \bigcup_{\mathbf{X}(j-1) \in \Pi(\mathbf{X}(j))} \left(\mathbf{E}(\mathbf{X}(j-1)) \oplus \{\Delta J_i(\mathbf{X}(j-1), \mathbf{U}'(j-1))\} \right), \quad (11)$$

where $\Pi(\mathbf{X}(j))$ – the set of states at the stage $(j-1)$ from which the transition into the state $\mathbf{X}(j)$ is possible; $\mathbf{U}'(j-1) \in \mathbf{D}_{\mathrm{U}}^{S}(\mathbf{X}(j-1))$ – controls which allow an aircraft to transit from the state $\mathbf{X}(j-1) \in \Pi(\mathbf{X}(j))$.

The solution of equation (11) also defines the set $\mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}(j))$ of Pareto-optimal controls that allow an aircraft to transit into the state $\mathbf{X}(j)$ and the corresponding set of states $\Pi_{\mathbf{E}}(\mathbf{X}(j)) \in \Pi(\mathbf{X}(j))$.

The set of Pareto-optimal conflict-free trajectories is defined as

$$\mathbf{P} = \{\mathbf{T} \in \mathbf{K} \mid \mathbf{J}(\mathbf{T}) \in \mathbf{E}(\mathbf{X}_k)\},$$

where $\mathbf{K}$ – the set of full trajectories $\mathbf{T} = \{\mathbf{X}_0, \mathbf{X}(1), \ldots \mathbf{X}_k\}$ by which an aircraft transits from the initial state $\mathbf{X}_0$ into the final state $\mathbf{X}_k$.

Each Pareto-optimal conflict-free trajectory $\mathbf{T} \in \mathbf{P}$ corresponds to the Pareto-optimal controls program $\mathbf{T}_{\mathrm{U}}$.

Pareto-optimal conflict-free trajectory $\mathbf{T} \in \mathbf{P}$ and corresponding program $\mathbf{T}_{\mathrm{U}}$ are determined using following backward procedure.

At the last stage each trajectory $\mathbf{T} \in \mathbf{P}$ contains the final state $\mathbf{X}_k$ that related with the state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ at the penultimate state by control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}_k) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}_k)$. For each state $\mathbf{X}(k-1)$ and corresponding control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}_k)$ the following trajectory and program are determined:

– trajectory $\mathbf{T}$ which contains the state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ and final state $\mathbf{X}_k$: $\mathbf{T} = \{\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k), \mathbf{X}_k\}$;

– program $\mathbf{T}_{\mathrm{U}}$ which contains the control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}_k) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}_k)$ that provides the transition of an aircraft from the state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ into the final state $\mathbf{X}_k$: $\mathbf{T}_{\mathrm{U}} = \{\mathbf{U}_{\mathbf{E}}(\mathbf{X}_k) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}_k)\}$.

At the penultimate stage a certain state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ can be related with several states $\mathbf{X}(k-2) \in \Pi_{\mathbf{E}}(\mathbf{X}(k-1))$ at stage $(k-2)$ by Pareto-optimal controls $\mathbf{U}_{\mathbf{E}}(\mathbf{X}(k-1)) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}(k-1))$. For each state $\mathbf{X}(k-2)$ and corresponding control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}(k-1))$ the following trajectory and program are determined:

– trajectory $\mathbf{T}$ which contains the state $\mathbf{X}(k-2) \in \Pi_{\mathbf{E}}(\mathbf{X}(k-1))$, state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ and final state $\mathbf{X}_k$: $\mathbf{T} = \{\mathbf{X}(k-2) \in \Pi_{\mathbf{E}}(\mathbf{X}(k-1)), \mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k), \mathbf{X}_k\}$;

– program $\mathbf{T}_{\mathrm{U}}$ which contains the control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}(k-1)) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}(k-1))$ and control $\mathbf{U}_{\mathbf{E}}(\mathbf{X}_k) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}_k)$ that provides the transition from the state $\mathbf{X}(k-1) \in \Pi_{\mathbf{E}}(\mathbf{X}_k)$ into the final state $\mathbf{X}_k$: $\mathbf{T}_{\mathrm{U}} = \{\mathbf{U}_{\mathbf{E}}(\mathbf{X}(k-1)) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}(k-1)), \mathbf{U}_{\mathbf{E}}(\mathbf{X}_k) \in \mathbf{D}_{\mathrm{U}}^{\mathbf{E}}(\mathbf{X}_k)\}$.

