UDC 629.735.33-519.05(045)

MATHEMATICAL MODELING OF THE TERMINATION FLIGHT CONTROL ALGORITHM FOR UNMANNED AERIAL VEHICLE

Kondrashov Ya. V., PhD; Arutyunyan A. K., Kravchyshyn I. O.
The National university of Civil Aviation
int2080@ukr.net

Эффективность алгоритма синтеза управления оценивается путем математического моделирования по методической точности наведения планирующего летательного аппарата в режиме его спуска на радиомаяк всенаправленного действия. Показано, что в результате синтеза управления формируется сходящийся устойчивый процесс изменения параметров движения, а точность наведения определяется, в основном, навигационными ошибками определения текущих фазовых координат аппарата.

Ключевые слова: терминальное управление, посадка, математическое моделирование, беспилотные летательные аппараты.

The efficiency of the algorithm of control synthesis is evaluated by means of mathematical modeling by a methodical error of the guidance of the gliding aircraft descending to the omnidirectional radio beacon. It's shown, that the control synthesis results in formation of convergent stable process of motion parameters variation, and the guidance error is generally determined by navigational errors of current phase coordinates determination.

Key words: termination control, landing, mathematical modeling, unmanned aerial vehicles.

Introduction. Problem definition

Controlled descent (flight) to the specified area of earth surface (circumterrestrial space) of unmanned gliding aerial vehicle (UAV) with a large lift-drag ratio (K >1) is considered. Such vehicles by means of aerodynamic maneuver in the atmosphere can significantly change their descent (flight) trajectory.

The problem of the control is guidance of the vehicle to the point with specified geographical coordinates in presence of random disturbances acting on the UAV during the descent (flight). That's why the most reasonable is termination control of UAV, based on prediction of coordinates of a point of landing (flight).

Because of this reason on sizeable part of the starting trajectory the control of UAV is autonomous. Prediction is realized by integration of system of differential equations of motion with initial conditions, determined by autonomous navigation system. Accumulated, as a result of control synthesis during landing (flight), navigation errors lead to appearance of not excluded by control system dispersion of landing (flight) points.

Nowadays designed methods of forming of non-autonomous control are based on principle of tracking of a priori given nominal programs of relative motion of aircraft and destination point parameters change. Concerning the task of landing these methods, particularly the method of proportional approach, even with nominal conditions of motion, doesn't provide necessary accuracy of guiding because of necessary overload exceeding of G tolerance in neighborhood landing spot and breach of convergence of control correction processes.

To increase accuracy of UAV guidance at the final part of landing (flight) non-autonomous control is reasonable, using information about relative position and motion of aircraft and destination point [1]. At this paper the method of synthesis of non-autonomous termination dual-channel control of UAV $U = f(\gamma_a, K)$ by dynamic roll angle $\gamma_a(t)$ and aerodynamic characteristic $K(t)$ under guidance to omnidirectional beacon situated in defined point of space.

Formations of the non-autonomous control, which are designed nowadays, are based on the principle of tracking of a priory set nominal programs of parameters variation of the relative motion of the vehicle and objective [2].

Concerning the problem of descent given methods, particularly the method of proportional approach, even at nominal conditions of a motion do not provide required accuracy of the guidance because of excess of the permissible g-forces by necessary ones at the neighborhood of the landing point and violation of convergence of the control processes [3].

To provide guidance of the vehicle with sufficiently high accuracy method of non-autonomous multistep adaptive termination control in a zone of close-range guidance, starting at the moment of grabbing of beacon signal by radio equipment of the vehicle [4—7].

At the moment of grabbing on a board of UAV inertial coordinate system $0x_iy_iz_i$ is formed, an origin of the system matches with center of inertia of the vehicle, axis $0y_i$ is directed by radius-vector $\vec{\mathbf{r}}$, and vertical plane $0x_iz_i$ is superposed with radio beacon at a landing point $C(\varphi_c, \lambda_c)$ with geographic latitude φ_c and longitude λ_c.

