SHOCK-TURBULENCE INTERACTION

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У статті досліджено взаємодію ізотропної турбулентності з ударною хвилею. Проведено пошук та аналіз існуючих робіт. У центрі уваги дослідження — подання ударної турбулентності. Представлені послідовності прямого чисельного моделювання канонічного взаємодії ударної турбулентності. Стверджується, що попередні дослідження цієї проблеми недостатні. Підтверджено, що ударна хвиля нестаціонарна.

The interaction of isotropic turbulence with a shock wave is investigated. The searching and analyzing of works has been prepared. The focus of the present study in this sphere is the instantaneous picture of shock-turbulence interaction. Own results of computational method are represented and compared with results of other scientists. The assumption that shock wave is non-stationary has corroborated in both cases. The assumption of sound propagation due to the interaction of turbulence with shock wave is investigated.

Problem description

Shock/turbulence interaction is a fundamental phenomenon in fluid mechanics that occurs in a wide range of interesting problems in various disciplines of science. Examples include supernovae explosions, inertial confinement fusion, hypersonic flight and propulsion, and shockwave lithotripsy (used to break up kidney stones). In many such applications the shock/turbulence interaction includes additional complexities, e.g., real gas effects, multiple species, non-uniform mean flow, or streamline curvature [1]. The most fundamental problem, where these additional complexities have been removed, is arguably that of isotropic turbulence passing through a nominally normal shockwave in a perfect gas. Given the historical success in studying building-block problems in fluid mechanics, canonical shock/turbulence interaction is the focus of the present study.

Ribner (1954) studied the problem analytically by solving the linearized Euler equations with linearized shock jump conditions for incoming purely vortical turbulence [3]. This linear interaction analysis (LIA) relies on several assumptions, most notably that the turbulence comprises a small perturbation relative to the shock and that nonlinear effects in the post-shock evolution are small (as well as the standard assumption of a difference in time scales). Rapid distortion theory (RDT) relies on the same assumptions, but additionally neglects both the post-shock linear evolution and all effects of the turbulence on the shock. In addition, the Rankine-Hugoniot shock jump conditions are incorporated into LIA but not RDT; one consequence is that LIA captures the generation of sound and entropy waves from incoming purely vortical turbulence [4].

Lee et al. (1993, 1997) [5] performed a set of landmark direct numerical simulations (DNS) of canonical shock/turbulence interaction.

The first of these papers considered shocks at Mach numbers up to 1.2 where the viscous structure of the shock was resolved; these were therefore truly direct solutions of the Navier-Stokes equations. In the second paper they verified that these “true” DNS results at Mach 1.2 could be replicated by instead capturing the shock (at considerably lower cost), provided suffcient grid resolution in the shock-normal direction at the shock. This methodology was then used to compute cases at Mach numbers up to 3. When comparing the results to LIA predictions, they found that LIA realistically represents many features, including the amplification of transverse vorticity, the amplification and post-shock evolution of the Reynolds stresses, and the decrease in transverse Taylor length scale.

The present study builds on these previous studies by Lee et al. In this paper, DNS in the extended sense of capturing the shock while directly resolving all scales of turbulence is used. It will be shown that a simple argument about the Kolmogorov scale implies that DNS (with a captured shock) requires a refined grid in both the shock-normal and the transverse directions to fully resolve the viscous scales of turbulence. This is verified by a grid convergence study, and implies that the calculations in Lee et al. (1997) [5] were, most likely, under-resolved. The present DNS data is fully resolved, which leads to larger differences between the data and LIA. The Reynolds stresses are more anisotropic in the present DNS, and there are qualitative differences in the Taylor length scales. This raises the interesting question of whether under-resolution of the post-shock turbulence in DNS essentially neglects some phenomenon that is also neglected in the LIA.