This sequence of described actions is performed at all the stages from last to first.

The selection of optimal conflict-free trajectory $\mathbf{T}^*$ from the set of Pareto-optimal trajectories $\mathbf{P}$ is performed by solving the optimization problem [16]:

$$\mathbf{T}^* = \arg \min_{\mathbf{T} \in \mathbf{P}} \max_{\mathbf{W} \in \mathbf{D}_W} \sum_{i=1}^{3} w_i \bar{c}_i(\mathbf{T}), \quad (12)$$

$$\bar{c}_1(\mathbf{T}) = 0{,}5\frac{J_1(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_1(\mathbf{T})}{\max_{\mathbf{T}\in\mathbf{P}} J_1(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_1(\mathbf{T})} + $$

$$+ 0{,}5\frac{J_2(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_2(\mathbf{T})}{\max_{\mathbf{T}\in\mathbf{P}} J_2(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_2(\mathbf{T})},$$

$$c_i(\mathbf{T}) = \frac{J_{i+1}(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_{i+1}(\mathbf{T})}{\max_{\mathbf{T}\in\mathbf{P}} J_{i+1}(\mathbf{T}) - \min_{\mathbf{T}\in\mathbf{P}} J_{i+1}(\mathbf{T})}, i = \overline{2,3},$$

where $\bar{c}_1$, $\bar{c}_2$, $\bar{c}_3$ – normalized estimations on the flight regularity c_1, flight economy c_2 and the complexity of maneuvering c_3 criteria respectively with the domain of allowable values $\mathbf{D}_{\bar{c}} = \{\bar{c} \mid \bar{c} \in [0,1]\}$; w_i – the weighting coefficients reflecting the relative importance of criteria and forming a vector $\mathbf{W} = \{w_i\}, i = \overline{1,3}$ with the domain of allowable values $\mathbf{D}_w$ and minimal value w_0:

$$\mathbf{D}_w = \left\{ \mathbf{W} \middle| \sum_{i=1}^{3} w_i = 1; w_i \ge w_{i+1}, i = \overline{1,2}; w_3 \ge w_0 > 0 \right\}.$$

5. Computer simulation

The conflict situation between two aircraft Boeing 737-800 flying in airspace with reduced vertical separation minima (RVSM) was simulated. The value of the lateral separation minimum is equal to $d_s = 18{,}5$ km (10 nautical miles) and the value of vertical separation minima is equal to $\Delta h_S = 300$ m (1000 feet). The geometric method for prediction of separation violations was used. The initial parameters of the aircraft flight and characteristics of predicted conflict situation are presented in Table 1.

It was assumed that to avoid the conflict the aircraft №1 should make manoeuvres. The aircraft №2 flies by planned trajectory.

The process was discretized in time on 5 stages. The discretization step for the first 4 stages is equal to $\Delta t = 60$ s.

It was assumed that the aircraft №1 can change heading, speed and vertical speed to avoid the conflict. Being in a certain state $\mathbf{X}(j-1)$ at stages $j = \overline{1,4}$ aircraft is able:

– to make a left/right turn with bank angle $\gamma = 20^\circ$ (turning time is limited to 15 s) or to do not change the heading;

– and to increase/decrease the airspeed on $\Delta V = 5$ m/s or to do not change it;

– and to increase/decrease the vertical speed on $\Delta V_h = 3$ m/s or to do not change it.

The minimal value of weighting coefficients in the optimization problem (12) is equal to $w_0 = 0{,}1$.

As a result of the simulation the optimal conflict-free trajectory $\mathbf{T}^*$ was determined.

The graphically this trajectory is shown in Fig. 1. The dependences of aircraft altitudes from time are shown in Fig. 2. The dependences of horizontal and vertical distance between aircraft from time are shown in Fig. 3. The program of assigned airspeed and vertical speed changes for aircraft №1 is represented in Table 2. The efficiency parameters of trajectory $\mathbf{T}^*$ are represented in Table 3.