Direction vector sight line $\vec{D}^0(t)$ and pointing direction distance to it $D(t)$ are external information. Internal information is a vector of current phase coordinates of the vehicle

$$\mathbf{x}^n = (V_k^n, \theta^n, \psi^n, h^n, \varphi_c^n, \lambda^n),$$

which is determined in autonomous regime by navigation system and includes earth speed V_k^n, slope angle of trajectory θ^n, track angle φ^n, altitude h^n, geocentric altitude φ_c^n and latitude λ^n, see fig. 1.

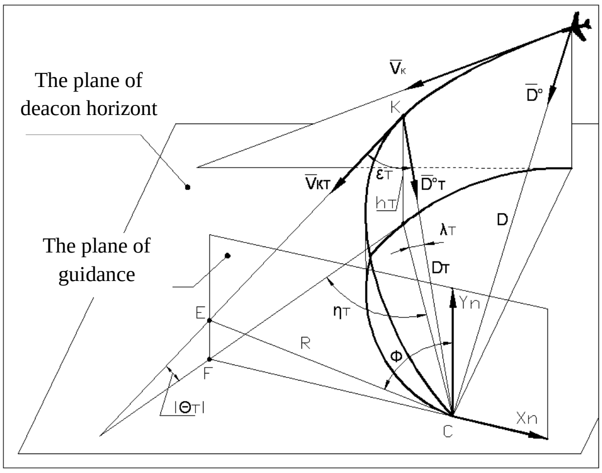

Fig. 1. Scheme of descent of UAV

For realization of multistep synthesis the initial length of guidance $T_0 = t_{k0} - t_0$ from moment of beacon gripping t_0 to predicting moment t_{k0} of reaching by vehicle the guidance sphere of radius $r_c = R_0 + h_k$ on the surface, which is located at a height h_k above ground level with radius R_0 the beacon is located, is divided on N_0 intervals

$$\Delta T_\nu = t_\nu - t_{\nu-1},\ \nu = \overline{2, N_1+1},\ t_{N1} = t_{k1},\ T_0 = \sum_{\nu=1}^{N_0} \Delta T_\nu .$$

In the end of the interval ΔT_1, analyzing the predicted time t_{kj} of guidance sphere reaching, the descent duration $T_1 = t_{k1} - t_1$ is divided on N_1 intervals

$$\Delta T_\nu = t_\nu - t_{\nu-1},\ \nu = \overline{1, N_0},\ t_{N0} = t_{k0},\ T_1 = \sum_{\nu=2}^{N_1+1} \Delta T_\nu$$

Thus for each current moment of correction t_j the rest predicted duration of descent $T_j = t_{kj} - t_j$ divided on N_j intervals

$$\Delta T_\nu = t_\nu - t_{\nu-1},\ \nu = \overline{j+1, N_j+j},\ t_{Nj} = t_{kj},\ T_j = \sum_{\nu=j+1}^{N_j+j} \Delta T_\nu$$

In the end of each interval the correction of control program is done and in the rest interval $t \in [t_j, t_{kj}]$ a descent is performed with help of command program

$$u_{\text{com}}^{(j)}(t \geq t_j) = (\gamma_{a\ \text{com}}^{(j)}(t \geq t_j), K^{(j)}(t \geq t_j)).$$

The aim of control on each step ΔT_j, $j = 1, 2, \ldots$ is forming of trajectory, going through the point of landing *C* in predicted moment of time of descent t_{kj} to the surface of the sphere. In case of two-parametric control deviation of predicted final position of the vehicle from the required one is recorded on plane of guidance by two coordinates – distance *R* and angle Φ, see fig. 1, which define the position cross point of a vector of velocity of the vehicle $\vec{V}_{kt} = \vec{V}_k(t_{kj})$ with this plane. The plane of guidance is perpendicular to both the plane of the horizon of a beacon and projection of a vector of velocity of the vehicle on this plane at the final moment of a descent. In the guidance plane an orthogonal system of coordinates $Cx_пy_пz_п$ is built, axes $Cy_п$ of which is directed by radius-vector of a point *C*.

