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Lanchester's model of armed confrontation on the plane with the inclusion of UAVs

The project focuses on using AI to model combat scenarios involving UAVs and robotic systems. AI algorithms optimize strategies in real-time, adapting to battlefield conditions like troop positioning and threats. Mathematical models and machine learning algorithms are created and enhance decision-making, improving efficiency and minimizing losses.

The relevance of Lanchester-type models is evident from regular publications in military periodicals, highlighting their effectiveness for operational forecasting during hostilities [1][4]. These models, part of broader mathematical systems, are essential for quick assessments of force ratios and predicting outcomes of planned actions. Analytical modeling is primarily used for such tasks, incorporating dynamic coefficients based on available intelligence about the enemy [2]. The process is modeled by a system of differential equations, with the enemy's likely actions considered in simplified scenarios [3]. Lanchester's deterministic model for heterogeneous troops forms the basis of such analysis ([4]. p.10):

$$\frac{dB_i}{dt} = -\sum_j \alpha_{ji} R_j(t) \qquad (1.1)$$

$$\frac{dR_j}{dt} = -\sum_i \alpha_{ij} B_i(t) \qquad (1.2)$$

We will consider the case of three types of armed forces, where the indices *i, j* can take the values 1, 2, 3, and the damage matrix [*ij*] is square. It is well-known that in this model, the functions B_i and R_j can take on non-integer values. To describe the phenomenon of Poisson disposal, let $X_i(\lambda_i)$ and $Y_j(\mu_j)$ be independent random variables distributed according to the Poisson law. Assuming now that B_i and R_j are integers, we consider the equation in the form:

$$B_i(t+\Delta t) = B_i(t) - X_i(\Delta t \frac{B_i(t)}{B(t)} \sum_{i=1}^{3} \alpha_{ij} R_j(t)) \qquad (1.3)$$

$$R_j(t+\Delta t) = R_j(t) - Y_j(\Delta t \frac{R_j(t)}{R(t)} \sum_{i=1}^{3} \alpha_{ji} B_i(t)) \qquad (1.4)$$

Since $\Delta t > 0$ is small, the Poisson variables X_i and Y_j are similarly insignificant, which means that they are expected to take on zero values. The matrix $[\alpha_{ij}]$ used is:

$$[\alpha_{ij}] = \begin{bmatrix} 1;\ 3;\ 0{,}5; \\ 0{,}2;\ 1;\ 2; \\ 2;\ 1;\ 0{,}5 \end{bmatrix}. \qquad (1.5)$$

This matrix reflects a rock-paper-scissors dynamic, which is quite typical for the types of forces: each type of force has its own strengths and weaknesses: typically for infantry, tanks, and anti-tank units [5].

We examine Lanchester models for heterogeneous troops using Poisson disposal, focusing on the best and worst battle scenarios in terms of time and casualties. The best-case scenario for the blue side (Figs. 2 and 3) shows short battles with minimal troop losses:

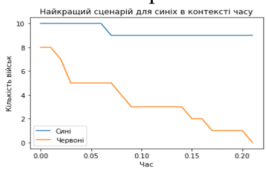


Fig. 2

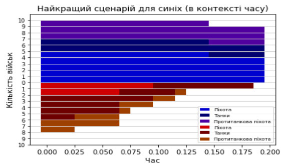


Fig. 3

while the red side's best scenario (Figs. 4 and 5) demonstrates similar trends.

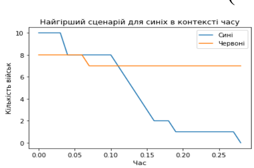


Fig. 4

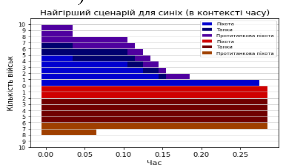


Fig. 5

In terms of minimizing losses, the blue side's best-case scenario (Figs. 6 and 7) involves prolonged combat with stable troop numbers.

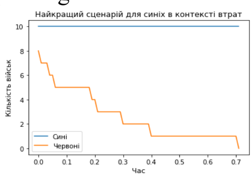


Fig. 6

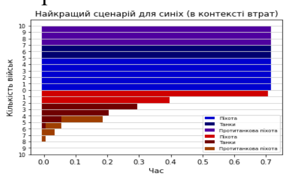


Fig. 7

For the red side, this trend is reflected in Figs. 8 and 9. In battles focused on time, infantry losses are quicker, while in casualty-driven scenarios, anti-tank infantry losses dominate.

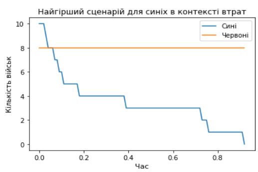


Fig. 8

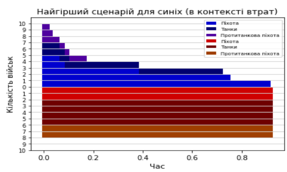
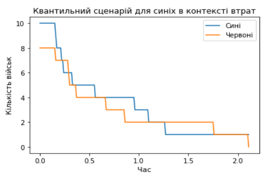


Fig. 9

The quantile scenario, representing an average battle outcome, is depicted in Figs. 10 and 11, showing equal chances for both sides.



