UDC 517.958: 531.34
DOI: 10.18372/2306-1472.67.10430

Vitaliy Mamchuk[1]
Ivan Lastivka[2]
Oleksandr Bezverkhyi[3]

RESEARCH AND MATHEMATICAL MODELING OF TURBULENT BOUNDARY LAYER AT POSITIVE PRESSURE GRADIENT

[1,2]National Aviation University
1, Kosmonavta Komarova avenue, Kyiv, 03058, Ukraine
[3]S. P. Timoshenko Institute of Mechanics
3, P. Nesterova str., Kyiv, 03057, Ukraine
E–mails: [1]vimamchuk@ukr.net, [2]iola@nau.edu.ua, [3]o_bezver@ukr.net

Abstract

Purpose: *Mathematical modeling of complex turbulent near-wall flows, that occur during the flow of airfoils, is impossible without understanding the nature of the flow in boundary layer. From a mathematical point of view, the calculation of such flows, because in practical problems they regarded as turbulent, and the characteristics of turbulence are largely dependent on the geometry of the profile of the longitudinal component of the average velocity of the near-wall flow. Based on this, the purpose of this work is studying and mathematical modeling of turbulent near-wall flows in the interaction with the real streamlined surface, that has certain features, such as the curvature, roughness, etc., as well as the study and research of the influence of the pressure gradient on the empirical coefficients, parameters of the flow, velocity profiles and friction stress.* ***Methods:*** *We performed the calculations using numerical finite-difference marching method with algebraic model of turbulent viscosity coefficient.* ***Results:*** *In this paper we present some results of the numerical study of the effect of the positive pressure gradient on the empirical coefficients of the transition zone and the law of the near-wall and the outer-wall areas.* ***Discussion****: Comparison of the calculated results with the experimental data shows that the proposed approaches provide an opportunity to simulate the flow as close as possible to their physical properties. Presented mathematical model for the calculation of turbulent boundary layers and near-wall flows makes it possible to calculate such a complex and valuable from a practical point of view type of the flow as the aerodynamic trail behind the streamlined body.*

Keywords: boundary layer; pressure gradient; turbulent viscosity; velocity profile; wall stream.

1. Introduction

Mathematical modeling of turbulent near–wall complex streams that occur when using devices and vehicles that use the wing as a mover or carrier or controls, is not possible without understanding the nature of the flow in the boundary layer. Boundary wall streams are one of the effective controls of boundary layer, which is widely used in aerial hydrodynamics. From a mathematical point of view, the calculation of such flows is a serious problem even for today [1–18], because of practical problems must be considered as turbulent, and the characteristics of turbulence are largely dependent on the geometry of turbulent wall stream of nonmonotonic longitudinal averaged speed profile. In addition, the real streamlined surface has certain characteristics (curvature, roughness, etc.) and interacts with the flow, affects the formation of the wall of stream flow.

2. Research and mathematical modeling of turbulent flows

In modern research the use of numerical methods using computer technology opportunities significantly expanded classes of solvable hydroaerodynamics problems, particularly problems of turbulent flows. [2–12]. Analysis of the results of calculations of the turbulent boundary layer showed that when there is a sharp change in pressure gradient, estimates deviate from experimental [2, 3, 9]. Deviation of the calculated data from experimental is particularly evident in sections of the boundary layer. This deviation of calculated results from experimental data is primarily due to unaccounted amendments to spatiality and influence of Reynolds normal stress [2, 3, 9]. Important role as ignoring the impact of the pressure gradient and low Reynolds numbers on empirical steel (factors) that are used in both algebraic and differential models in [5, 9, 10, 13]. Detailed analysis of the calculation results of the work of many authors indicates the need for a comprehensive study and research of the influence of the pressure gradient on empirical coefficients, parameters of flow, velocity profiles and friction stress [2, 3, 5, 8, 9, 13, 14]. In studies and calculations introduced amendments to the gradient, usually in empirical coefficients or not taken into account [3, 4, 14, 15]. Amendment to Van Drista factor taken into account in [2, 3, 5, 11].