The focus of the present study is the instantaneous picture of shock/turbulence interaction, and how this changes as the degree of nonlinearity (broadly, the strength of the turbulence relative to the strength of the shock) is increased. Zank et al. (2002) argued that the nonlinearity of shock/turbulence interaction should be taken into account in analytical theories, and developed a simple model based on the inviscid Burger’s equation. Given a jump in mean

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velocities, the model quantitatively predicts increased mean shock speed and decreased turbulence amplification ratio as the incoming turbulence intensity is increased. In addition, the model predicts that the shock becomes unstable at a critical turbulence intensity.

Hesselink & Sturtevant (1988) performed experiments where a normal shock propagated through a region of randomly mixed helium and R12, and documented instantaneous pressure profiles through the shock that were drastically different from the classical picture. They found what they called “peaked” and “rounded” profiles, and explained these features by shock focusing in the random medium. Interestingly, the same features appear in the present DNS data as well, especially at higher levels of nonlinearity. This raises at least two possibilities: either that nonlinearity in a single fluid problem gives rise to similar dynamics as shock focusing in a two-fluid medium, or that these peaked and rounded profiles really are effects of nonlinearity and not directly a multi-fluid effect. Hesselink and Sturtevant did not document the velocity fluctuations induced by the array of jets used to create the random medium, and the present DNS data is for a single fluid only.

Past work on canonical s hock-turbulence interaction

Thuse we may subdivide past work on shock-turbulence interaction on three groups: theoretical, experimental and computational.

Theoretical

Ribner (NACA 1953, NACA 1954, AIAA J 1987): linear interaction analysis including Rankine-Hugoniot relations; Lele (PhysFl 1992): turbulent shock relations using RDT, predicted modified mean shock jumps; Zank et al (PhysFl 2002): simplified theory, predicted “unstable” shock at high turbulence intensity.

Experimental: wind tunnels

Barre et al (AIAA J 1996): grid turbulence passing through a Mach 3 shock, hot-wire and LDV measurements.

Experimental: shock tubes

Hesselink & Sturtevant (JFM 1988): weak shocks propagating through random medium, showed “peaked” and “rounded” instantaneous pressure profiles.

Computational (DNS):

Lee et al (JFM 1993): resolved viscous shock structure, $M = 1.2$, $Re\lambda = 20$, $Mt = 0.1$, found modified instantaneous profiles of dilatation. Lee et al (JFM 1997): captured shock up to $M = 3$, found good agreement with Ribner’s linear theory. Mahesh et al (JFM 1997): influence of entropy fluctuations [5].

Motivation

Improve fidelity compared to past computational work: Past DNS failed to resolve the post-shock viscous dissipation due to computational limitations.

Study nonlinear interaction regime (high Mt compared to M):

- Linear theory assumes $Mt = 0$
- Past DNS had $Mt = 0.1$ (linear regime for $M > 1.05$)

Study fundamental interaction-physics:

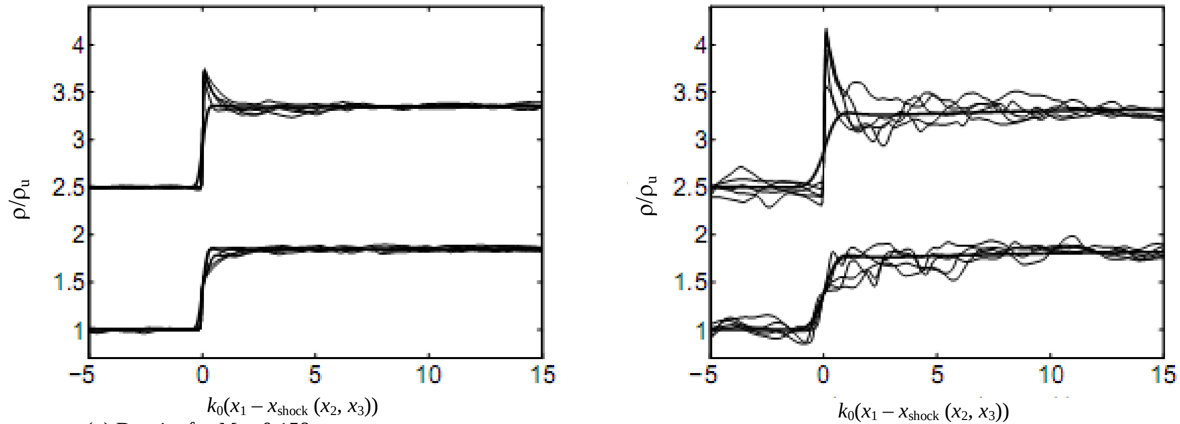
- The instantaneous interaction-process largely remains a mystery.

