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PERIODIC SIGNAL FILTRATION USING DIGITAL FILTERING SYSTEM CALCULATION OPTIMIZED BY APPROXIMATION

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The article assesses the filtration of periodic signals of different shapes with the use of optimized calculation of discrete low pass filtering system (LPFS) orders. To assess the feasibility of the calculation of the optimized discrete filtering system order, a series of experiments has been carried out on generation of noisy periodic signals, calculation of optimal and optimized LPFS orders. After calculation of MSE between filtered and original signals, it was observed that the optimal and optimized filters allow us to get comparable noisy signal filtration quality.

Keywords: discrete signal; blind filtration; optimized by approximation.

Наведено оцінку оптимальності фільтрації періодичних сигналів різної форми при оптимізованому розрахунку порядку цифрової фільтруючої системи нижніх частот (ФСНЧ). Проведено серію експериментів з генерування зашумлених періодичних сигналів, розрахунком оптимального і оптимізованого порядків ФСНЧ. Після розрахунку значень середньоквадратичної похибки відфільтрованих сигналів було помічено, що оптимальна і оптимізована фільтруючі системи дають змогу отримати порівнянну якість фільтрації зашумленого сигналу.

Ключові слова: цифровий сигнал; сліпа фільтрація; апроксимування; оптимізований розрахунок.

Introduction

Various technical problems require filtration of noisy signals. One of signal filtration options is low-frequency signal filtration using a low-pass filtering system which is designed to separate out the low-frequency component of a signal represented by a time series of discrete noisy samples.

For the best extraction of the low-frequency signal component, it is important to choose the correct order of the discrete filter. A class of problems exists in which there is no information about the original noise-free signal; in this case, filter adjustment is executed by the use of unsupervised (“blind”) learning [1].

In [2], the authors used optimized order calculation for the Simple Moving Average (SMA) filter in order to filter periodic noisy signals retrieved from the accelerometer. After applying the calculated orders of the SMA filter, periodic oscillations corresponding to test human walking steps were separated.

In this work, we have developed software library to simulate signal generation and discrete processing in order to assess the results of applying the optimized SMA filtering system order calculation based on approximation. For this purpose, the original periodic signals of different shapes were generated, and their modified copies were obtained by adding a random noise component to the original signal.

In *Filtered and Approximated Values Calculation* section, the calculations of filtered and approximated values for noisy signal are described.

In *Optimal Filtering System Order Calculation* section, the calculation of the optimal SMA filtering system order based on the minimal MSE with respect to the original noise-free signal is described.

In *Optimized Filtering System Order Calculation* section, the calculation of the optimized SMA filtering system order based on the minimal MSE with respect to the approximated signal is described.

In *Filtration of Periodic Signals* section, the results of experiments that were performed to compare the values of orders obtained by means of the optimized calculation and the optimal calculation are described.

In *Conclusions* section, the authors generalize the results obtained in *Filtration of Periodic Signals* section.

Filtered and approximated values calculation

In this paper, the average values of a specified number of sampling point values are considered as the filtered ones, which are calculated by means of the simple moving average algorithm (SMA).

The calculation of the filtered value by means of the SMA can be expressed as follows [3]:

$$y_{f\_m}[k] = \frac{1}{2m+1} \sum_{i=-m}^{m} y[k+i], \qquad (1)$$

where k is a sampling point $(k = \{m, m+1, m+2, ..., N-m\})$, m is the SMA filtering system order ($m \in Z$), $y[k+i]$ is the value of a noisy signal at the sampling point $k+i$, $y_{f\_m}[k]$ is the filtered signal value at the sampling point *k*.

