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D.A. Poltoratskyi
(National Aviation University, Ukraine)

Design and application of the Walsh-like bases with linear coherence for DFT processor frequency scales

New unique bases Walsh-Cooley and the Walsh-Tukey are designed to provide linear coherence to the frequency scales of the Discrete Fourier Transform (DFT) processor. The performance and practical application of the DFT processor, using these bases, are demonstrated through a comparative analysis, showcasing the benefits of these new bases in improving computational outcomes.

Introduction into the Walsh-like bases.

Walsh systems are widely used in signal processing due to their orthogonal properties, which aid in transforming discrete signals efficiently. While Walsh-like bases outperform the Discrete Fourier Transform (DFT) in processing digital, pulse, or rectangular signals, the DFT is more suited for sinusoidal signals because of its sinusoidal basis functions. Traditional Walsh bases, such as Walsh-Hadamard and Walsh-Paley, lack linear coherence to frequency scales, resulting in inconsistent frequency resolution. This paper introduces two Walsh-like bases, Walsh-Cooley and Walsh-Tukey, which address this issue by providing linear coherence, enhancing their utility in digital signal processing.

A complete system of Walsh functions creates a Walsh's space basis, where each function is called a basis function of the N order. All basis functions are represented as symmetric matrix of N order. Matrix for Walsh-Cooley and Walsh-Tukey bases are shown in expressions (1) and (2):

$$W_C = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & +1 & +1 & +1 & -1 & -1 & -1 & -1 \\ +1 & -1 & +1 & -1 & -1 & +1 & -1 & +1 \\ +1 & +1 & -1 & -1 & -1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & -1 & +1 & +1 & -1 \end{bmatrix} \quad (1)$$

$$W_T = \begin{bmatrix} +1 & +1 & +1 & +1 & +1 & +1 & +1 & +1 \\ +1 & -1 & -1 & -1 & -1 & +1 & +1 & +1 \\ +1 & -1 & -1 & +1 & +1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & -1 & +1 & -1 & -1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 \\ +1 & +1 & -1 & +1 & -1 & -1 & +1 & -1 \\ +1 & +1 & -1 & -1 & +1 & +1 & -1 & -1 \\ +1 & +1 & +1 & -1 & -1 & -1 & -1 & +1 \end{bmatrix} \quad (2)$$

The transition from the time domain to the frequency domain is achieved by multiplying the input signal samples by a matrix of size N. This operation is equivalent to performing a matrix-vector multiplication. Very important parameter here is the size of matrix which can be calculated in the next way:

$$N = 2^k, k \in \mathbb{N} \quad (3)$$

Design of the Walsh-Cooley basis.

The Walsh-Cooley matrix is a special version of the Walsh matrix ordered using the Gray code. The Walsh-Cooley matrix is based on the Hadamard matrix, a square matrix whose elements are +1 and -1 and which is orthogonal. To get the Walsh-Cooley matrix, you need to apply Gray's code to rearrange the rows of the Hadamard matrix. A Gray code is a sequence of binary numbers where each subsequent number differs from the previous one by changing only one bit. The result of rows rearrangement is the Walsh-Cooley matrix. Algorithm complexity is quadratic time $O(N^2)$.

Design of the Walsh-Tukey basis.

The algorithm of the Walsh-Tukey matrix designing shown on Figure 2. Result of the algorithm is the N-order matrix. Firstly, we fill-in initial zero and first rows. Where zero rows fully consist of the 0 (+1) values and the first row contains half of the N non-zero values 1 (-1), other places are filled by zero elements. Each row in the matrix equals the column with the same index. By next three steps we do: filling even rows, filling odd rows, rows supplementation. Even rows are filled using expression:

$$(2k, t) = (k, 2t) \quad (4)$$

Which means that the first half of the even row copies even elements of the row whose index is two times less than current. Outstanding elements can be easily added following the rule that the second half of even rows copy the first one.

Odd rows can be filled using expression (2). Where the odd row is fully copied from the selected even row:

$$row_{odd} = (N + 1) - row_{even} \quad (5)$$

And the final third step is a search over rows where elements of one half of the row are copying elements from another for even rows and for odd rows elements of the one half are inverted to another.