Use insights to improve reduced-order modeling

- How to predict the modification of turbulence across the shock in RANS (larger kinetic energy, smaller length scales)?
- How do existing LES subgrid-models perform on the strongly out-of-equilibrium post-shock turbulence?

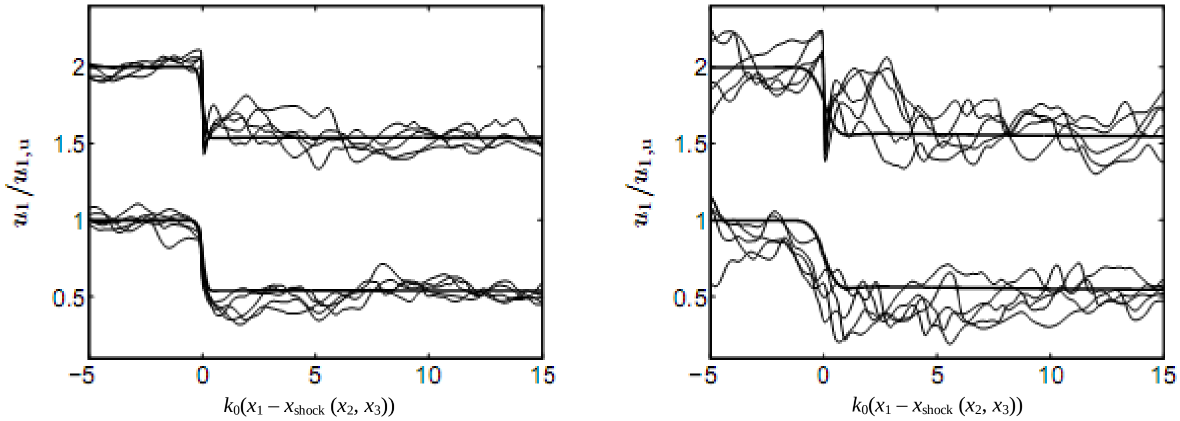
Instantaneous results

The instantaneous structures of the shock/turbulence interaction are investigated in my work “Interaction of turbulent flow with shock wave”. The computational (fig. 1) and some theoretical results are represented there. J. Larsson studies how these are affected by the turbulent Mach number.



(*a*) Density for M_t = 0.158

(*b*) Density for M_t = 0.387



(*c*) Density for M_t = 0.158

(*d*) Density for M_t = 0.387

Fig. 1. Instantaneous profiles along the streamwise direction for M = 1.50 at different transverse coordinates (x2, x3). Each figure shows five profiles through “weak” and “strong” (with offset) interactions, respectively. The streamwise coordinate has been shifted by the instantaneous position of the shock x1,shock(x2, x3) for each profile. Mean profiles shown by thick lines[6].

On average the turbulence causes the shock jumps in density, pressure and velocity to be smaller than the laminar Rankine-Hugoniot conditions, but the instantaneous picture is more complex. In this section he visualizes and investigates some instantaneous structures. Specifically,compares instantaneous interactions that are “strong” and “weak” in the sense that they have fluid compressions and density/pressure jumps that are larger or smaller than on average [2].

