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Modeling of optimization problems in the electric power industry by function minimization methods

The proposed article considers the application of methods for minimizing functions of many variables for solving optimization problems in the electric power industry. It is noted that these methods are the random search method, the direct search method, and the statistical gradient method. The methods of linearization of optimization problems of nonlinear programming are analyzed.

I. Introduction

When searching for solutions to optimization electric power problems, methods of minimizing functions of many variables are used. Probabilistic modeling of random processes of power consumption makes it possible to implement the random search method as a method of minimizing functions of many variables. The random search method is characterized by the introduction of an element of randomness into the search algorithm. This method is based on the generation of the sequence $\{u_k\}$ according to the algorithm:

$$u_{k+1} = u_k + \alpha_k \varepsilon, k = 0,1, \ldots,$$

$\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ −where α_k- positive value; realization of *n* - dimensional random variable ε with a known distribution law.

A variation of the random search method without learning is the statistical gradient algorithm [1]. It should be noted that random search algorithms without training do not have the ability to analyze the results of previous iterations, as well as to determine the direction of decrease of the minimized functions. These algorithms are characterized by slow convergence.

II. Problem statement

Simulation of complex electric power systems in the search for solutions to minimization problems involves the use of random search algorithms with training. These algorithms are characterized by the property of self-learning in the process of finding a minimum. The learning of the algorithm occurs in the process of changing the law of distribution of a random vector $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ depending on the iteration number and the results of previous iterations. At the same time, those directions in which the function under study is decreasing have become more probable, while other directions have become less probable. That is, random vectors $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n)$ with different distribution laws should be analyzed at different stages of this method. In this case, the iterative process of finding the minimum can be represented as [1]:

$$u_{k+1} = u_k + \alpha_k \varepsilon_k, k = 0,1, \ldots,$$

In the process of searching for an extremum according to these algorithms, the randomness factor decreases, the degree of determinism of the minimization algorithm increases, while random search factors predetermine its flexibility.

Minimization methods are also characterized by the area of convergence of the method, the stability of the method to errors, the breadth of the class of problems to which it is applied. The optimization problem of minimization in the presence of random errors predetermines the use of stochastic programming methods, as well as stochastic approximation. Iterative algorithms, during the implementation of which it is possible to obtain a vector of controlled variables—estimates x*, which correspond to the minimum value of the objective function of several variables, form the basis of direct search methods [2]. When solving problems of unconstrained optimization, it is recommended to use direct search methods, which are based on estimating the values of the objective function; gradient methods using the exact values of the first derivatives of the objective function; second-order methods that use estimates of both the first derivatives and the second derivatives of the objective function. In the case when, in the process of searching for a solution to an optimization problem, it is difficult to find an analytical expression for the derivatives of the objective function, then when using gradient methods, it is advisable to implement the procedure of difference approximation of derivatives. When analyzing multimodal objective functions, it is advisable to carry out the procedure for identifying local minima.

III. linearization methods for nonlinear programming problems

The search for solutions to optimization problems without constraints, as well as optimization problems with linear constraints, can be carried out using efficient algorithms. In this case, it is expedient to present the problem of conditional optimization as a parametric problem without restrictions. The search for a solution to the problem of conditional optimization is recommended to be carried out on the basis of methods of unconditional optimization as a sequence of optimization problems without restrictions [2]. In this case, it is recommended to apply an approach based on the linearization procedure, according to which the optimization problem can be reduced to a problem with linear constraints. The application of the linearization procedure allows one to apply linear programming methods both for finding a solution to a sequence of linear programming problems and for implementing iterative procedures of the simplex method. The linearization method used for the conditional optimization problem is as follows: the general nonlinear problem of mathematical programming is replaced by a problem that is formed on the basis of the linearization procedure for the functions that are part of the original problem. As a consequence, it seems possible to search for a solution to the resulting linear programming problem. In this case, the desired solution of the optimization problem is an approximation to the solution of a nonlinear problem.

The mathematical model of a nonlinear programming problem with linear constraints can be represented as [2]:

$$f(x) \to min$$

under restrictions

$$Ax \leq b,$$
$$x \geq 0,$$

where the objective function $f(x)$ is non-linear. The domain of admissible solutions is a polyhedron. Since the objective function is approximated by a non-linear dependence, then, as a consequence, the corner point of the admissible solution area may not be the optimal solution. Moreover, if the objective function is not convex, then this problem can have many local minima. A mathematical model of a linear programming problem that has an optimal solution at an admissible corner point of a bounded admissible region can be obtained based on the linearization procedure at an admissible point x^0 of nonlinear programming with linear constraints. The desired mathematical model has the form:

$$\tilde{f}(x, x^0) \rightarrow min$$

under restrictions

$$Ax \leq b,$$

$x^0 x \geq 0.$

When solving a linearized problem, a solution $\tilde{x}^*$ can be obtained, while the original nonlinear programming problem can have many local minima. Since the points and $\tilde{x}^*$ are admissible, the following inequality holds:

$$\tilde{f}(x^0; x^0) > \tilde{f}(\tilde{x}^* ; x^0).$$

The nonlinear objective function f(x) is approximated at the point x^0 by a linear function:

$$\tilde{f}(x; x^0) = f(x^0) + \nabla f(x^0)(x - x^0),$$

$x^0$where is the linearization point;

$\nabla f(x^0)$ - the first derivative of the objective function at the linearization point.

