UDC 629.76

A. Kovalenko, A. Klipa. PhD
(National Aviation University, Ukraine)

Apogee Determination in Student Rocket Development

The article discusses methods for determining the apogee of a student rocket's flight. Analytical and numerical approaches are presented for calculating the maximum altitude reached by the rocket, taking into account various factors affecting the flight trajectory.

Precise determination of the apogee of a flight is important in establishing a student rocket due to safety implications and for achieving set objectives. The apogee is an instant, highest altitude point that the rocket achieves at its trajectory. Being able to predict this point with accuracy allows for optimization both in the design of the rocket and of its payload.

Theoretical Foundations

This process involves essentially the classical mechanics of determining the apogee of the rocket flight, given by the laws of motion of a body under gravity and aerodynamic action. It is very significant to predict as precisely as possible what maximum altitude or apogee the rocket will cover during the mission planning phase so that the rocket meets its objectives with safety considerations being well observed.

Basic Motion Under Gravity

If air resistance is to be neglected, as one might do in some idealization, the maximum altitude h reached by a rocket, given initial velocity and launch angle, is a function of those two: its initial velocity and its angle of launch. A formula for this can be derived from the kinematic equations of motion:

$$h = \frac{V_0^2 \sin^2 \theta}{2g}$$

where V_0 represents the initial velocity, θ represents the launch angle, and g represents the acceleration due to gravity. This equation considers the rocket is taken in a vacuum without any other influences such as air resistance acting on it. Under these ideal conditions, apogee would be dependent on the square of the initial velocity and the square of the sine of the launch angle. It really indicates that finding an optimal combination of those two parameters mentioned is crucial for obtaining maximum altitude. However, under more realistic conditions, consideration should be given to the effect of the aerodynamic drag that decreases the apogee height.

This equation assumes a vacuum environment where no other forces, such as air resistance, act upon the rocket. Under these simplified conditions, the apogee depends directly on the square of the initial velocity and the square of the sine of the launch angle, highlighting the importance of optimizing both parameters to maximize altitude.

However, in real conditions, it is necessary to consider aerodynamic drag, which reduces the apogee height.

Methodology for Calculating the Apogee

The numerical simulation of flight trajectories with variable air resistance is a far more precise way of calculating the apogee of the trajectory. The differential equations describing the rocket motion are solved by the fourth-order Runge-Kutta method.

$$\begin{cases} \dfrac{dv}{dt} = -g - \dfrac{C_d A v^2}{2m} \\ \dfrac{dx}{dt} = v \end{cases}$$

where v is the velocity, t describes time, Cd is the drag coefficient, ρ air density, A cross-sectional area of the rocket, and m mass of the rocket. Due to the fact that the drag force includes the v^2 term, these equations are coupled and nonlinear; for that reason, analytical solutions corresponding to most practical situations are not possible. That is why numerical methods become indispensable with respect to such equations for solving, with a view to predicting the apogee of the rocket with good accuracy.

Practical Implementation

In the student rocket design process, all necessary specifications and performance criteria were conducted with large numerical calculations using MATLAB software. The critically important parameters, such as mass, shape, cross-sectional area, and drag coefficient, were duly considered with the motive of building a more accurate simulation model. Detailed analysis of the rocket mass—structural parts and propellant—associated with total weight and its consequences in acceleration and velocity during the phases of flight. The aerodynamic shape was formulated with minimum air resistance, yet stable; a lot of special attention has been paid to the configuration of the nose cone and fuselage. Optimization of the cross-sectional area was a major compromise factor between drag reduction and accommodation of space necessary for internal systems and payload. The estimation of aerodynamic forces acting on the vehicle under conditions of desired speed and altitude was fairly precise, as this was all based on the rocket geometry and material properties which allowed determination of the drag coefficient. With the findings indicated in the simulation results, it follows that the launch conditions were not sufficient to reach the intended apogee, but forces had to be compromised. That is, it actually pointed out that better performance of the rocket requires initial thrust and better mass distribution. This kind of finding subsequently led the group to further develop the launch parameters such that the rocket would hit a stated apogee during the final set of flying tests. In consequence, making the iterative adjustments on the model by reviewing simulation feedback made the development process quite efficient because it ensured high reliability and performance for the student rocket.

Conclusion

Precise determination of the apogee point is one of the most important issues in the successful development of a student rocket. Meeting the required apogee guarantees the mission's objectives of the rocket, which may vary from some scientific experiments on altitude attainment to the testing of some propulsion systems or even engineering principles. The precision of the prediction and attainment of the apogee directly influences the overall performance and reliability of the rocket and hence is a critical focus area during the design and testing phases. Numerical methods and simulation tools, such as MATLAB, therefore play an important role in this process. These computational techniques allow the engineer and the student to model a realistic rocket flight trajectory due to the myriad complicated factors causing effects on the nature of the rocket's ascent. Variables that can be captured during simulation range from aerodynamic drag, changing mass due to fuel consumption, changeable air density with altitude, and even variable launch angle effects. An integrated approach that enables them to catch most of the problems and optimize the design parameters before any physical test, reducing failure risks and allowing the development process to be more efficient.

References

1. Sutton G.P. *Rocket Propulsion Elements*. John Wiley & Sons. – New York, 2017. – 784 p.
2. Barrowman J.S. *The Practical Calculation of the Aerodynamic Characteristics of Slender Finned Vehicles*. National Aeronautics and Space Administration. – Washington D.C., 1967. – 45 p.
3. Humble R.W., Henry G.N., Larson W.J. *Space Propulsion Analysis and Design*. McGraw-Hill. – New York, 1995. – 736 p.