The task of control synthesis is formed by the following way. For each discrete moment of time t_j, $j = 1, 2, \ldots$ is needed to determine the optimal command control

$$u_{\text{com}}^{(j)}(t \geq t_j) = (\gamma_{a\ \text{com}}^{(j)}(t \geq t_j),\ K^{(j)}(t \geq t_j)).$$

For whole rest interval of descent $t \in [t_j, t_{kj}]$, minimizing a predicted length *R* at the final moment of time of descent on a destination sphere

$$u_{\text{com}}^{(j)}(\gamma_{a\ \text{com}}^{(j)},\quad K_{\text{com}}^{(j)}) = \arg\min_u R(y(t_{kj}), u).$$

In the result of multistep correction the command program of control is formed in a form of sequence of intermediate programs

$$u_{\text{com}}(t \geq t_0) = (u_{\text{com}}^{(0)}, u_{\text{com}}^{(1)} u_{\text{com}}^{(2)}, \ldots, u_{\text{com}}^{(j)}, \ldots).$$

Here $u_{\text{com}}^{(0)}(t \geq t_0) = (\gamma_a^{(0)}(t \geq t_0),\quad K^{(0)}(t \geq t_0))$ is a command program, formed in the end of the zone of autonomous control.

Mathematical modeling

The efficiency of the algorithm of control synthesis is evaluated by a methodical error of the guidance of the gliding aircraft at nominal conditions of motion by mathematical modeling of descent at the short-range guidance to the omnidirectional radio beacon.

The modeling is carried out under conditions of ideal navigation. Herewith current phase coordinates of the vehicle are

$$\mathbf{x}^n = (V_k^n, \theta^n, \psi^n, h^n, \varphi_c^n, \lambda^n),$$

which are determined by autonomous navigation system, when modeling are determined by integration of system of differential equations of motion

$$\dot{\mathbf{x}} = f(\mathbf{x}, K, Q_x, \gamma_a, t)$$

in non-central field of the rotating Earth.

The modeling is carried out for hypothetic gliding aircraft with variable lift-drag ratio $K_{\text{nom}}(M)$ and drag coefficient $C_{xa\,\text{nom}}(M)$, fig. 2.

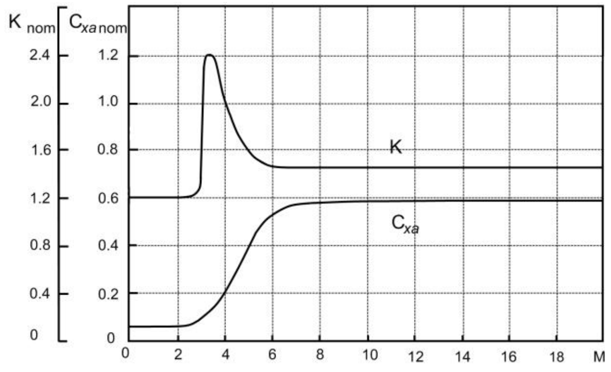

Fig. 2. Aerodynamic characteristics

The aircraft is guided to the radio beacon located at the point $C(\varphi_c, \lambda_c)$ with geographic latitude $\varphi_c = 36{,}864°$ and longitude $\lambda_c = 63{,}643°$ at the sphere of guidance at the altitude $h_k = 4$ km above the Earth surface. The point C is a landing point of the aircraft at descent at nominal conditions of motion and constant roll angle $\gamma_{a\,\text{nom}} = -22{,}8°$. The methodical error of the method of the synthesis of termination control is characterized by deviation of the landing point from the desired one.

At every step of the control ΔT_j the navigation values of the phase coordinates of the vehicle $x^H(t_j) = x(t_j)$ are determined and direction of the beacon sight line $\vec{\mathbf{D}}^0_j = \vec{\mathbf{D}}^0(t_j)$ and distance to the beacon $D_j = D(t_j)$ are modeled.