In this work, in order to approximate signal samples, the empirical regression equation is used. Calculation of approximated values (conditional expectations) can be expressed by the following relationship [4]:

$$y_{a\_n}[i] = b_0 + b_1 \cdot i, \tag{2}$$

where

$$b_1 = \frac{n\sum_{i=0}^{n-1} iy[i] - \sum_{i=0}^{n-1} i \sum_{i=0}^{n-1} y[i]}{n\sum_{i=0}^{n-1} i^2 - \left(\sum_{i=0}^{n-1} i\right)^2}, \tag{3}$$

$$b_0 = \frac{1}{n}\left(\sum_{i=0}^{n-1} y[i] - b_1 \sum_{i=0}^{n-1} i\right), \tag{4}$$

and $i = \{0,1,2,3,...,n-1\}$ is the sampling point number, n is the approximation order (a quantity of samples (values) of a noisy signal $n = \{2,3,4,...\}$, i.e., the size of the approximation interval), $y[i]$ is the value of a noisy signal at the sampling point i, $y_{a\_n}[i]$ is the signal value after approximation.

The optimal filtering system order calculation

If the values of the original (noise-free) signal are known, we can calculate the optimal value of the SMA filtering system order to bring nearer the filtered noise signal to the original (noise-free) one as much as possible.

The optimal SMA filtering system order *m* can be calculated by minimizing the MSE value when comparing the original $y_{sig}[k]$ and filtered $y_{f\_m\,optimal}[k]$ signals, i.e., using the following relation [5]:

$$MSE(Y_{sig}, Y_{f\_m\_optimal}) = \frac{\sum_{k=m}^{N-m}\left(y_{sig}[k] - y_{f\_m\_optimal}[k]\right)^2}{N-2m}, \tag{5}$$

where k is the sampling time point, $y_{sig}[k]$ is the value of the original noise-free signal at the sampling point k, $y_{f\_m\,optimal}[k]$ is the value of the filtered signal at the sampling point k, m is the order of optimal SMA filter, and N is the total number of sampling points.

The optimized filtering system order calculation

If we do not know the values of the original noise-free signal, it is necessary to use methods of "blind" filtration. One of them is the method of the discrete filtering system order calculation optimized by approximation. The calculation of the SMA filtering system order optimized by approximation involves the following steps:

1. Choosing the order m of SMA filter and estimation of filtered values for sampling time points, in accordance with (1).
2. Choosing the approximation interval n of sampling time points and calculation of the approximated sample values for the time point intervals, in accordance with (2).
3. Calculating the MSE values between the approximated and filtered signals using the following relationship:

$$MSE(Y_{app}, Y_{f\_m\_optimiz}) = \frac{\sum_{k=m}^{N-m}\left(y_{app}[k] - y_{f\_m\_optimiz}[k]\right)^2}{N-2m}, \tag{6}$$

where k is the sampling time point, $y_{app}[k]$ is the value of the approximated signal at the sampling point k, $y_{f\_m\,optimal}[k]$ is the value of the filtered signal at the sampling point k, m is the order of the optimized SMA filter, and N is the total number of sampling points of the noised original signal (a noise component was added to the original noise-free signal).

Next, consider the results of experiments which aimed at comparing the values of the optimal and optimized orders of SMA filter.

Filtration of Periodic Signals

To bring the laboratory experiment nearer to the real signal-to-noise ratio, the experiment on capturing human walking steps was performed. For this purpose, the accelerometer Kionix KXTF9-1026 [6] was used.

Fig. 1 shows the real captured accelerometer signal and the signal obtained after filtering. The data were captured by walking for 5 seconds; 8 full steps was made during motion.

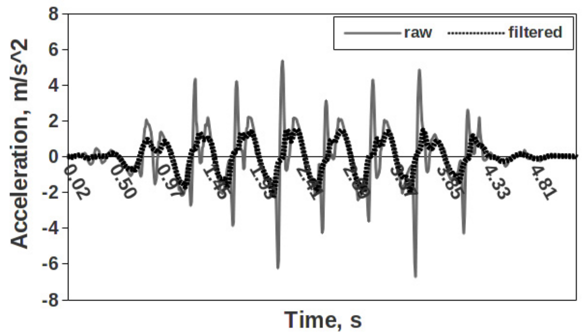

Fig. 1. Raw and filtered signals (8 steps, 5 seconds)

In Fig. 2, if all crossings of zero mark to consider as a human step response, all noisy signal zero crossings could be recognized as separate human steps. Using the proposed method of a noise component filtration allowed us to avoid false step detections.

