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Long-term strength of quasilinear composite elastomers of the periodic structure

A model and criterion of long-term strength for composite elastomers of periodic structure is constructed. The parameters of the stress concentration at the boundary of the components are determined. The possibility of predicting the long-term strength of the hereditary material is shown. The computer software Wolfram Mathematica 14.1 is used for numerical analysis and illustrations.

Introduction

Composite elastomers of a periodic structure in the modern classification of mechanical metamaterials belong to a group with certain mechanical properties [1, 2, 6] arising from the geometry of their subunits, and not only from the composition of the material [1, 3, 4]. The concept of metamaterial has been introduced from electromagnetics and acoustics to mechanics. Composite structures are the object of research in the mechanics of long-term deformation and fracture [1, 5, 7]. Still, methods for three-dimensional analysis and design of these engineering microstructures are only emerging.

Generally, metamaterials have a variety of significantly improved mechanical properties, including zero or negative Poisson's ratio, vanishing shear modulus, significantly nonlinear time-dependent behavior, and other features that distinguish them from conventional natural and composite structures. In contrast to purely elastic or viscous heterogeneous media, the connection of conservative and dissipative deformation mechanisms can lead to new effective properties that are absent at the scale of individual components. This can be confirmed using the correspondence principle [3], which allows you to transform a viscoelastic problem into a symbolically elastic one in the domain of variables of the Laplace–Carson transform. It is of practical interest to use the complex given properties characterizing the reaction of the composite medium to harmonic loads [6].

State of the Problem

From the principle of Boltzmann superposition, the reaction to stress of a linear hereditary viscoelastic material with a given deformation history $\mathbf{e}(u)$, $u \in [0;t]$ with the initial condition $\boldsymbol{\sigma}(t=0)=0$, we can write [3, 6]

$$\boldsymbol{\sigma}(t) = \frac{d}{dt}[\int_0^t \mathbf{R}(t-u)\mathbf{e}(u)]du$$

with $\mathbf{R}(t)$ the viscoelastic stiffness tensor (relaxation function) or, in a short form of notation

$$\boldsymbol{\sigma}(\mathbf{x},t) = (\mathbf{R} * d\mathbf{e})(\mathbf{x},t). \qquad (1)$$

Here $*$ the product of the integral convolution, tensors are in bold throughout the text. Similarly, the response of the deformation $\mathbf{e}(t)$ to the stress history $\boldsymbol{\sigma}(u), u \in [0;t]$, and the initial condition $\mathbf{e}(t=0)=0$, is written

$$\mathbf{e}(t) = (\mathbf{J} * d\boldsymbol{\sigma})(t) \tag{2}$$

$\mathbf{J}(t)$ is the viscoelastic compliance tensor (creep function), the general form of which is

$$\mathbf{J}(t) = \mathbf{S} + \frac{1}{\boldsymbol{\eta}_r} t + \int_0^{\infty} \mathbf{H}(\tau)(1 - e^{-t/\tau}) d\tau$$

with $\mathbf{S}$ elastic compliance, $\frac{1}{\boldsymbol{\eta}_r}$ retarded viscous compliance and $\mathbf{H}$ retardation spectrum. The relaxation and retardation spectra characterize the viscoelastic transient process of the long-term reaction of the material.

We apply the formulation of the problem in terms of quasi-linear hereditary creep with fractional defining equations [3, 6]. The constitutive equation has the form (1) with a fractional relaxation function $\mathbf{R}_{\mu}(t)$

$$\mathbf{R}_{\mu}(t) = \mathbf{C}_r + \int_0^{+\infty} \mathbf{G}(\tau) E_{\mu}[-(t/\tau)^{\mu}] d\tau,$$

where $E_{\mu}(t)$ is the Mittag-Leffler function [3, 6]. It is obvious that for $\mu = 1$ the relaxation function $\mathbf{R}_{\mu}(t)$ corresponds to the classical relaxation function $\mathbf{R}(t)$. Deformation in hereditary materials is determined by the stress history $\boldsymbol{\sigma}(u),\ (u \in [0,t])$, as well as the initial conditions $\boldsymbol{\sigma}(0) = \mathbf{0}$. For a linear medium, after integration in (2, it is possible to write down in the general tensor form [4, 6]