Taking into account the fact that when descending the accuracy of prediction of the termination parameters of the motion $y(t_{kj})$ should increase, the step of control correction is taken variable, linearly decreasing from value $\Delta T_1 = 10$ s to 2 s.

$$\Delta T_j = 2 + 8T_{j-1}/T_0 .$$

When carrying out prediction of terminal parameters of motion by equations in finite-difference representation the predicted duration of descent T_j is divided to $G = 50$ intervals of constant duration

$$\Delta\tau_j = T_j/G ,$$

that allows to shorten discrecity of motion parameters computation with descent of the aircraft.

The instant of grabbing of the beacon signal is defined from the condition of sight of the beacon by the vehicle at ascend of the vehicle above the horizon plane by angle $\beta_з = 5°$.

One of the basic parameters, characterizing convergence and stability of the aircraft self-guidance process, is variation of the aerodynamic overload directed by the normal to the self-guidance plane line $\vec{\mathbf{N}}$ [4], fig. 3

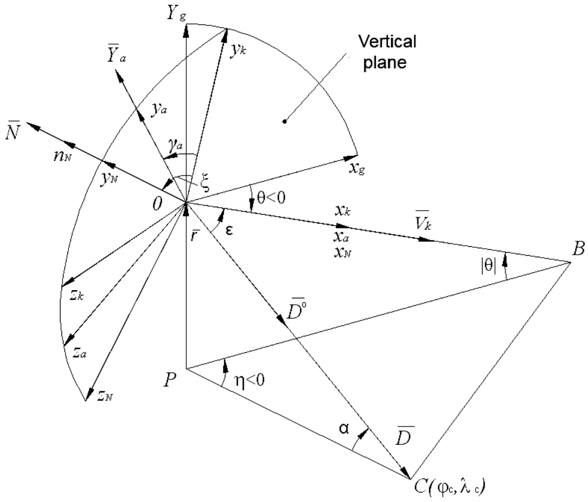

Fig. 3. Coordinate system

$$n_{Nj} = \frac{K_j \sigma_{xj} q_j}{g_0} \cos(\xi_j - \gamma_{aj}), \quad j = 1, 2, \ldots$$

where $\sigma_{xj} = C_{xj} S/m$ — ballistic coefficient; C_{xj} — drag coefficient; S — midsection area; $q_j = \rho_j V_{kj}^2 / 2\tau$ — dynamic pressure; ρ_j — atmosphere density; V_{kj} — actual speed of the vehicle; g_0 — gravitational acceleration at the surface of the Earth, ξ_j — orientation angle of the normal vector in the trajectory coordinate frame.

To determine the angle ξ_j in the inertial reference frame unit vector $\vec{j}^0 = (j^0_{xi}, j^0_{yi}, j^0_{zi})$ is formed, which is an ort $0y_k$ axis of trajectory reference frame basic vector $\vec{j}^0 = (0,1,0)$

$$\begin{Vmatrix} j^0_{xi} \\ j^0_{yi} \\ j^0_{zi} \end{Vmatrix} = Q \begin{Vmatrix} 0 \\ 1 \\ 0 \end{Vmatrix} .$$

Two vectors $\vec{j}^0 = (j^0_{xi}, j^0_{yi}, j^0_{zi})$ and $\vec{\mathbf{N}} = (N_{xi}, N_{yi}, N_{zi})$ define main value of the angle $\xi_{0j} \in [0, 2\pi]$.

$$\sin\xi_{0j} = \frac{\left|\vec{j}^0 \cdot \vec{N}\right|}{\left|\vec{j}^0\right| \cdot \left|\vec{N}\right|} \operatorname{sign}(N_{xi}), \quad \cos\xi_{0j} = \frac{\left(\vec{j}^0 \cdot \vec{N}\right)}{\left|\vec{j}^0\right| \cdot \left|\vec{N}\right|} .$$

To fix a position of the vector $\vec{\mathbf{N}}$ relative to $0y_k$ axis of the trajectory reference frame a rule for determination of sign of the angle ξ_j is introduced: the angle is positive $\xi_j > 0$ if a rotation of the $0y_k$ axis to superimpose with the vector is carried out counter-clockwise, as it seems from the end of vector of $0x_k$ axis

$$\xi_j = \begin{cases} \xi_{0j}, & \text{if} \quad \xi_{0j} \le \pi, \\ \xi_{0j} - 2\pi, & \text{if} \quad \xi_{0j} > \pi. \end{cases}$$