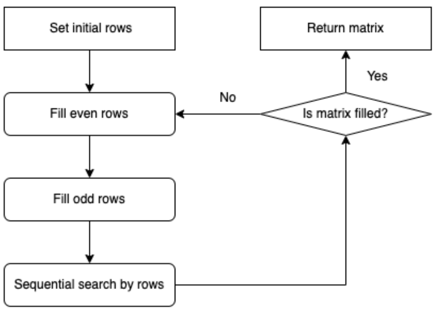

Fig. 1. Block diagram of the Walsh-Tukey system designing algorithm

If the matrix is not filled after the third step, the algorithm should be repeated from the first step. Algorithm complexity is $O(\log N)$.

Walsh-like bases application in digital signal analysis.

Let's take as an example the transformation of the square shaped signal using Walsh-Cooley basis. Input signal parameters are next: frequency is 62,5 Hz; sampling frequency is 1000 Hz; length is 128 points.

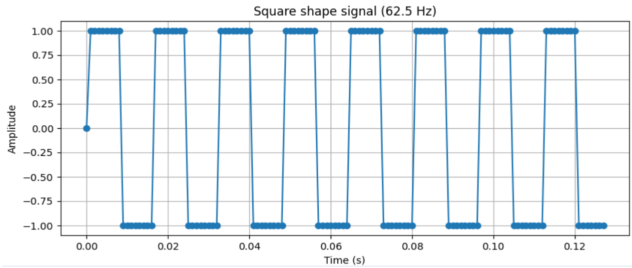

Fig. 2. Generated square shaped signal

Let's transform signal using Walsh-Cooley matrix, where $N = 128$. Signal spectrum is shown on Figure. 3.

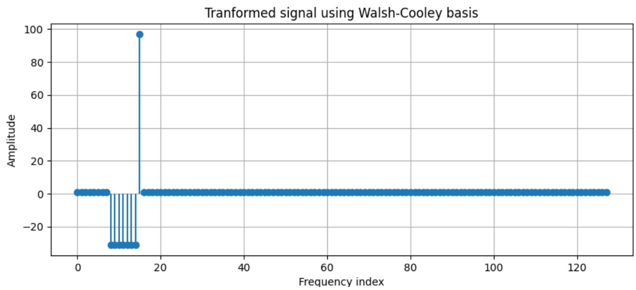

Fig. 3. Transformed signal using Walsh-Cooley basis

In Walsh-Cooley (frequency) domain the signal is divided in frequency indexes. The first index with higher amplitude is 8, which we can map on frequency value using expression (6):

$$f_k = \frac{k * f_s}{N} \quad (6)$$

So, $f_8 = 62{,}5\ Hz$ that equals to main frequency of the signal, other amplitude jumps show other harmonics. This feature to map Walsh-Cooley frequency index to a real frequency value is achieved because of linear coherence to the frequency scales. Transformation in Walsh-Tukey basis has the same steps and only difference is used matrix.

Conclusions.

In this paper, we introduced the Walsh-Cooley and Walsh-Tukey bases, demonstrating their potential to provide linear coherence to the frequency scales of the DFT processor. These bases address the limitations of traditional Walsh systems, particularly in achieving consistent frequency resolution. Through a comparative analysis and practical application, we have shown that Walsh-like bases offer computational efficiency, ease of hardware implementation, and improved signal processing for digital, pulse, and rectangular signals. The proposed bases open new avenues for optimized digital signal processing, providing a balanced trade-off between resource usage and performance, especially when working with non-sinusoidal signals.

References

1. J. Walsh, "A closed set of normal orthogonal functions", Amer. J. Math., v. 45, pp. 5-24, 1923.Hadcock R.N. Joints in Composite Structures. Proc. Conf.Vehicle Design AFFDL-TR-72-13. – 1972. – P. 791–811.
2. Белецкий А. Я. Оптимальные Уолша и Уолше-подобные базисы дискретного преобразования Фурье. / А. Я. Белецкий. // Захист інформації. — 2018. – Т. 20, № 2. – С. 104-119.