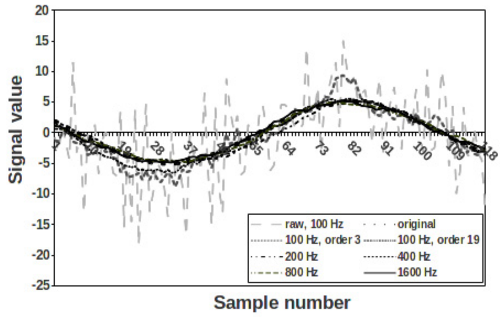

Fig. 2. Raw and filtered sinusoidal signals (using optimized orders). For demonstration purpose unoptimized order m = 3 is shown

The signal-to-noise ratio of the captured accelerometer signal has been calculated by the following expressions:

$$SNR = \left(\frac{A_{filtered}}{A_{noise}} \right)^2, \quad (7)$$

where

$$A_{filtered} = \sqrt{\frac{1}{N-2m} \sum_{k=m}^{N-m} y_{f\_m}[k]}, \quad (8)$$

$$A_{noise} = \sqrt{\frac{1}{N-2m} \sum_{k=m}^{N-m} \left(y_{f\_m}[k] - y[k] \right)^2}, \quad (9)$$

where k is a sampling point; $y[k]$ is the sample of the noisy signal; $y_{f_m}[k]$ is the sample of the filtered signal; m is the filtering system order; N is the total sample number.

The sampling frequency was equal to 125 Hz (the maximum available for the experimental purposes). After calculating, the optimized SMA filtering system order m = 8 was obtained (using (6)).

After filtering the noised signal, the value of signal-to-noise ratio equal to 0.49 was calculated.

The filtration for different signal sampling rates

To assess the degree of closeness between the original signal and the signal filtered by the SMA filter with optimized order calculated using the proposed method, a series of experiments with periodic signals of different shapes and sampling rates of noised signal has been performed. For experimental purposes, signals of four shapes (sine, triangle, square, saw tooth) were generated.

The generated signals characteristics are listed in Table 1.

Table 1

Characteristics of test signals for different signal sampling rates

Signal frequency, Hz	1
Sampling rate, Hz	100; 200; 400; 800; 1600
Noise component distribution	Gaussian
Signal-to-noise ratio	0.49
Signal detection time, s	1

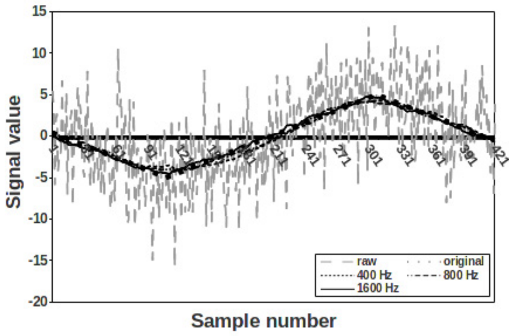

Fig. 3. Raw and filtered triangle signals

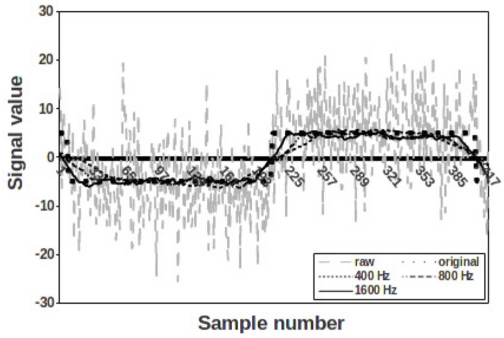

Fig. 4. Raw and filtered square signals

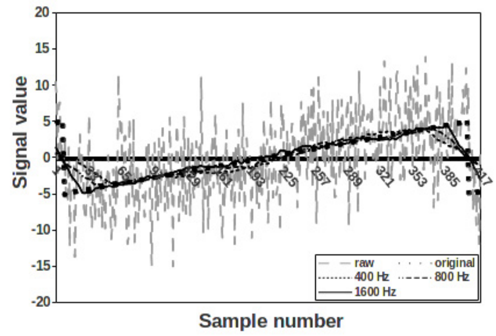

Fig. 5. Raw and filtered sawtooth signals

For a sinusoidal signals with a sampling rates 200, 400, 800, 1600 Hz, obvious discrepancies between original noise-free and filtered signals are not observed. At the frequency of 100 Hz the signal amplitude distortion is observed, while the signal shape remains sufficiently close to the shape of the original signal.