As a result of mathematical modeling of the descent with termination control at the short-range part of the guidance dependences of distance to the beacon $D(t_c)$, altitude of the vehicle $h(t_c)$, normal to the guidance plane overload $n_N(t_c)$, the angles of sight $\varepsilon(t_c)$, course $\eta(t_c)$ and orientation of self guidance plane $\xi(t_c)$, command roll angle $\gamma_{a\,\mathrm{com}}(t_c)$ and modulating function $\omega(t_c)$ on duration of descent $t_c = t - t_0$ are obtained, fig. 4, 5, 6.

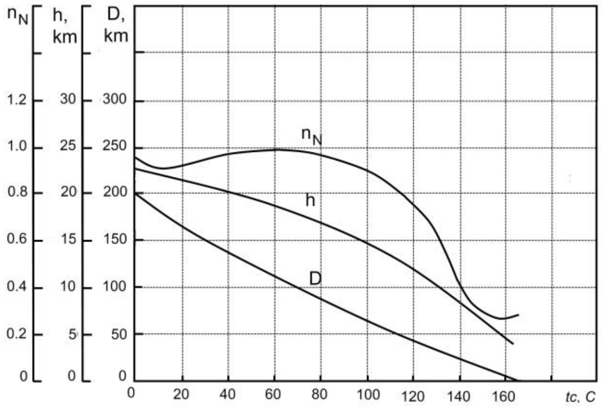

Fig. 4. Variation of navigation parameters

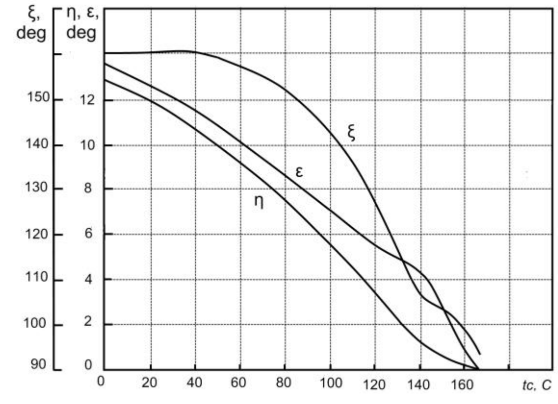

Fig. 5. Variation of navigation parameters

The process of control results in formation of stable monotonic decreasing change of trajectory parameters. The self-guidance plane changes its location in space and at the end of the descent takes a vertical position. At the end of descend the velocity vector is directed along with the sight line. Variation of the normal overload is significant. As opposed to traditional self guidance methods, particularly the method of proportional approach [3], the overload decreases when descending, it provides controllability of the aircraft at all part of the short-range guidance.

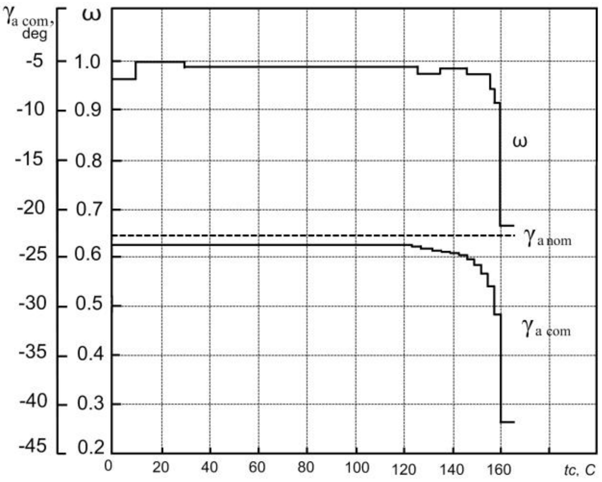

Fig. 6. Control programs

As a result the deviation of the landing point is not greater than 0,06 km. This deviation is methodical error of the guidance, which characterizes the efficiency of the method of control.