It should be noted that the approximate formulas to account for the influence of the pressure gradient by a factor of Van Drista in Sebes [11] and by Casey studies [5] give significantly different numerical values. It is known that the effect of the pressure gradient in the near–wall region appears much weaker than outside. Therefore, the influence of the pressure gradient by a factor external area should be considered first. Analysis of the results of research by Stanford conference materials [16] showed that the rate of the outer regions of this model with increasing pressure gradient drops from 0.09 to 0.045. Since the area of the law is contained between the walls and the outer transition area, the effect of the pressure gradient on empirical factor should be taken into account and it [5]. In numerical calculations using algebraic models tend to "merge" the solutions of the wall and the outer regions [2–6, 10, 11]. This merging of solutions is conducted by pre–selected ordinate, or by finding the equation of turbulent viscosity coefficient values of the wall and the outer regions, each of which was given the different formulas [2–6, 10, 11]. The study, presented in [6] indicate incorrect merging tasks at a rate of turbulent viscosity two–layer scheme: ordinate splice point increases with pressure gradient, not decreases. So natural is the intention to have the formula for the coefficient of turbulent viscosity, which continuously describing the entire boundary layer [7, 8].

This paper sets out some of the results of the study of the effect of the pressure gradient on empirical coefficients areas and regions; analysis of numerical calculations gradient turbulent boundary layer using algebraic models in which the turbulent viscosity is a continuous function. The problem as to hold more volume study of the effect of positive pressure gradient on empirical coefficients of the transition zone wall and the wall area of the law, and external and assessment of its impact on each of them and simultaneously to all.

In numerical calculations is used the method of lines and following semi–empirical formula for the coefficient of turbulent viscosity ε_T [7, 8]:

$$\varepsilon_T = \chi\rho\upsilon^*\Delta\gamma(\bar{y})th\left(\frac{l\,\bar{\tau}_w}{\chi\Delta}\right)\frac{\partial u}{\partial y}, \quad (1)$$

where

$$\bar{y} = \frac{y}{\delta};\ \bar{\tau}_w = \frac{\tau_w}{\tau_\omega} = 1 + \Phi\bar{y},\ \Phi = \frac{\delta}{\tau_\omega}\frac{\partial p}{dx};$$

ρ – density of liquid; γ – function of intermittency factor; χ – empirical constant; Δ – Klauzer's length scale; υ^* – dynamic speed; l – stirring path length; τ_ω – напруження тертя на стінці; τ_w – tension of friction on wall; δ – thickness of the boundary layer.

Studies have shown that satisfactory formula (1) gives the comparison with experiment on a flat plate [13], if we take

$$\gamma(\bar{y}) = \sqrt{1-\bar{y}}, \quad (2)$$

$$\bar{l} = k\bar{y}th\frac{sh^2(\chi_1 y^+)th[sh^2(\chi_2 y^+)]}{ky^+\sqrt{\bar{\tau}_w}},$$

where $$\bar{l} = \frac{y}{\delta},\quad y^+ = \frac{y\upsilon^*}{\nu}. \quad (3)$$

Numerical calculations for a flat plate with the use of this model showed good coincidence with experimental results [2] and the results of this calculation and conducted in [7, 8]. The advantage of the formula (1) in that it involves the approximate analytical solutions in the areas of wall and outer regions [7, 8]. The lack of experimental results for the coefficient of turbulent viscosity in a wide range of positive pressure gradient didn't allow to directly ensure in effect of the latter on factors χ, k and χ_1. So, approximate equations for friction stress profiles and speeds were built and using them an impact assessment on the pressure gradient χ was conducted. To obtain approximate analytic solutions in the external area the expression $\sqrt{1-\bar{y}}$ was laid out in a row and taken into account the two members of the series. Then, with the method of [8] the velocity profile was found. The resulting formula was used in numerical calculation using the Stanford conference materials [16]. The results of numerical experiments the following dependency was obtained:

$$\chi = 0{,}0095 + \frac{1}{74{,}6 + (2{,}4+\beta)^2},\ \beta = \frac{\delta^*}{\tau_\omega}\frac{\partial p}{\partial x}. \quad (4)$$