$$\mathbf{e}(\mathbf{x},t) = \int_0^t \mathbf{J}(t-u) \frac{d}{du} \boldsymbol{\sigma}(\mathbf{x},u) du,$$

where $\mathbf{J}(t) = \mathbf{J}^e(\boldsymbol{\sigma}) g(t)$ is the nonlinear creep tensor function (retardation). In a short symbolic form [3], we have

$$\mathbf{e}(\mathbf{x},t) = \frac{d}{dt}(\mathbf{J} * \boldsymbol{\sigma})(\mathbf{x},t) = (\boldsymbol{\sigma} * \dot{\mathbf{J}})(\mathbf{x},t), \tag{3}$$

where the asterisk denotes the integral convolution operation. Thus, the expression $(\boldsymbol{\sigma} * \dot{\mathbf{J}})(t) = (\boldsymbol{\sigma} * d\mathbf{J})(t)$ is a Stieltjes convolution [7].

Harmonic loads of multi-component elastomers

The local problem for harmonic loads is solved according to the principle of correspondence to the transformation of the fractional viscoelasticity problem into the symbolic elasticity problem. From time to time, the problem is transferred to the LC region and allows you to determine the moduli that characterize the viscoelastic reaction of the material. Consider an inhomogeneous medium occupying the volume Ω, and consisting of N homogeneous phases with the characteristic function

$\chi^{(s)}(\mathbf{x})$ and the volume $\Omega^{(s)}$, $(s \in [0;N])$. In addition, it is assumed that $\Omega^{(s)} << \Omega$ and that the phases are perfectly connected. The fractional hereditary relaxation function of the phase (s) is denoted by $\mathbf{R}_{\mu}^{(s)}(t)$. It follows that the local relaxation tensor $\mathbf{R}_{\mu}(\mathbf{x},t)$ has the form

$$\mathbf{R}_{\mu}(\mathbf{x},t) = \sum_{s=1}^{N} \mathbf{R}^{(s)}(t)\chi^{(s)}(\mathbf{x})$$

with $\chi^{(s)}(\mathbf{x}) = 1$ if $\mathbf{x} \in \Omega^{(s)}$ and 0 otherwise. The average volume values for Ω and $(\Omega^{(s)}$ are denoted by $\langle \cdot \rangle$ and $\langle \cdot \rangle^{(s)}$, respectively. By definition, the characteristic function, the volume fraction of the phase (s) is $c_s = \langle \chi^{(s)} \rangle$. The volumetric averages of the function f for the Ω composite and for the Ω^r phase are denoted respectively.

The reaction of a viscoelastic inhomogeneous medium to sinusoidal load is transformed into a spectral region by considering the Laplace–Carson transform of the basic equation for a purely imaginary transformation variable $z = i\omega, (i^2 = -1)$.

Assuming the total strain load $\overline{e}(t) = LC(\overline{e}^A) * e^{i\omega t}$, the local problem corresponding to the steady-state mode at angular frequency ω is written as

$$\boldsymbol{\sigma}^*(\mathbf{x},i\omega) = \mathbf{R}^*(\mathbf{x},i\omega)\mathbf{e}^*(\mathbf{x},i\omega), \qquad div\ \boldsymbol{\sigma}^*(\mathbf{x},i\omega) = \mathbf{0},$$

$$curl(curl\ \mathbf{e}^*(\mathbf{x},i\omega)) = \mathbf{0}, \qquad \forall(\mathbf{x},\omega) \in \Omega \times [0;+\infty] \tag{4}$$

with given boundary conditions $\langle \mathbf{e}^* \rangle = \overline{\mathbf{e}}^*$.

The constitutive equation in the complex domain is written

$$\langle \boldsymbol{\sigma}^* \rangle(i\omega) = \tilde{\mathbf{R}}_{\mu}^*(i\omega)\langle \mathbf{e}^* \rangle(i\omega), \qquad \forall \omega \in [0;+\infty[, \tag{5}$$

Then the relaxation tensor is represented as the sum of real and imaginary parts

$$\tilde{\mathbf{R}}_{\mu}^*(i\omega) = \tilde{\mathbf{R}}_{\mu'}(\omega^{\mu}) + i\tilde{\mathbf{R}}_{\mu'}(\omega^{\mu}). \tag{6}$$