$f(x^0) > f(x^0) + \nabla f(x^0)(\tilde{x}^* - x^0)$There is an inequality [2]:

$\nabla f(x^0)(\tilde{x}^* - x^0) < 0.$or

$(\tilde{x}^* - x^0)$The direction of descent is given by the vector

One-dimensional search is advisable to carry out on the segment:

$$x = x^0 + \alpha(\tilde{x}^* - x^0), 0 \leq \alpha \leq 1.$$

As a consequence, the solution of the linear programming problem allows us to determine the direction of the search for the optimal solution. The mathematical model of the general problem of nonlinear programming can be represented as follows:

$$f(x) \rightarrow min$$

under restrictions:

$$g_j(x) \geq 0, j = \overline{1, J};$$

$$h_k(x) = 0, k = \overline{1, K};$$

$$x_i^{(U)} \geq x_i \geq x_i^{(L)}, i = \overline{1, N}.$$

The application of the linearization procedure in estimating the solution $\hat{x}^{(t)}$ makes it possible to search for a linear programming problem whose mathematical model has the following form:

$$f(x^{(t)}) + \nabla f\left(x^{(t)}\right)(x - x^{(t)}) \rightarrow min$$

under restrictions:

$$g_j\left(x^{(t)}\right) + \nabla g_j\left(x^{(t)}\right)\left(x - x^{(t)}\right) \geq 0, j = \overline{1, J};$$
$$h_k\left(x^{(t)}\right) + \nabla h_k\left(x^{(t)}\right)\left(x - x^{(t)}\right) = 0, k = \overline{1, K};$$
$$\hat{x}^{(t+1)} x_i^U \geq x_i \geq x_i^L, i = \overline{1, N}$$

x_iIt is expedient to search for a new approximation by applying linear programming methods [2]. For convergence to the optimal solution, the following is sufficient: the set of points $\{\hat{x}^{(t)}\}$ must satisfy the condition that the values of the objective function and the constraint residual at the point $x^{(t+1)}$were less than their values at the point $x^{(t)}$. It should be noted that it is advisable to carry out the linearization procedure only in the vicinity of the base point $x^{(t)}$. At the same time, in the vicinity of the base point of the linearized subproblem, the variables
must satisfy the constraints:

$$-\delta_i \leq x_i - x_i^{(t)} \leq \delta_i, i = \overline{1, N},$$

where $\delta_i$is the step parameter.

h_iFor the convergence of the algorithm, it is advisable to choose the step parameter in such a way that the value of the objective function decreases, while at each iteration the residuals on the restrictions also decrease. The application of the linearization procedure to find the direction of descent serves as the basis for the development of efficient optimization algorithms. The analysis of a non-linear programming method involves consideration of the steepest descent phase, as well as the linear programming phase. The steepest descent phase is characterized by an increase in the degree of admissibility of the solution vector x, taking into account the constraints (x)=0 and $g_i(x) \geq 0$. The steepest descent procedure unconditionally minimizes the sum of squared residuals. The sum of squared residual errors is estimated before the start of each phase of the steepest descent and each stage of the linear programming method, and regardless of whether the previous flow vector of solutions x belongs to the admissible region of solutions of the nonlinear programming problem. Acceleration or deceleration of computational procedures at the stage of linear programming is achieved based on the change in $\Delta x_{j_{max}}$. If the desired point, given by the vector x, oscillates in the vicinity of a ridge or valley, then the parameter is analyzed, which changes sign if two successive steps Δx_j are opposite in sign. This parameter also affects the value of the counterγ_j , used to determine the need to change the step length limit . The value of the counter is determined by a number indicating the number of times to move along the variable x_j in the same direction.

When the condition

$$\left|x_j^{(k)} - x_j^{(k-1)}\right| \leq \varepsilon$$

the iterative process of finding the optimal solution ends. Provided that the analyzed point belongs to the admissible area of the solution of the optimization problem, the process of searching for the optimal solution passes into the phase of linear programming.

The algorithm of the nonlinear programming method predetermines the superposition of the steepest descent phase and the linear programming phase. If the objective function and constraints of the optimization problem are non-linear, then it is recommended to search for a solution to the non-linear programming problem based on the generalized reduced gradient method. According to this method, it is advisable to use linear-approximating algorithms. In this case, the mathematical model of the nonlinear programming problem can be represented as: minimize f(x), x$\in E^n$ under restrictions:

$$h_i(x) = 0, i = \overline{1,m},$$
$$L_j \leq x_i \leq U_j, j = \overline{1,n}.$$

Constraints - equalities express the dependence between the variables of a nonlinear programming problem in an implicit form. As a consequence, difficulties arise in reducing the dimension of the problem. However, the reduction in the dimension of the problem is possible due to the method of limited variations; in this case, the reduced gradient can be used as a criterion in the search for optimality. The reduced gradient method can be implemented when searching for a solution to an optimization quadratic programming problem.

IV. Conclusions

Simulation of complex electric power systems in the search for solutions to minimization problems involves the use of random search algorithms with training. These algorithms are characterized by the property of self-learning in the process of finding a minimum.

The optimization problem of minimization in the presence of random errors predetermines the use of stochastic programming methods, as well as stochastic approximation.

The application of the linearization procedure allows one to apply linear programming methods both for finding a solution to a sequence of linear programming problems and for implementing iterative procedures of the simplex method.

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