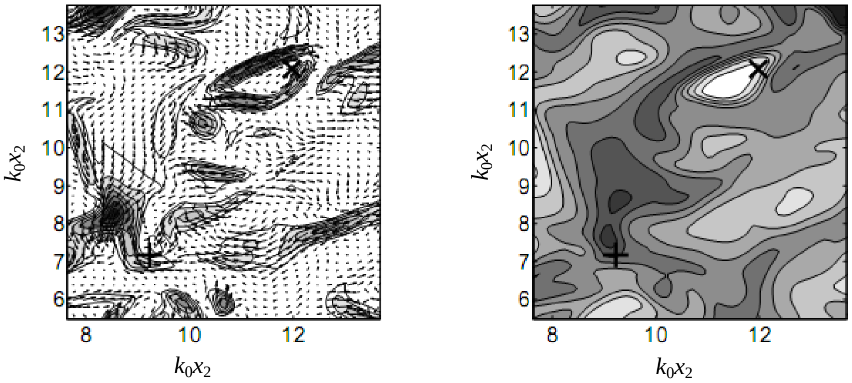
Larson then finds the points (x2, x3) of the strongest/weakest interaction as the min/max of shock. After excluding a circle near these points, the process is repeated to find the next strongest/weakest points, and so on. Profiles through the five strongest/weakest interaction points are shown in Fig. 1 at two different turbulent Mach numbers: M_t = 0.158 and 0.387, corresponding to turbulence intensities of 6 % and 15 %, respectively. For clarity, the streamwise coordinate x1 has been shifted in the figures such that the instantaneous shock-positions line up. We note that the resulting “strong” and “weak” profiles closely resemble the “peaked” and “rounded” ones found by Hesselink & Sturtevant (1988).

Larson first notes that the high-intensity case (naturally) displays much larger excursions from the mean, and that the mean shock thickness is larger. The stronger-than-average interactions are qualitatively similar at both values of M_t The instantaneous density jump is larger than the average, in fact almost twice as large for the M_t = 0.387 case, and there is an immediate expansion and decrease of ρ behind the shock toward the mean level. This expansion is also seen in the velocity profiles, which all show positive streamwise acceleration both upstream and (especially) downstream of the shock. Apart from explaining the post-shock decrease in density, this acceleration also suggests that the stronger-than-average interaction is associated with eddy structures that locally accelerate the fluid. The fact that the acceleration appears stronger behind the shock would be consistent with an eddy oriented in the transverse plane, since the shock compression would increase the transverse vorticity (and thereby the induced acceleration).

To investigate this hypothesis Larson considers the transverse vorticity in a slice immediately behind the shock in Fig. 2. The slice is taken from the same snapshot as the profiles in Figs. 1(*b*) and 1(*d*); the locations of one strong and one weak interaction are also shown in Fig. 2.

Both the strong and weak interactions occur near strong eddies. The weak interaction, in particular, occurs near the head of a hairpin- or ring-like eddy. The vorticity vectors in Fig. 2(*a*) show that the eddy near the weak point acts to decrease the streamwise velocity, while the eddy near the strong interaction acts to increase it.

This leads to low- and high-speed jets, as shown in Fig. 2(*b*). Therefore the hypothesized relationship between strong transverse eddy structures, local acceleration and instantaneously strong/weak shock-interactions seems reasonable.



(*a*) Contours and vectors of vorticity

(*b*) Contours of streamwise velocity

Fig. 2. Instantaneous slice immediately behind the shock (looking upstream) for $(M, M_t) = (1.50, 0.387)$, with dark regions denoting higher values. The locations of one instantaneously strong (plus at ≈ (9, 7)) and one weak (cross at ≈ (12, 12)) interaction are also marked [6]

A further visualization is shown in Fig. 3 for the same case. The shock displays two structures that do not occur at lower Mt in the region around (k0x1, k0x2) ≈ (0, 13).

First, there is a region of low momentum where the shock is instantaneously replaced by a smooth compression (a "weak" interaction). Secondly, the shock has branched out in a Y-shape immediately above this low-momentum region.