It should be noted that for factor k was used known Horstmann

$$k = 0,4 + 0,182275(1 - e^{-0,32068\beta}). \qquad (5)$$

Approximate analytical studies conducted in the near–wall region using materials yielded the following value to account for low Reynolds numbers and the pressure gradient by a factor χ_1, namely:

$$\chi_1 = \chi_{1R}\chi_{10}(1 + 15,0895 p^+ r_1 r_2), \qquad (6)$$

where

$$r_1 = 1 - 0,5e^{-0,1436\beta} + 0,5e^{-0,3531\beta},\ p^+ = \frac{\nu}{\rho\upsilon^{*3}}\frac{\partial p}{\partial x},$$

$$r_2 = 1 - e^{-76,1528\frac{p^+}{\beta}} + e^{-361,4064\frac{p^+}{\beta}},$$

$$\chi_{1R} = 1 + 0,1\left(1 - e^{-\frac{14}{1+z^2}}\right),\ z = 10^{-3}\,\mathrm{Re}_{\delta^{**}},$$

$$\mathrm{Re}_{\delta^{**}} = \frac{\delta^{**}u_e}{\nu}.$$

In the method of lines $\frac{\partial u}{\partial x}$ is replaced by the ultimate difference, that is $\frac{\partial u}{\partial x} = \frac{u(x,y) - u(x-\Delta x, y)}{\Delta x}$. As a result of differential equations in partial derivatives

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{dx} + \frac{1}{\rho}\frac{\partial \tau}{dy}, \qquad (7)$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0$$

is reduced to a system of ordinary differential equations. Previously the system of equations (7) was dimensionlessed and introduced new variables as follows:

$$y_1 = \frac{u}{u_e},\quad y_2 = \frac{\tau}{\rho u_e^2},\quad y_3 = \frac{\upsilon}{u_e},\quad X = \frac{x}{L},\quad Y = \frac{y}{L},$$

where u_e – speed of outer flow, L – characteristic size.

Thus, we obtain the system of ordinary differential equations

$$\frac{dy_1}{dY} = f,$$

$$\frac{dy_2}{dY} = y_1\frac{y_1 - y_1^*}{\Delta X} + y_3 f + \frac{y_1^* - 1}{u_e}\frac{\partial u_e}{\partial X}, \qquad (8)$$

$$\frac{dy_3}{dY} = \frac{y_1^* - y_1}{\Delta X} - \frac{y_1}{u_e}\frac{\partial u_e}{\partial X},$$

where

$$f = \frac{\rho u_e L y_2}{\mu + \varepsilon_T},\ y_1^* = y_1(X - \Delta X, Y).$$

Boundary conditions for the system (8):

$$y_1 = 0,\ y_2 = y_{2\omega},\ y_3 = 0 \quad \text{when } Y = 0,$$
$$y_1 \to 1,\ y_2 \to 0,\ y_3 \to 0 \quad \text{when } Y \to \infty. \qquad (9)$$

The second boundary condition at Y = 0 is unknown and must be determined in the process of solving. This difficulty is overcome by "ranging" in this marginal condition, and by building iterative process based on the parameters δ and υ^*. In each fixed section $X = const$ the Cauchy problem for systems of ordinary differential equations with initial conditions $y_1 = 0,\ y_2 = y_{2\omega},\ y_3 = 0$ in the $Y = 0$ is solved. Integration of differential equations (8) was conducted using Runge–Kutta fourth order method to values Y_k, that exceed the thickness of the boundary layer. For boundary layer thickness was set to a value ordinates for which the longitudinal speed was $0,995u_e$. To implement this method using a computer in the original section, the values $\delta,\ \Delta,\ \upsilon^*$ were set from the experiment, and velocity profiles were determined by the Pertsha's formula [3]. In the first stage of the research were taken the experimental data, for which there are approximation for external speed.

