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V. Kovalchuk, PhD
(State University of Infrastructure and Technologies, Ukraine)

Two-dimensional numerical simulations of Hopf bifurcations for pendulum systems

This article examines the approach to predicting the dynamic behavior of pendulum systems when essential parameters change. Analytical and numerical methods are used to study flutter bifurcations of an inverted pendulum with a follower force. The proposed method demonstrates higher accuracy when calculating the limit cycle.

Mathematical modeling of the pendulum system with a follower force

Pendulum systems are classic models of theoretical mechanics and nonlinear theory of oscillations, the study of which allows us to identify many important nonlinear dynamic effects. In various modern branches of applied mechanics, such systems are used as an effective models for various processes and structural elements. We observe the practical use of pendulum systems in many technical and transport vehicles. In particular, in the technique for solving problems related to stability of movement and unwanted oscillations, pendulum extinguishers are used.

In the current transport industry, a lot of attention is paid to the design and research of high-speed vehicles. In particular, the demand for faster vertical-lift air vehicles is growing [1]. Accordingly, the development and improvement of propeller-driven aircraft require additional research into the features of dynamic instability, which may arise as a result of the interaction between additional inertial, elastic, and aerodynamic loads.

It was found experimentally that when the variable (control) parameters of the relevant systems exceed the critical values that determine the limit of stability, such an undesirable phenomenon as vortex flutter may occur. As a result of this phenomenon, aeroelastic systems recover more slowly to equilibrium after disturbances [1].

Modeling and prediction of the boundary of the stability region corresponding to flutter bifurcations is of great importance for rational design and improvement of the object being studied (a structural element of a vehicle, for example).

Analytical and numerical simulations are important for the study of flutter and other features of the behavior of dynamic systems and, in particular, for global bifurcation studies of the influence of essential parameters of these systems [2].

This work is devoted to the study of Hopf bifurcations in the phase space of an inverted double pendulum with a follower force. This force can be considered imperfect in the sense that the direction of its action does not coincide with the axis of the second (upper) link of the pendulum. This approach is considered in [3], where the influence of the system parameters on the value of the critical load is analyzed and it is shown that near the boundary of the domain of flutter instability, the value of the critical load can be increased by reducing the stiffness of the hinge at the point of attachment of the pendulum.

This study uses Hopf's theory of bifurcations, which is an important tool for studying the dynamics of nonlinear systems. The application of this theory makes it possible to analyze in detail the stability of solutions differential equations and system behaviour with successive changes in the values of essential parameters.

The investigated pendulum has two degrees of freedom. The differential equations of the perturbed motion of the pendulum in the normal Cauchy form can be written in the form:

$$
\begin{aligned}
x_1' &= x_2, \\
x_2' &= F_2\left(x_1, x_2, x_3, x_4\right), \\
x_3' &= x_4, \\
x_4' &= F_4\left(x_1, x_2, x_3, x_4\right).
\end{aligned}
$$

Here $x_1 = \varphi_1$, $x_2 = \varphi_1'$, $x_3 = \varphi_2$, $x_4 = \varphi_2'$ – the state variables of the studied mechanical system.

Numerical analysis of the domain of stability of a double pendulum

Integration of the indicated equations can be carried out using the Maple application package. Numerical modeling of phase portraits in the MATLAB environment shows that qualitative changes in the dynamic behavior of the system occur when the essential parameters of the pendulum change. In particular, Hopf bifurcations are observed when crossing the boundary of the stability region, which corresponds to a pair of purely imaginary roots of the characteristic equation.

It is shown that in the critical case of one pair of purely imaginary roots of the characteristic level, the boundary of the stability region can be described by the equation:

$$A \cdot P^2 + 2B \cdot P \cdot c + C \cdot c^2 + 2D \cdot P + 2E \cdot c + F = 0 . \quad (1)$$

Here P and c – system parameters that we consider essential. At the same time, P is the module of the follower force applied to the upper end of the investigated inverted pendulum. The second parameter is c – the stiffness coefficient of the elastic fastening of the upper end of the pendulum.

When carrying out numerical calculations, the transition to dimensionless quantities was made. To do this, we take the parameters of the lower link of the pendulum as basic parameters: m_1 (mass, kg), l_1 (length, m), c_1 (rigidity of elastic fastening, N·m). All other system parameters are related to the specified ones. In particular, for essential parameters we have:

$$\overline{P} = P \cdot \frac{l_1}{c_1}, \quad \overline{c} = c \cdot \frac{l_1^2}{c_1}.$$

The character of the line described by equation (1) is determined by the discriminant of the senior terms of the equation, i.e., the value of δ:

$$\delta = \begin{vmatrix} A & B \\ B & C \end{vmatrix}.$$

Numerical simulations of the division of the first quadrant of the plane of essential parameters ($c > 0$, $P > 0$) into domains with different characteristics of stability of the vertical equilibrium position of the pendulum were carried out. It is shown that at different values of the tracking force asymmetry parameters, the boundaries of the stability region are elliptical and hyperbolic lines. When the imaging point of the system passes through these lines, Hopf bifurcations occur. The corresponding stable and unstable limit cycles can be constructed using MATLAB tools. During the research, the influence of the orientation parameters of the follower force on the configuration and stability of the limit cycles was analyzed.

Conclusions

Hopf bifurcations occurring in the phase space of a dynamic system simulating an inverted double pendulum with a tracking force are considered. Numerical modeling by means of Maple and MATLAB application programs was used to study areas of system stability. Qualitative changes observed in the dynamic behavior of the system after Hopf bifurcations are analyzed.

References

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