Table 1. The initial parameters of aircraft flight and characteristics of predicted conflict situation

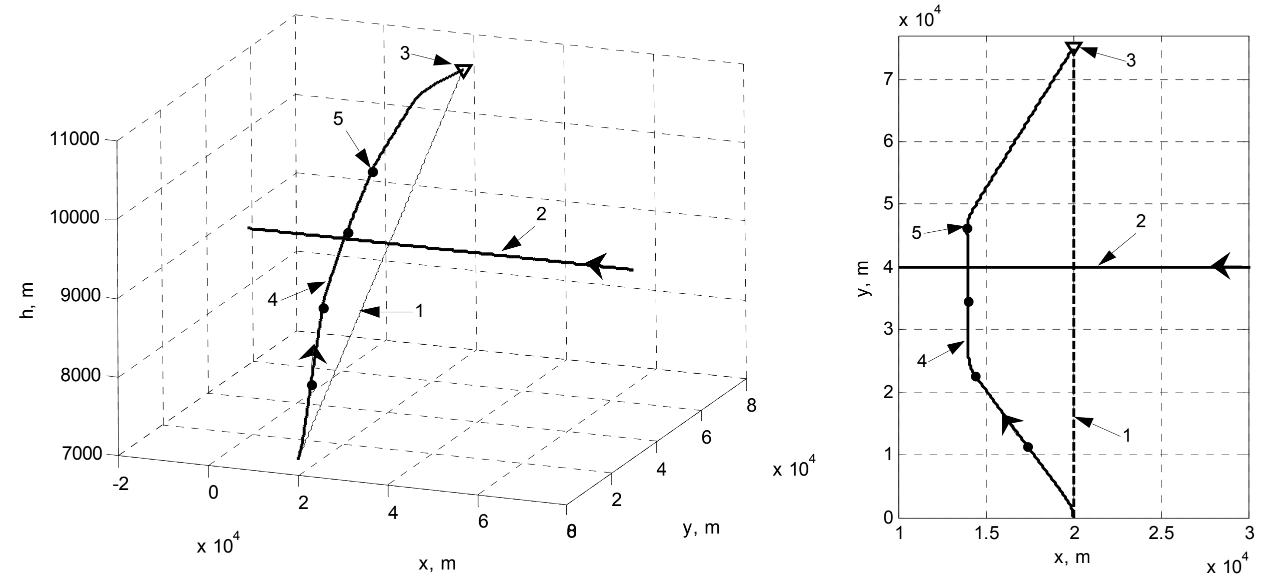
Parameter	Aircraft №1	Aircraft №2
Heading φ, degrees	0	270
Airspeed V, m/s	195	220
Vertical speed V_h, m/s	9	0
Initial coordinates $(x_0; y_0)$, km	(20; 0)	(75; 40)
Initial altitude h_0, m	7200	9150
Assigned flight level (altitude h_a, m)	350 (10650)	300 (9150)
Distance to the control point L_0, km	75	–
Planned time of control point overflight t_k, s	385	–
Time interval of separation violation $[t_{los}, t_{end}]$, s	[183, 250]	
Predicted minimum horizontal distance between aircraft $d_{\min 0}$, m	6548	
Predicted minimum vertical distance between aircraft $\Delta h_{\min 0}$, m	0	

Table 2. The program of assigned airspeed and vertical speed changes for aircraft №1

Stage	1	2	3	4	5
Airspeed V_a, m/s	195	195	195	195	195
Vertical speed V_{ha}, m/s	12	12	12	9	9

Table 3. The efficiency parameters of optimal conflict-free flight trajectory

Parameters	Value
Deviation from the planned flight time, s	5,4
Deviation from the assigned flight level at control point, m	0
Additional fuel consumption, %	0,8
Number of flight profile changes	6



a) b)

Fig. 1. The aircraft trajectories: *a* – in three-dimensional space; *b* – in horizontal plane; 1 – planed trajectory for the aircraft №1; 2 – planed trajectory for the aircraft №2; 3 – control point on the route; 4 – optimal conflict-free trajectory for the aircraft №1; 5 – states at the stages.