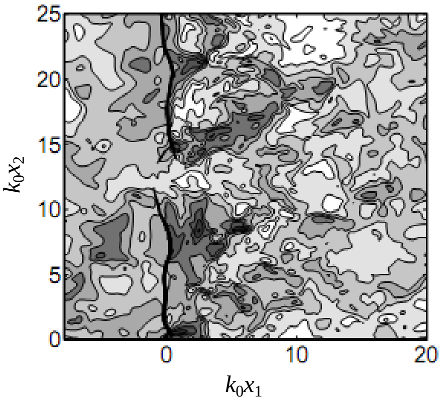


Fig. 3. Instantaneous slice for $(M, M_t) = (1.50, 0.387)$. Streamwise momentum ρu1 in gray scale, with dark regions denoting higher momentum. Contours of high fluid compression (outlining the shock) are overlaid in thick lines [6].

The Fig. 4 depicts the contours of Mach numbers at supersonic turbulent flow ($M = 1.5$). This is the result of computer modeling. We can say, that comparing it with the Larson's results (fig. 3) a lot of similar moments are detected. The shock wave is nonstationar, it has curvilinear shape.

Assumption of sound propagation during interaction of turbulence with shock wave

When the turbulence is absent we can use known balance equations.

$$\rho_1 V_1 = \rho_2 V_2\ ; \qquad (1)$$

$$p_1 + \rho_1 V_1{}^2 = p_2 + \rho_2 V^2{}_2\ ; \qquad (2)$$

$$k/(k-1) * \rho_1/p_1 + V^2{}_1 = k/(k-1) * \rho_2/p_2 + V^2{}_2 \qquad (3)$$

Using equations of Mass Conservation (1), Impulse Conservation (2) and Energy Conservation (3), we obtain kinematics energy in the right hand side of turbulence equation.

This energy obtained in downstream flow, may be obtained as a Sound Propagation as an S.

Analytic work addressing the interaction of turbulence with a shock wave has concentrated on investigating the generation of sound by jet engines.

Hydrodynamics modes (acoustic, vertical, and entropic) can experience considerable amplification on passage through a shock, while downstream of a shock there exists a critical angle for incident acoustic modes where the reflection coefficient is 1.

$$k/(k-1) * p_1/\rho_1 + V^2{}_1/2 + 1/2*\xi_{T1} =$$

$$= k\ /\ (k-1) * \rho_2/p_2 + V^2{}_2/2 + 1/2*\ \xi_{T2} + S. \qquad (4)$$

Where ξ_{T1} — Turbulent Flow Energy, ξ_{T2} — Sound Energy.

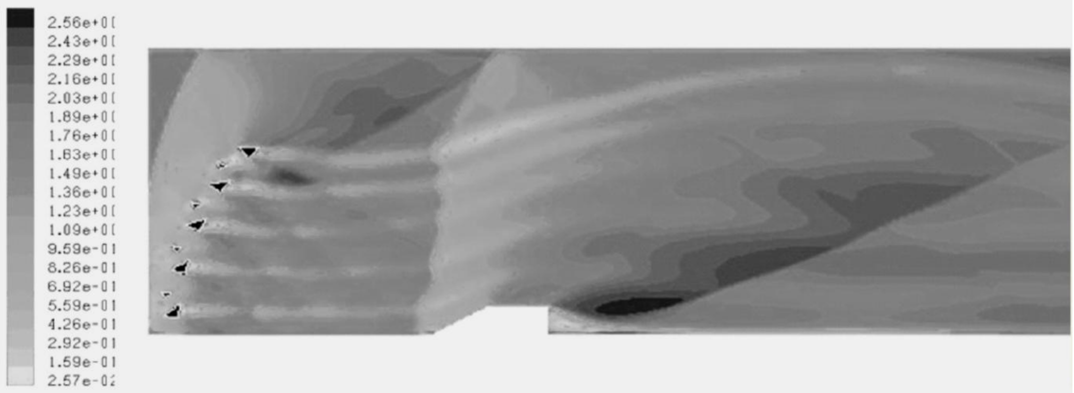


Fig. 4. Instantaneous slice for $M = 1.5$ in a turbulent flaw in gray scale, with dark regions denoting higher Mach number