3. Analysis of research results.

Figure 1–3 is given as an example comparison of the calculated curves with the experimental results of Ludwig and Tillman (*id.* 1200).

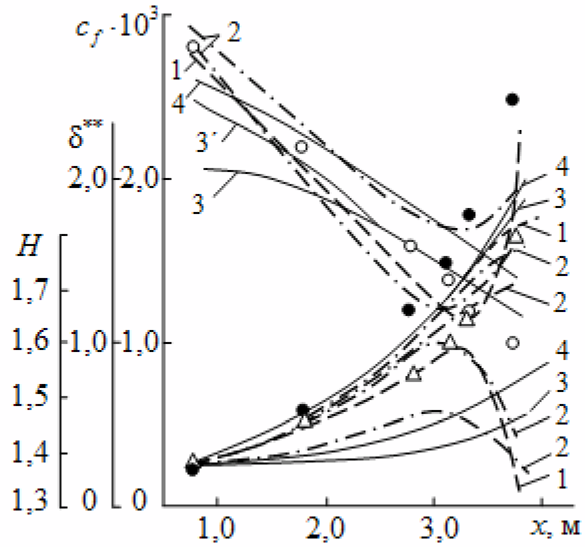

Fig. 1. Comparison of the calculated curves for the parameters c_f, H and δ^{**} experiment with Ludwig and Tillman [16], and the calculations of other authors [12, 15]:

1 – calculation for [3]; 2 – calculation for [9]; 3 – valid calculation excluding amendments at a constant $\chi_{10} = 0,064$, 3' – in $\chi_{10} = 0,072$; 4 – calculation amended by formulas (4), (5), (6); ○ – research c_f; ● – research δ^{**}; Δ – research H.

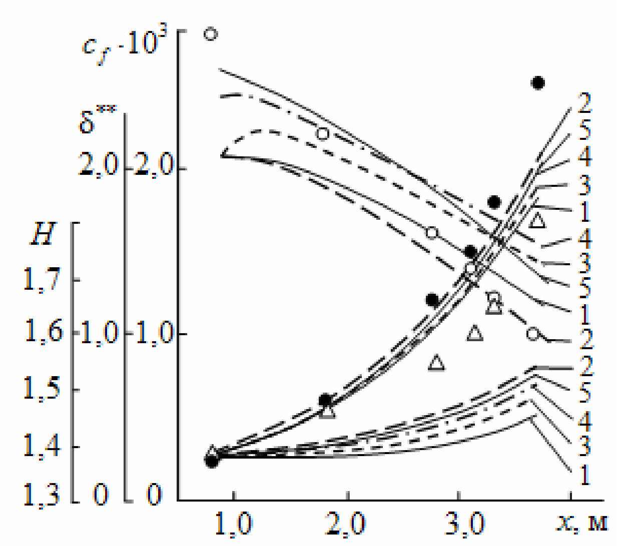

Fig. 2. Comparison of the calculated curves for the parameters c_f, H i δ^{**} and experiment with Ludwig and Tillman [16] taking into account and excluding adjustments for gradient:

1 – calculation excluding amendments; 2 – calculation including amendments for $b\chi$ by formula (4); 3 – calculation including amendments for bk by formula (5); 4 – calculation including amendments $b\chi_1$ by Casey [2]; 5 – calculation including amendments by formulas (4), (5), (6); ○ – research c_f; ● – research δ^{**}; Δ – research H.

Calculated curves 3, 3' in Fig. 1 (excluding adjustments for pressure gradient and small Reynolds number) is in good agreement with the experimental results. The introduction of amendments to improve the coordination can be achieved between the calculated and experimental data. This is evident from a comparison of calculated curves 3, 4 and experiment.

Analysis of the results presented graphically in Fig. 2 indicates that the incorporation of amendments to a separate gradient factor leads to qualitatively and quantitatively different accounting curves. Therefore, by introducing amendments we can refine the calculated curves, as the impact of each on the integral curve is different.