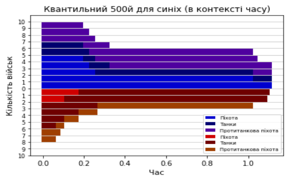


Fig. 10 Fig. 11

A battle fraction analysis (Fig. 12) provides insights into combat duration dynamics, which are crucial for military strategy.

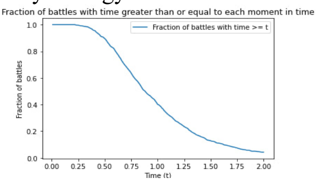
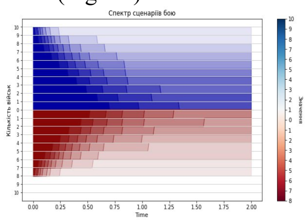


Fig. 12

Lastly, a heatmap (Fig. 13) illustrates troop dynamics over time, supported by average values (Fig. 14).



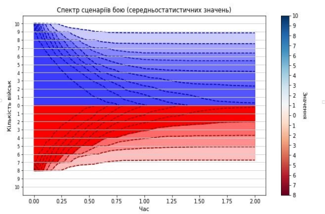


Fig. 13 Fig. 14

We construct a Lanchester space model that considers not only the temporal dynamics but also the location and movement of forces. The Euclidean distance metric is used for troop movement in a 400 by 400 grid, ensuring rapid movement to the center. The direction of movement for blue and red forces is represented by D_B and D_R respectively, where $B_{i(x;y)}$ and $R_{j(x;y)}$ are the positions of blue and red units, and N is the field size:

$$D_B = \left[\frac{\frac{N}{2} - B_{i(x;y)}}{\sqrt{\left(\frac{N}{2} - B_{i(x;y)}\right)^2 + \left(\frac{N}{2} - R_{j(x;y)}\right)^2}} \right] \qquad (1.6)$$

$$D_R = \left[\frac{\frac{N}{2} - R_{j(x;y)}}{\sqrt{\left(\frac{N}{2} - B_{i(x;y)}\right)^2 + \left(\frac{N}{2} - R_{j(x;y)}\right)^2}} \right] \qquad (1.7)$$

Movement is modeled as a two-dimensional random walk, with random displacements incorporating uncertainties. The final movement vector V_f is defined as:

$$V_f = (1 - p) * V_m + p \cdot R \qquad (1.8)$$

where p represents the probability of random movement, and R is a uniform distribution vector for movement in any direction. Elimination of combat units is determined by calculating the probability for each unit to be destroyed based on the surrounding conditions. The risk posed by a blue unit at coordinates (x, y) and belonging to troop type $i \in \{1,2,3\}$ to a red unit at coordinates (x, y) of type $j \in \{1,2,3\}$ is given by the formula:

$$P_j = \frac{\alpha_{i,j}\Delta t}{N_{Red,r_{attack}}(x,y)F(\bar{x},\bar{y})},$$

In the second case, we will consider two types of air forces, namely reconnaissance drones and kamikaze drones, by taking into consideration their interaction and impact on combat. First and foremost, we will create a model for reconnaissance drones, which provide a coefficient of information before and during combat. They gather information about ground forces, which allows kamikaze drones to inflict more damage later. Therefore, the formula for calculating the amount of information I that a reconnaissance drone collects during a certain time Δt is based on the theory of Poisson event flow and probability distributions (2.10):

$$I(t + \Delta t) = \frac{e^{\Delta t \lambda_d k}(\Delta t \lambda_d k)^k}{k!} \qquad (1.10)$$

From now on, we will concentrate on kamikaze drones, which are unmanned aerial vehicles designed to attack targets in order to inflict maximum damage. Such drones are commonly used to deliver the maximum possible strike at a single point in time. Therefore, their number decreases in proportion to the attacks they carry out, and they cannot accumulate information or maintain their numbers. Based on these assumptions, equations can be created:

$$B_d(t + \Delta t) = B_d(t) - X_d(\Delta t \frac{B_r(t)}{B(t)} \sum_{i=1}^{3} \beta_{ij} R_j(t)) + \frac{e^{\Delta t \lambda_d k}(\Delta t \lambda_d k)^k}{k!} \qquad (1.11)$$

$$R_d(t + \Delta t) = R_d(t) - Y_d(\Delta t \frac{R_r(t)}{R(t)} \sum_{i=1}^{3} \beta_{ji} B_i(t)) + \frac{e^{\Delta t \lambda_d k}(\Delta t \lambda_d k)^k}{k!} \qquad (1.12)$$

Conclusions: We investigated changes in combat using the Lanchester model with Poisson disposal, focusing on troop dynamics over time. The study addressed heterogeneous troops, Poisson disposals, and applied Monte Carlo methods to solve differential equations. We utilized Python to simulate combat flow, predict outcomes, and generate detailed graphs, allowing for year-long predictions in small increments. Methods such as modeling, analysis, and systematization were employed to develop a mathematical Lanchester model for force dynamics. Additionally, programs for automated histograms, statistical models, and visual troop positioning were created. The model effectively represents combat dynamics and confirms that troop numbers and positioning significantly influence outcomes. Spatio-temporal simulations verified that location plays a critical role in attrition-based conflict modeling.

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