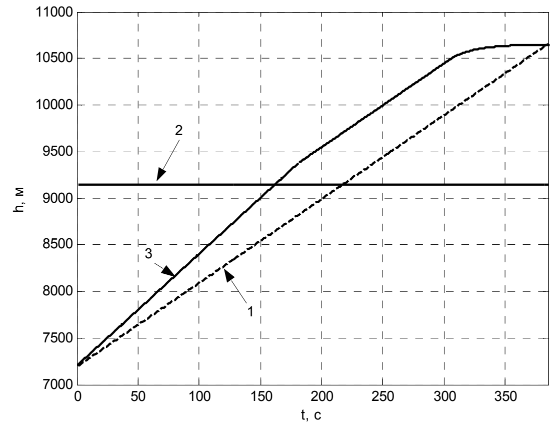
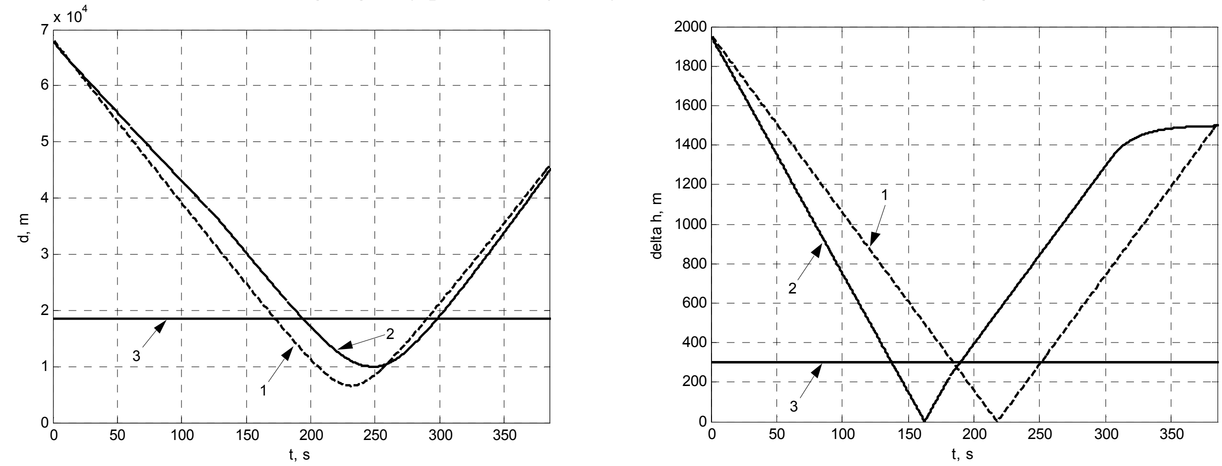


Fig. 2. The dependences of aircraft altitudes from time: 1 – altitude of aircraft №1 during flight by planned trajectory; 2 – altitude of aircraft №2 during flight by planned trajectory; 3 – altitude of aircraft №1 during conflict resolution.



a) b)

Fig. 3. The dependence of distances between aircraft from time: *a* – horizontal distance *d*; *b* – vertical distance Δh; 1 – during flight by planed trajectories; 2 – during conflict resolution; 3 – separation minimum.

The results of computer simulation show that optimal conflict-free trajectory $\mathbf{T}^*$ ensures the conflict avoidance and complies with defined optimality criteria.

6. Conclusions

The method of multi-objective resolution of conflict between two aircraft in three-dimensional space has been developed. Described method provides the synthesis of conflict-free flight trajectory according to criteria of flight regularity, flight economy and the complexity of maneuvering. The synthesis of Pareto-optimal trajectories is carried out using the multi-objective dynamic programming. The selection of optimal conflict-free trajectory from the set of Pareto-optimal trajectories is carried out using the convolution of optimality criteria.

The advantages of the method are:

– heading, speed and altitude change maneuvers are used for conflict avoidance;

– the simulation of trajectories is performed using the special model of controlled aircraft motion;

– the multi-objective optimization of conflict-free trajectories is applied;

– the sequential synthesis of conflict-free flight trajectories is performed using the dynamic programming that enhances the computational efficiency.

Proposed method can be used for development of advanced conflict resolution tools for automated ATC systems.

References

[1] *Eby, M.S.* A Self-Organizational Approach for Resolving Air Traffic Conflicts. The Lincoln Laboratory Journal. - 1994. - Vol. 7 (2). - P. 239-254.

[2] *Eby, M.S.*; *Kelly, W.E.* Free Flight Separation Assurance Using Distributed Algorithms. Proc. of IEEE Aerospace Conf., Snowmass. - 1999. - P. 429-441.

[3] *Kosecka, J.; Tomlin, C.; Pappas, G.; Sastry, S.* Generation of Conflict Resolution Maneuvers for Air Traffic Management. Proceedings of The 1997 IEEE/RSJ International Conference On Intelligent Robot And Systems. - 1997. - Vol.3 - P. 1598 - 1603.

[4] *Zeghal, K. A*. Review of Different Approaches Based on Force Fields for Airborne Conflict Resolution. Proc. AIAA Guidance, Navigation, and Control Conf. - Boston, 1998. - P. 818-827.