From Fig. 3 we see that the calculated curves defect rate is in good agreement with experiment. It should also be noted that the calculation makes it possible to find bends in speed profiles.

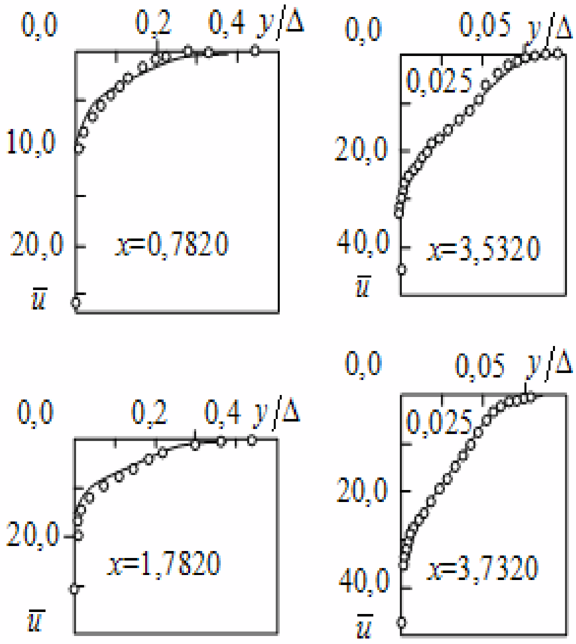

Fig. 3. Comparison of the calculated defect rate curves (solid line) with experiment of Ludwig and Tillman (circles) [16], $\bar{u} = u - u_e / \upsilon^*$.

From the above rises the need to create a model that would allow to count as broad a class of turbulent near-wall flow, especially such a complex and important from a practical point of view reaches as aerodynamic trace. Aerodynamic trace – is a zone of inhibited gas flow to some extent by the streamlined body that represents the boundary layer, which came from the surface of the body [17, 18]. In [8] the one of the aforementioned models, namely:

$$\mu_t = \chi\rho\delta u_e \gamma \ln\left(\frac{l\sqrt{\tau}}{\chi\delta u_e / \nu_*} + 1\right), \; l = ky; \qquad (10)$$

$$k = 0,223 + 0,121\, arcg \frac{du_e}{dx},$$

$$\chi = 0,01 + \frac{1}{75 + (2,5 + \Phi_1)^2}. \qquad (11)$$

Fig. 4 compared numerical calculations with experimental data from [12] – the interaction of aerodynamic trace of a boundary layer, aerodynamic footprint – the result of the merger boundary layers formed on the top and bottom surfaces symmetrical profile NASA 0012 (chord length 100 mm, d – height gap between the streamlined surface and profile), set at zero angle of attack.

Comparative analysis shows almost exact reproduction trends deformation speed profiles along the coordinate $\bar{x}$.

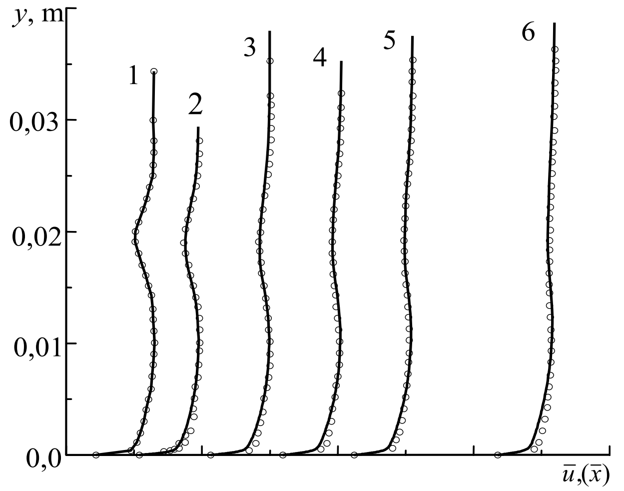

Fig. 4. The distribution of speed $\bar{u} = f(y)$ in aerodynamic profile of NASA 0012, which is immersed in the turbulent boundary layer flat in longitudinal section $x = const$, *lines* – numerical calculations; ○ – experimental data [12]

Calculations performed without iterative finite–difference method with second order accuracy in both directions.