Due to the assumption we should also obtain equation, initial form of which was modified Rankie-Hugoniot equation:

$$p_2(\rho_2)= p_2\left(p_2\right)=\left[\frac{6\frac{p_2}{p_1}-1}{\left(\frac{k+2}{k-1}\right)-\frac{p_2}{p_1}}+\right.$$

$$\left.+\frac{2\rho_1 V_1^2\left(E_{turb}+S\right)\left[1+\frac{k+1}{k-1}\times 2\left(E_{turb}+S\right)V_1\right]}{\left(\frac{k+1}{k-1}\right)-\frac{r_2}{\rho_2}}\right]\times p_1 .$$

According to the assumption we found the equation of determination the difference of the turbulent energy in downstream and upstream flows:

$$\Delta E_{turb}=\frac{1}{2}V_1^2\left(M^{\alpha}\frac{V_2^2}{V_1^2}-1\right)\left(u_1\right)^2 .$$

Summary and future work

A sequence of direct numerical simulations (DNS) of canonical shock/turbulence interaction is presented. Care is taken to ensure fully developed isotropic turbulence upstream of the shock, and a systematic grid refinement study shows that the viscous dissipation is fully resolved on the finest grids. Thus the DNS databases are ideally suited for exploration of the fundamental physics and dynamics of shock/turbulence interaction. It is argued that previous DNS studies of this problem may have been under-resolved in the post-shock region, since the Kolmogorov scale decreases during the interaction. A simple estimate of this change is given; it agrees to reasonable accuracy with the DNS data. A more quantitative assessment would require higher Reynolds numbers, since the viscous decay is significant in the present data. The estimate suggests a smaller change in the Kolmogorov scale at Mach numbers above 3.6, which should be investigated in future work.

Contrary to the previous DNS by Lee et al. (1997), and contrary to the linear interaction analysis of Ribner (1954), the streamwise Taylor length scale is consistently larger than the transverse scale in the present data.

While the present result is some what counterintuitive, it is entirely consistent with the notion of a return to local isotropy at the smallest scales, and the lack of a return to isotropy at the larger scales.

These processes are nonlinear, and therefore absent from linear analysis. It is speculated that under-resolution in the post-shock region may have under-predicted this nonlinear development in previous DNS studies. Instantaneous profiles through the shock can be quite different from the average profiles, especially at higher levels of the turbulent (compared to the mean) Mach number. Locally, the shock compression may be twice as strong as on average, or so weak that it is effectively a smooth compression. It is conjectured that these excursions from the average behavior can be connected to local eddy structures with strong transverse vorticity that cause local acceleration/deceleration that the shock responds to. The most important future work is to analyze the data in more depth, and more quantitatively. The amplification ratios of velocity and vorticity variances should be compared to linear analysis, and the instantaneous flow-fields around extreme interaction events should be probed in greater detail.

REFERENCES

1. *Касьянов В. А.* Моделирование полета / В. А. Касьянов. — К. : НАУ, 2004. — 400 с.

2. *Larsson J., Gustafsson B.* 2008 Stability criteria for hybrid difference methods. J. Comput. Phys. 227, 2886–2898.

3. *Ribner H. S.* 1954 Shock-turbulence interaction and the generation of noise. NACA. Report 1233.

4. *Татарский В. И.* Распространение волн в турбулентной атмосфере / В. И. Татарский. — М. : Наука // Главная редакция физико-математической литературы, 1967. — 548 с.

5. *Lee, S., Lele, S. K., Moin, P.* 1997 Interaction of isotropic turbulence with shock waves: effect of shock strength. J. Fluid Mech. 340, 225–247.

6. *Larsson, J., Lele, S. K., Moi, P.* 2007 Effect of numerical dissipation on the predicted spectra for compressible turbulence. Annual Research Briefs, Center for Turbulence Research, Stanford, CA, 47–57.

7. *Шквар Е.О.* Потоково-розподілені алгоритми аеродинамічних задач / Е. О. Шквар. — К. : Інститут гідромеханіки, 2008. — 360 с.

Стаття надійшла до редакції 09.12.09.