[5] *Bicchi, A; Pallottino, L.* On Optimal Cooperative Conflict Resolution for Air Traffic Management Systems. IEEE Transactions on Intelligent Transportation Systems. - 2000. - Vol. 1, No. 4.– P.221-232.

[6] *Hu., J.; Prandini, M.; Sastry, S.* Optimal Maneuver for Multiple Aircraft Conflict Resolution: A Braid Point of View. Proc. of the 39th IEEE conf. on decision and control. - Sydney. - 2000. - Vol. 4. - P. 4164-4169.

[7] *Hu, J.; Prandini, M.; Sastry, S.* Optimal Coordinated Maneuvers for Three Dimensional Aircraft Conflict Resolution. Journal of Guidance, Control, and Dynamics. - 2002. - Vol. 25, No. 5 - P. 888-900.

[8] *Cetek, C.* Realistic Speed Change Maneuvers for Air Traffic Conflict Avoidance and their Impact on Aircraft Economics. International Journal of Civil Aviation. - 2009. - Vol. 1 (1). - P. 62-73.

[9] *Cafieri, S.*; Durand, N. Aircraft deconfliction with speed regulation: new models from mixed-integer optimization. Journal of Global Optimization. - 2014. - Vol. 58(4). - P. 613-629.

[10] *Ehrmanntraut, R.; Christien, R.* Analysis of Aircraft Conflict Geometries in Europe. Digital Avionics Systems Conference, 24-28 Oct. 2004. - Vol.1 - P. 3.E.2-1– 3.E.2-7

[11] *Paglione, M.M.; Santiago, C.; Crowell, A.; Oaks, R.D.* Analysis of the Aircraft to Aircraft Conflict Properties in the National Airspace System. AIAA Guidance, Navigation and Control Conference and Exhibit, 18-21 August 2008, Honolulu, Hawaii.

[12] *Vasyliev, D.V.* Multi-Objective Synthesis of Conflict-Free Aircraft Trajectories. Science-based Technologies. - 2014. - 1. - P. 37-40. (in Ukrainian)

[13] *Bellman, R.* Dynamic Programming. - Princeton, New Jersey: Princeton University Press, 1957, Six Printing, 1972.

[14] *Kogan, D.I.* Dynamic Programming and Discrete Multi-Objective Optimization: Tutorial. - Nizhny Novgorod: Publishing House of Nizhny Novgorod State University, 2005. - 260 p. (in Russian)

[15] *Vasyliev, D.V.* Mathematical Model of Controlled Aircraft Motion for Analysis of Air Navigation Service Processes. Systems of Arms and Military Equipment. - 2013. - 2. - P. 63-67. (in Ukrainian)

[16] *Vasyliev, D.V.* Model of Multi-Objective Selection of Trajectories for Aircraft Conflicts Resolution. Systems of Information Processing. - Kharkiv: KhUAF, 2013. - Issue 4 (111). - P. 85-88. (in Ukrainian)

Received 12 May 2016

Д.В. Васильєв

Метод багатокритеріального розв’язання конфліктної ситуації між двома повітряними суднами у тривимірному просторі на основі динамічного програмування

Державне підприємство обслуговування повітряного руху України (Украерорух), Аеропорт, м. Бориспіль, Київська обл., Україна, 08307
Національний авіаційний університет, просп. Космонавта Комарова, 1, м. Київ, Україна, 03058
E-mail: dvasyliev@nau.edu.ua