Here $\tilde{\mathbf{R}}_{\mu'}(\omega^{\mu})$, $\tilde{\mathbf{R}}_{\mu''}(\omega^{\mu})$ are the accumulation and dissipation moduli proportional to the stored and dissipated energy. Since $i^{\mu} = e^{i\pi\mu/2}$, then

$$\tilde{\mathbf{R}}_{\mu'}(\omega^{\mu}) = \tilde{\mathbf{C}}_r + \int_0^{+\infty} \frac{1}{q}[\theta cos(\frac{\pi\mu}{2}) + \theta^2]\tilde{\mathbf{G}}(\tau)d\tau$$

$$\tilde{\mathbf{R}}_{\mu''}(\omega^{\mu}) = \int_0^{+\infty} \frac{1}{q}[\theta sin(\frac{\pi\mu}{2})]\tilde{\mathbf{G}}(\tau)d\tau.$$

At the same time,

$$\theta = (\omega\tau)^{\mu}, \qquad q = 1 + 2\theta cos(\frac{\pi\mu}{2}) + \theta^2.$$

The effective scattering tensor $\tilde{\boldsymbol{\eta}}(\omega^{\mu})$, which characterizes the attenuation, is determined by

$$\tilde{\boldsymbol{\eta}}(\omega^{\mu}) = \mathbf{R}''(\omega^{\mu})[\mathbf{R}'(\omega^{\mu})]^{-1}. \tag{7}$$

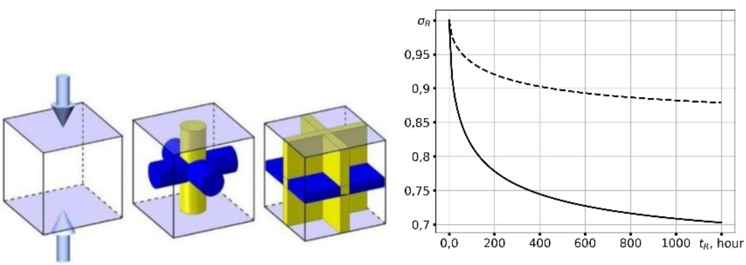
The gradients of displacements of the second approximation are decomposed into dilatational and deviator parts. Then for the deviators of the second approximation the condition of incompressibility will coincide in shape with the analogous condition of the linear theory of viscoelasticity [2, 6]. Consider the case when the material is composed of periodically spaced cells. A periodic environment is obtained due to periodic cell transfer along different directions in space [7].

For most metal elastomers the linearity range is relatively small, and satisfactory results can be obtained at low stress and only for short loading duration. So we use here quasilinear equations of viscoelasticity given in [3, 6]. It is known that constitutive equations formed by multiple integrals make it impossible to identify integral kernels and determine their parameters in typical experiments. An approach based on the similarity of isochronous creep diagrams is more promising for building nonlinear models of hereditary creep [7]. In [3], this algorithm was further developed due to the expansion of the initial condition of similarity, which includes the diagram of instantaneous deformation as an isochrone for the zero moment. The extension of the similarity condition made it possible to build a nonlinear creep model with a time-invariant nature of the nonlinearity, which is determined by the instantaneous deformation diagram.

Examples

It is of practical interest to use such complex given properties to characterize the reaction of the composite medium to harmonic loads. For example, a typical solid propellant material consists of polymeric binder and other additives for improved bonding and burning [2, 6].

Fig1.



(a) (b)

Solid propellants show distinct nonlinear time-and temperature-dependent viscoelastic behavior because of unexpected damage propagation under various loading and pressurizing conditions. Therefore, the study of the mechanical properties and viscoelastic behavior for the structural integrity of propellant grain is the goal of practical importance in the strength and durability of aviation machines.

Figure 1a shows a schematic diagram of options for improving the efficiency of composite elastomer metastructures In fig. 1b shows comparative graphs of the dependence of the critical stress σ_R on time t matrix (dashed line) and the reinforced composite (solid).

The software complex Wolfram Mathematica 14.1 [8] (license 8801-3966) was used for numerical analysis and preparation of illustrative.

Conclusions:

The method of successive approximation is used to obtain the full system of the hereditary creep equations of the second order. The creep functions of the laminate composite are found. Also, interface stress concentration parameters are determined. The examples show the importance of the mutual influence of nonlinear elastic and creep properties of the components on long-term fracture parameters. A practical result is the possibility of finite element modeling the long-term strength of composite multilayered structures.

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