From the above experiments, it can be concluded that the 100 Hz sampling rate could be sufficient to bring the filtered sinusoidal signal close to the noise-free original signal. This fact allows us to use the proposed method for the purpose of filtering the signal received from the accelerometer Kionix KXTF9-1026 at a sampling rate of 125 Hz, including detecting human steps.

For triangle signal, the quality of filtration is similar to the quality of the sinusoidal signal filtration. For rectangular and sawtooth signals, increasing the sampling rate of a noisy signal allows to bring the filtered signal nearer to the original noise-free signal shape.

The filtration for different signal-to-noise ratios

To assess the degree of closeness of the SMA filtering system order to the optimal order obtained using the proposed method, a series of experiments with periodic signals of different shapes and signal-to-noise ratios has been performed. For experimental purposes, signals of four shapes (sine, triangle, square, saw tooth) were generated.

The generated signals characteristics are listed in Table 2.

Table 2

Characteristics of test signals for different signal-to-noise ratios

Signal frequency, Hz	1
Sampling rate, Hz	1600
Noise component distribution	Gaussian
Signal-to-noise ratio	0.1; 0.2; 0.3; 0.5; 1.0; 3.0
Signal detection time, s	2

Fig. 6 shows the values of the SMA filtering system order for the sinusoidal signal at different signal-to-noise ratios of the [0.1, 3.0] interval for the optimal order (by original noise-free signal) and the optimized order (by approximated signal).

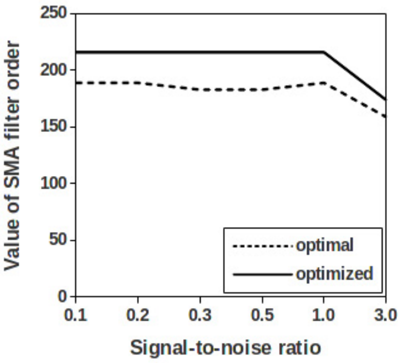

Fig. 6. The values of the optimal and the optimized orders of the SMA filter for sinusoidal signal

From Fig. 6, one can see that the maximal difference between the optimal and optimized SMA filtering system orders is about 27 while a quarter of the signal period includes 400 sample points.

Next, it was necessary to test the hypothesis that the signals of other shapes (triangular, square, and sawtooth) also allow us to calculate relatively near-optimal values of the SMA filtering system order.

Fig. 7 shows the difference between the values of the optimal and optimized SMA filtering system orders for noisy signals of different shapes and various signal-to-noise ratios of the range [0.1, 3.0].

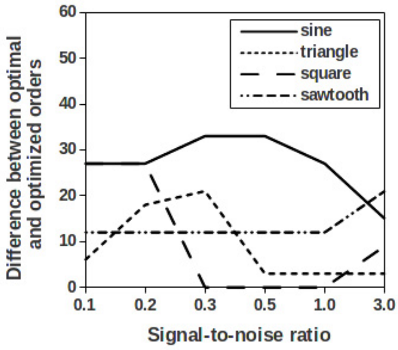

Fig. 7. The difference between the values of optimal and optimized orders of SMA filter for noisy signals of various shapes

From Fig. 6 and Fig. 7, it follows that for optimized calculation of the SMA order most of the calculated values differ from the optimal order within the range [0, 35] for sampling rate equal to 1600 Hz.

Next, it was necessary to verify the assumption that the optimized SMA filter output values are near to the optimal SMA filter output values compared to original noise-free signal values, i.e., the difference between MSE values is minimal.