Operability of the algorithm of termination control synthesis is defined by availability of guidance of the vehicle to the arbitrary landing points in the vicinity of the nominal point $C(\varphi_c, \lambda_c)$. That is why descent to the landing points located at latitude of the nominal one with deviation along the parallel $\Delta L = R_0(\lambda - \lambda_c)$ by distance from — 150 km to 0 km was modeled. The fig. 7, 8 shows command programs of the variation of roll angle $\gamma_{a\,\mathrm{com}}(t_c)$ and modulating function $\omega(t_c)$ at guidance to the landing points at distances — 50 km and — 100 km from the nominal one respectively. The error of guidance of the vehicle was to the chosen points was not greater than 0,04 km.

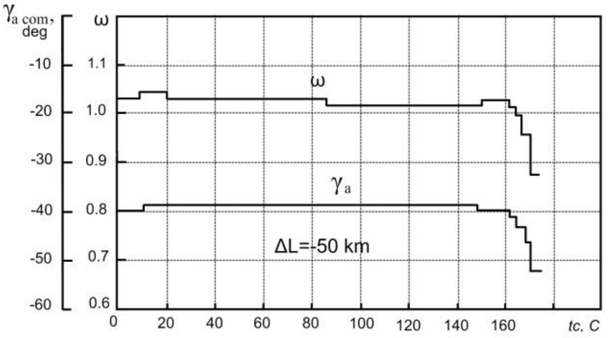

Fig. 7. Control programs

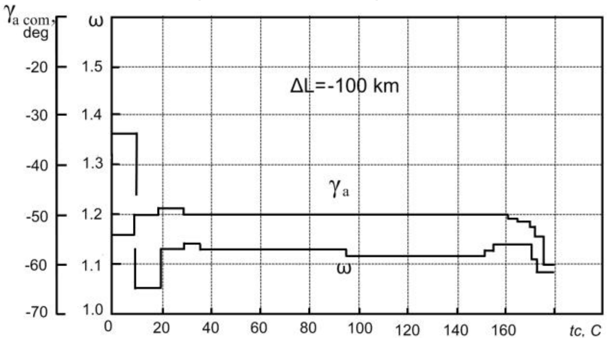

Fig. 8. Control programs

Conclusion

Results of the modeling shows that formation of the non-autonomous control of the roll angle and aerodynamic characteristic, based on multistep correction of the control program with use of hitting trajectories, allows to realize guidance of the vehicle with high methodical accuracy.

Convergent stable process of motion parameters variation is formed as a result of the control synthesis. At the end of the descent the velocity vector is close or directed along the sight line, and the self-guidance plane takes vertical position.

The domain of possible landing points is restricted by the range of possible variations of roll angle and aerodynamic characteristic. At high methodical accuracy of the guidance, at action of disturbances the accuracy of guidance is generally determined by navigation errors of current phase coordinates determination. Hereby, the method of termination control is efficient mean of the control synthesis for short-range guidance of the vehicle to the specified landing point.

REFERENCES

1. *Кондрашов Я. В.* Метод управления полетом легких летательных аппаратов/ тез. докл. Междунар. науч.-техн. конф. «Проблемы совершенствования радиоэлектронных комплексов и систем обеспечения полетов»: КИИГА, Киев, 22-24.09.1992. — С. 23—24.

2. *Кринецкий Е. И.* Системы самонаведения/ Е. И. Кринецкий. — М. : Машиностроение, 1970. — 236 с.

3. *Каменков Е. Ф.* Маневрирование спускаемых систем/ Е. Ф. Каменков. — М. : Машиностроение, 1983. — 184 с.

4. *Kondrashov Ya., Arutyunyan A., Kravchyshyn I.* The method of prediction of unmanned aerial vehicle motion parameters. Матеріали IX міжнар. наук.-техн. конф. «АВІА-2009». —Т. 2. — К. :НАУ, 2009. — С. 9.29—9.32.

5. *Kondrashov Ya., Arutyunyan A., Kravchyshyn I.* Principles of termination control of unmanned aerial vehicles flight path/ Наукоємні технології. — К. : НАУ, 2009. — С. 45—48.

Стаття надійшла до редакції 11.05.10.