Numerical calculations confirmed and feasibility of modifications related to the application of the modified cubic spline function [8] for setting the initial and boundary conditions at the outer edge of the turbulent boundary layer, in particular, they have successfully been used for interpolation table function that describes the shape of an airplane wing profile. Spline function with good accuracy interpolates experimental velocity profile at any point. Others require interpolation "stitching" in certain areas "pieces" of certain polynomials, which in turn requires "adjusting" their options.

4. Consclusions

1. In the illustration (Fig. 4) are showed comparison of calculated distributions of speed, $\bar{u}(y)$ $\bar{u} = u/u_e$, $\bar{x} = x/L$, $\bar{y} = y/L$, $\bar{u} = \bar{u}(\bar{y})\big|_{x=x_0}$ – initial conditions, L – the characteristic size of the streamlined surface) with experimental velocity profiles at different sections along the direction of flow. Comparison override comparing characteristics, $\delta^* = \int_0^\delta (1 - u/u_e)\, dy$, $\delta^{**} = \int_0^\delta u/u_e (1 - u/u_e)\, dy$ thickness displacement and loss of momentum respectively, which were at every turn by the formula Wadle as nature allows you to track playback calculated by deformation velocity profile. The calculations were carried out based on our current model of turbulence.

2. Comparison of the calculated results with experimental data show that the proposed approaches allow to flow adequately simulate their physical properties.

3. A mathematical model for calculating turbulent boundary layers and layers of wall streams makes it possible to count such a complex and valuable species reaches almost as aerodynamic body should be located in the boundary layer of smooth surface that allows hope for further generalization of the proposed approaches to more sophisticated trends.

4. Comparison of calculations with known experimental data studies [17] and [4] gave encouraging results, that confirm whether to extend numerical experiments to clarify the formula (9), (10), which enable to avoid using coefficients χ_1 and χ_2.

References

[1] *Ahlbery J. H.* Theory of Splines and Their Applications / J. H. Ahlbery, E. N. Nilson, J.L. Walsh. – New York, 1967.

[2] Computation of turbulent boundary layers. [Ed. S. J. Kline, E. A. Morcovin, G. Sovran, G.J. Cockrell]. 1968: AFOSRIPF Stanford Conference. – Stanford : University, 1969. Vol. 1, 590 p.

[3] *Fedyaevskyy K. K.* Calculation of turbulent boundary layer of an incompressible fluid / K. K. Fedyaevskiy, A. S. Hynevskyy, A. V. Koles-nikov. – St.P. : Sudostroenye, 1973, 256 p. (in Russian).

[4] *Kocheryzhnykov G. V.* About experience of numerical integration of turbulent boundary layer. G. V. *Kocheryzhnykov*, S. K. Matveev Hydroaeromechanic and theory of elasticity. 1968, Vol. 13, P. 39–50. (in Russian).

[5] *Horstmen*. Turbulence model for calculating nonequilibristic streams at positive pressure gradient / Horstmen // Rocket technics and spacecraft. – 1977, Vol. 15, № 2, P. 5–7. (in Russian).

[6] *Lapyn U. V.* The problem of "merging" in theory nonequilibristic turbulent streams / U. V. Lapyn, A. L. Yarin // Mechanics of fluid and gas. – 1979, № 3, P. 33–41. (in Russian).

[7] *Mamchuk V. I.* Mathematical modeling of turbulent near wall flaws on the oscillating bodies / V. I. Mamchuk // Bulletin of Lviv. Univ, Sir. glue.

Math. and Information. 2002, Vol. 4, P. 131–136. (in Ukrainian).

[8] *Mamchuk V. I.* Turbulence model and the results of calculations of plane turbulent near–wall steams / V. I. Mamchuk // Bulletin KIUCA. – K. : KIUCA. – 1998, № 1, P. 291–294. (in Ukrainian).