The formula for calculation of the difference between the mean squared errors $|\Delta MSE|$ has the following form:

$$|\Delta MSE| = |MSE(Z, Y_{f\_optimiz}) - MSE(Z, Y_{f\_optimal})|, \quad (10)$$

where

$$MSE(Z, Y_{f\_optimiz}) = \frac{\sum_{k=m}^{N-m} \left(z[k] - y_{f\_optimiz}[k]\right)^2}{N-2m}, \quad (11)$$

$$MSE(Z, Y_{f\_optimal}) = \frac{\sum_{k=m}^{N-m} \left(z[k] - y_{f\_optimal}[k]\right)^2}{N-2m}, \quad (12)$$

and k is the sampling point, $z[k]$ is the value of the original noise-free signal at the sampling point k, $y[k]$ is the value of the processed signal at the sampling point k, N is the total number of sampling points, Z is original noise-free signal, Y is filtered noisy signal.

Fig. 8 shows the difference $|\Delta MSE|$ between the MSE values for signals with different shapes as a function of the signal-to-noise ratio [0.1, 3.0].

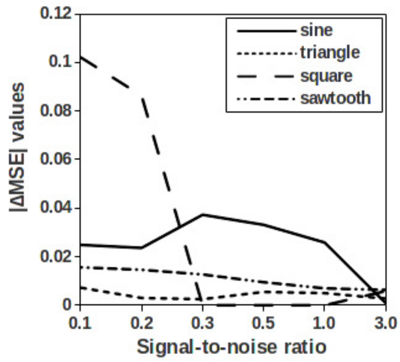

Fig. 8. The difference $|\Delta MSE|$ between the optimal and optimized orders of SMA filter

Based on the data shown in Fig. 8, one can conclude that for noisy signals of the mentioned shapes with the value of signal-to-noise ratio greater than 0.3, the use of optimized calculation of discrete filtering system order is efficient and allows us to calculate the SMA filtering system order near to the optimal one (with MSE values within range [0.0, 0.04]).

Conclusions

Filtration of noisy signals is needed when solving technical problems, including processing of signals received from inertial sensors (such as an accelerometer). Challenges of choosing the optimal order of the filter separating the useful signal from noise are caused by that sensitivity of sensors producing raw signals as well as the characteristics of these signals (including noise level) can vary within quite wide limits and are beforehand indeterminate. When constructing a noisy signal filter, the method of low-pass filtering system order calculation optimized by approximation can be used.

In order to assess the possibility of applying the method for calculation of the SMA filtering system order and for subsequent filtration of periodic signals with a signal-to-noise ratio [0.1, 3.0], the mathematical library composed of software modules was developed.

The developed library allowed us to carry out a series of experiments in which noisy periodic signals of various shapes (sine, triangle, square, and sawtooth) were generated, and corresponding optimal and optimized SMA filtering system orders (for given signal-to-noise ratios) were calculated, filtration with corresponding orders was executed, and estimated MSE values were compared.

After estimating the difference of the values of the optimal and optimized SMA filtering system orders, it has been shown that these orders differ on average by values within the interval [0, 35]. After sine, triangle and sawtooth noisy signals were filtered, the difference of MSE values (with respect to original noise-free signals) belongs to the [0.0, 0.04] interval. For the square signal, the difference of MSE values belongs to the [0.0, 0.1] interval.

It was concluded (Fig. 2) that the 100 Hz sampling frequency could be sufficient to bring the filtered sinusoidal signal shape close to noise-free original signal shape. This fact shows the possibility of using the above method for filtering close to harmonic signals received from the signal source at a restricted sampling rate.

For all signal shapes (Fig. 2 – Fig. 5), increasing the sampling rate of a noisy signal allows bring nearer filtered signal shape to the original noise-free signal shape.

It follows from Fig. 8 that, when increasing signal-to-noise ratio values (i.e., when reducing a noise amplitude), the values of $|\Delta MSE|$ are decreased.

The developed library can be used for experimental processing of discrete signals of various technical problems.

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