Мета: Глобальні тенденції зростання інтенсивності повітряного руху обумовлюють збільшення кількості конфліктних ситуацій між повітряними судами. Актуальною проблемою є розробка нових методів розв’язання конфліктних ситуацій, які повинні забезпечувати синтез безконфліктних траєкторій у тривимірному просторі у відповідності до різних критеріїв ефективності польотів. **Методи:** Розроблено метод багатокритеріального розв’язання конфліктної ситуації між двома повітряними суднами із застосуванням маневрування зміною курсу, швидкості та висоти польоту. Описаний метод на основі динамічного програмування забезпечує синтез оптимальної безконфліктної траєкторії відповідно до критеріїв регулярності, економічності польотів та складності маневрування. Наведено рівняння багатокритеріального динамічного програмування для визначення множини Парето-оптимальних оцінок безконфліктних траєкторій у неперервній та дискретній формі. Синтез Парето-оптимальних безконфліктних траєкторій здійснюється із застосуванням прямої процедури дискретного багатокритеріального динамічного програмування. Моделювання траєкторій польоту виконується із використанням спеціальної моделі керованого руху повітряного судна. Вибір оптимальної безконфліктної траєкторії з множини Парето-оптимальних виконується із застосуванням згортки критеріїв оптимальності. В рамках методу визначено наступні процедури: прогнозування порушень мінімумів ешелонування; дискретизації станів та керувань, інтерполяції оцінок ефективності траєкторій за встановленими критеріями оптимальності. **Результати:** Дослідження запропонованого методу виконано шляхом комп’ютерного моделювання, результати якого показали, що розрахована оптимальна безконфліктна траєкторія забезпечує усунення конфліктної ситуації та відповідає встановленим критеріям оптимальності. **Обговорення:** Основними перевагами методу є: застосування маневрів по зміні курсу, швидкості та висоти польоту для усунення конфлікту; багатокритеріальна оптимізація безконфліктних траєкторій; застосування динамічного програмування, що підвищує обчислювальну ефективність. Запропонований метод може бути використаний при розробці засобів розв’язання конфліктних ситуацій для автоматизованих систем управління повітряним рухом.

Ключові слова: багатокритеріальна оптимізація; безпека польотів; динамічне програмування; повітряне судно; розв’язання конфліктної ситуації; управління повітряним рухом

Д.В. Васильев

Метод многокритериального разрешения конфликтной ситуации между двумя воздушными судами в трехмерном пространстве на основе динамического программирования

Государственное предприятие обслуживания воздушного движения Украины (Украэрорух), Аэропорт, г. Борисполь, Киевская обл., Украина, 08307
Национальный авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03058
E-mail: dvasyliev@nau.edu.ua

Цель: Глобальные тенденции роста интенсивности воздушного движения обусловливают увеличение количества конфликтных ситуаций между воздушными судами. Актуальной проблемой является разработка новых методов решения конфликтных ситуаций, которые должны обеспечивать синтез бесконфликтных траекторий в трехмерном пространстве в соответствии с разными критериями эффективности полетов. **Методы:** Разработан метод многокритериального разрешения конфликтной ситуации между двумя воздушными судами с применением маневрирования по изменению курса, скорости и высоты полета. Описанный метод на основе динамического программирования обеспечивает синтез оптимальной бесконфликтной траектории в соответствии с критериями регулярности, экономичности полетов и сложности маневрирования. Приведены уравнения многокритериального динамического программирования для определения множества Парето-оптимальных оценок бесконфликтных траекторий в непрерывной и дискретной форме. Синтез Парето-оптимальных бесконфликтных траекторий осуществляется с применением прямой процедуры дискретного многокритериального динамического программирования. Моделирование траекторий полета выполняется с использованием специальной модели управляемого движения воздушного судна. Выбор оптимальной бесконфликтной траектории из множества Парето-оптимальных выполняется с применением свертки критериев оптимальности. В рамках метода определены следующие процедуры: прогнозирования нарушений минимумов эшелонирования; дискретизации состояний и управлений, интерполяции оценок эффективности траекторий по установленным критериям оптимальности. **Результаты:** Исследование предложенного метода выполнено путем компьютерного моделирования, результаты которого показали, что рассчитанная оптимальная бесконфликтная траектория обеспечивает устранение конфликтной ситуации и соответствует установленным критериям оптимальности. **Обсуждение:** Основными преимуществами метода являются: применение маневров по изменению курса, скорости и высоты полета для устранения конфликта; многокритериальная оптимизация бесконфликтных траекторий; применение динамического программирования, повышающего вычислительную эффективность. Предложенный метод может быть использован при разработке средств разрешения конфликтных ситуаций для автоматизированных систем управления воздушным движением.

Ключевые слова: безопасность полетов; воздушное судно; динамическое программирование; многокритериальная оптимизация; разрешение конфликтной ситуации; управление воздушным движением.

Vasyliev Denys (1987). PhD (Eng).
Instructor of Training and Certification Centre of Ukrainian State Air Traffic Services Enterprise (UkSATSE), Boryspil, Ukraine.
Associate Professor of Air Navigation Systems Department of National Aviation University, Kyiv, Ukraine.
PhD (Eng) on specialty "Navigation and Air Traffic Control" (2014)
Education: National Aviation University, Kyiv, Ukraine (2010).
Research area: situation analysis and decision-making in air traffic management.
Publications: 37.
E-mail: dvasyliev@nau.edu.ua