[9] *Novozhilov V. V.* Theory flat turbulent boundary layer of an incompressible fluid / V. V. Novozhylov. St. P: Sudostroenye, 1977, 164 p. (in Russian).

[10] *Pletcher*. Calculation of turbulent boundary layer at small Reynolds numbers. Rocket Technics and Space. 1976, Vol. 14, № 5, P. 181–183. (In Russian).

[11] *Sebesy*. Kinematic turbulent viscosity at small Reynolds numbers. Rocket technics and spacecraft. – 1973, Vol. 11, № 1, P. 121–123. (in Russian).

[12] *Tulapurkara G.* Interaction boundary layer with trace of bodies of different shapes / G. Tulapurkara, V. Ramzhy, R. Radzhasekar // Aerospace technic. – 1990, № 12, P. 3–10. (in Russian).

[13] *Hyntse Y. O.* Turbulence. Moscow: Fyzmathyz, 1963, 680 p. (in Russian).

[14] *Yoon* A. A. Theory and practice of modeling turbulent streams / A. A. Yoon. – Moscow: Book house "Liberkom", 2009, 272 p. (in Russian).

[15] *Mellor*. Incomeresible turbulent boundary layer at arbitrary pressure gradient with parting transverse flow / Mellor // Rocket technics and spacecraft. – 1977, Vol. 5, № 9, P. 43–54. (in Russian).

[16] Computation of turbulent boundary layers / Ed. P. E. Coles, E. A. Hirst. – 1968: AFOSR– IPF Stanford Conference. – Stanford : University, 1969, Vol. 2, 519 p.

[17] Aerodynamic traces in the gas compressor turbine engines : Monograph / [Y. M. Tereshchenko, M. S. Kulik, I. O. Lastivka and others]; ed. Y. M. Tereshchenko. – K. : NAU-Druk, 2012, 232 p. (in Russian).

[18] *Lastivka I. O.* The calculation of aerodynamic trace parameters behind compressor grating blade / I. O. Lastivka // Eastern European Journal of advanced technologies. – Kharkiv: Technological Center, 2011, № 4/7 (52), P. 47–50. (in Ukrainian).

Received 18 February 2016.

В.І. Мамчук[1], І.О. Ластівка[2], О.І. Безверхий[3]. **Дослідження та математичне моделювання турбулентного примежевого шару при додатному градієнті тиску**

[1,2]Національний авіаційний університет, просп. Космонавта Комарова, 1, Київ, Україна, 03058
[3]Інститут механіки ім. С.П. Тимошенка НАН України, вул. П. Нестерова, 3, м. Київ, Україна, 03057
E-mails: [1]vimamchuk@ukr.net; [2]iola@nau.edu.ua; [3]o_bezver@ukr.net

Мета: Математичне моделювання складних турбулентних пристінних течій, які виникають при обтіканні аеродинамічних профілів, неможливе без розуміння природи розвитку течії у примежевому шарі. З математичної точки зору розрахунок таких течій складає серйозну проблему і по сьогоднішній день, оскільки в практичних завданнях їх необхідно розглядати як турбулентні, а характеристики турбулентності значною мірою залежать від геометрії профіля поздовжньої складової осередненої швидкості пристінного струменя. Виходячи з цього, метою цієї роботи є дослідження та математичне моделювання турбулентних пристінних течій при взаємодії з реальною обтічною поверхнею, яка має свої певні особливості такі як кривизна, шорсткість тощо, а також вивчення й дослідження впливу градієнта тиску на емпіричні коефіцієнти, параметри течії, профілі швидкості і напруження тертя. **Методи:** Розрахунки виконано числовим скінченно-різнецевим маршевим методом з використанням алгебраїчної моделі коефіцієнта турбулентної в’язкості. **Результати:** У цій роботі викладено деякі результати числового дослідження впливу додатного градієнта тиску на емпіричні коефіцієнти перехідної зони і закону стінки пристінної та зовнішньої областей. **Обговорення:** Порівняння отриманих розрахункових результатів з експериментальними даними показує, що запропоновані підходи дають змогу моделювати течії максимально наближено їх фізичним властивостям. Запропонована математична модель для розрахунку турбулентних примежевих шарів і пристінних струменів дає можливість розраховувати і такий складний та цінний з практичної точки зору вид течії, як аеродинамічний слід за обтічним тілом, що вселяє надію на поширення даних підходів на більш складні види течій.

Ключові слова: градієнт тиску; примежевий шар; пристінний струмінь; профіль швидкості; турбулентна в’язкість.

В.И. Мамчук[1], И.А. Ластивка[2], А.И. Безверхий[3]. Исследование и математическое моделирование турбулентного пограничного слоя при положительном градиенте давления
[1,2]Национальний авиационный университет, просп. Космонавта Комарова, 1, Киев, Украина, 03058
[3]Институт механики им. С.П. Тимошенко НАН Украины, ул. П. Нестерова, 3, г. Киев, Украина, 03057
E-mails: [1]vimamchuk@ukr.net; [2]iola@nau.edu.ua; [3]o_bezver@ukr.net

Цель: Математическое моделирование сложных турбулентных пристенных течений, возникающих при обтекании аэродинамических профилей, невозможно без понимания природы развития течения в пограничном слое. С математической точки зрения расчет таких течений составляет серьезную проблему и на сегодняшний день, поскольку в практических задачах их рассматривают как турбулентные, а характеристики турбулентности в значительной степени зависят от геометрии профиля продольной составляющей осредненной скорости пристеночной струи. Исходя из этого, целью данной работы является исследование и математическое моделирование турбулентных пристенных течений при взаимодействии с реальной обтекаемой поверхностью, которая имеет свои определенные особенности такие как кривизна, шероховатость и т.д., а также изучение и исследование влияния градиента давления на эмпирические коэффициенты, параметры течения, профили скорости и напряжения трения. **Методы:** Расчеты выполнены численным конечно-разностным маршевым методом с использованием алгебраической модели коэффициента турбулентной вязкости. **Результаты:** В данной работе изложены некоторые результаты численного исследования влияния положительного градиента давления на эмпирические коэффициенты переходной зоны и закон стенки пристенной и внешней областей. **Обсуждение:** Сравнение полученных расчетных результатов с экспериментальными данными показывает, что предложенные подходы дают возможность моделировать течения максимально приближенно к их физическим свойствам. Предложенная математическая модель для расчета турбулентного пограничного слоев и пристенных струй дает возможность рассчитывать и такой сложный и ценный с практической точки зрения вид течения, как аэродинамический след за обтекаемым телом, что вселяет надежду на распространение данных подходов на более сложные виды течений.

Ключевые слова: градиент давления; пограничный слой; пристенный поток; профиль скорости; турбулентная вязкость.

Mamchuk Vitalij (1951). Ph.D., Associate Professor.
Higher and Computational Mathematics, Educational and Research Institute of Computer Information Technologies, National Aviation University, Kyiv, Ukraine.
Education: Ivan Franko National University of Lviv, Lviv, Ukraine (1973).
Research area: applied problems of mechanics
Publications: 73.
E-mail: vimamchuk@ukr.net

Lastivka Ivan (1956). Doctor of Technical Sciences, Professor.
Head of Department of Mathematics, Educational and Research Institute of Economics and Management, National Aviation University, Kyiv, Ukraine.
Education: Taras Shevchenko National University of Kyiv, Kyiv, Ukraine (1982).
Research area: models and methods to improve aerodynamic and aeroelastic performance of compressors of gas turbine engines.
Publications: 110.
E-mail: iola @nau.edu.ua

Bezverkhyi Oleksandr (1951). Doctor of physical and mathematical sciences, Professor.
Head of Department of electroelasticity of S.P.Timoshenko Institute of Mechanics, Kyiv, Ukraine
Education: Dnipropetrovsk State University, Dnipropetrovsk, Ukraine (1976).
Research area: hydroelectroelasticity
Publications: 140.
E-mail: o_bezver@ukr.net