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FEATURES OF THE INSTANTANEOUS FREQUENCY ESTIMATION ALGORITHM WITH PRE-FILTERING

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Features of the harmonic signal frequency estimation algorithm in different signal and interference situations are researched. Reasonability of using pre-filtering in order to improve estimation precision is proved. Research methodology is a statistical modeling.

Keywords: frequency, estimation, periodical signal, interference, pre-filtering.

Досліджено властивості алгоритму оцінювання частоти гармонічного сигналу у різних сигнально-завадових ситуаціях. Доведена доцільність використання попередньої фільтрації для покращення точності оцінок частоти. Методологія досліджень базується на статистичному моделюванні.

Ключові слова: частота, оцінка, періодичний сигнал, завада, попередня фільтрація.

Introduction

The instantaneous frequency (IF) is one of the basic signal parameters which provides important information about the time-varying spectral changes in non-stationary signals.

The concept of the IF finds its usage in various technical fields and applications such as seismic, radar, sonar, communications, electrical, mechanical and biomedical applications [1].

The specific case is the IF estimation in real time by short observation period, when its duration does not exceed one-two cycles of a signal and classical approaches like FFT, Hilbert transform or zero crossing detection are inapplicable.

At first, we should note that this paper is devoted to narrow enough problem and some terms will be used in a short form if it does not lead to ambiguity. For example, "estimation" means IF estimation.

Problem statement

In author's paper digital nonlinear algorithm for the IF estimation (the "algorithm") has been proposed and investigated [2].

It is based on an auto regression model of sine wave and supposes to calculate normalized frequency, which actually is a phase shift between samples

$$\gamma^*_{1(2)} = \arccos\left(\left(B(\bar{x}) \pm \sqrt{B(\bar{x})^2 + 2}\right)/2\right)$$

where $B(*)$ is signal statistics;

$$B(\bar{x}) = 0.5\sum_{i=2}^{N-1}\left[(x_{i+1} + x_{i-1})^2 - 2x_i^2\right] \Big/ \left[\sum_{i=2}^{N-1}\left(x_i\,(x_{i+1} + x_{i-1})\right)\right].$$

The next step is election of the value γ^* that is located in the method uniqueness zone: $(0,\ \pi/2)$.

And finally, the frequency is calculated as

$$f_S^* = \gamma^* f_T / 2\pi$$

where f_T is the sampling frequency.

This algorithm has been synthesized with idealized assumptions about the nature of the useful harmonic signal (the "signal").

In practice, the processing conditions can be significantly complicated by various distorting factors. Thus, the signal can be although periodic but non-sinusoidal, with the varied frequency or just absent. An interference is often correlated (the "interference") or even a harmonic.

The main purpose of this paper is to highlight and qualitative analyze the basic properties of the algorithm that are not reported in [2].

The results of numerical calculations have illustrative sense. Obviously, every aspect of practical application leads to conditions change and requires more detailed research.

Research methodology is a statistical modeling. We determine following general principles and conditions (if otherwise is not indicated in the text): the frequency of a signal S is always $f_S = 1\,\text{MHz}$, the variance of a white Gaussian noise (the "noise") $\sigma_g^2 = 1$, length of a window is equal to one period of the signal and contains 16 samples, an initial phase at each attempt is random.

Low-pass (LPF) and band-pass (BPF) Butterworth filters (the "filter") are used.

The “attempt” is execution of the estimation in a separate window.

In order to prevent correlation of various attempts, their windows do not overlap. In figures SNR is shown in absolute units, errors are normalized relative to nominal frequency.

Estimation errors by the harmonic interference

Appearance of an additional harmonic o with different frequency f_o and power P_o considerably worsen precision of estimation because the algorithm does not have any filtering properties.

Fig. 1 shows the graph of the estimation mean which depends on the frequency ratio and the signal to interference power ratio $\mathrm{m}_f(f_o/f_S, P_o/P_S)$ when the noise is absent.

Estimations randomness is caused by randomization of the signal and the interference initial phases and the standard deviation lies within 9 % zone relative to the nominal frequency. The obtained surface $\mathrm{m}_\omega(\cdot)$ is characterized by smoothness and high enough linearity of one-dimensional dependencies $\mathrm{m}_f(f_o/f_S \mid P_o/P_S = \mathrm{const})$.

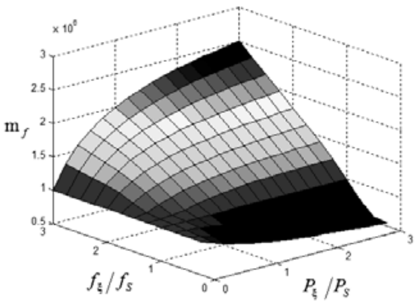

Fig. 1. Estimation mean for the harmonic interference

It was found that estimations practically does not depend on window size and the sampling frequency if it is much higher than frequencies f_S and f_o.

It should be noted that research of pre-filtering in this situation is pointless because it is only enough to determine the signal to interference power ratio from amplitude-frequency characteristic of the filter and directly address the function $\mathrm{m}_f(f_o/f_S, P_o/P_S)$.

Frequency deviation influence on precision

In the case of the locally non-stationary signal, when its actual IF is significantly variable within a single window, estimation depends on this variation degree.

For example, for linear frequency deviation estimation approximately equals middle between initial f_b and final f_e frequencies in the window:

$$f^* = (f_b + f_e)/2 + \Delta f^*.$$

Character of error Δf^* is shown in Fig. 2. Besides difference between window sizes is caused by different phase distances between samples.

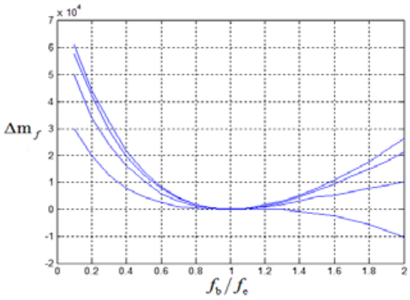

Fig. 2. Frequency estimation with deviation

Estimation of the noise process frequency

Let’s consider properties of the algorithm in estimation of noise processes if the signal is absent.

The set of noise processes was formed from a generative white Gaussian noise by 1st and 2nd order filters, as it is shown in the Table.

Bandwidth is shown in percent relative to the central frequency f_C, the cut-off frequency of a LPF is f_{co}.

Research results are presented for the nominal frequency 1 Hz.

Noise processes estimation

Noise process	Characteristic	Mean		Std. dev.	
	Window	32	16	32	16
	Filter				
White Gauss.	—	3.8	1.7	2.5	1.2
Low-frequency	LPF, 1-ord.	$0.8+0.7 f_{co}$	$0.7+0.5 f_{co}$	0.45	0.5
	LPF, 2-ord.	$0{,}8 f_{co}$	$0{,}7 f_{co}$	0.3	0.3
Narrow-band	BPF, 1-ord.,10%	$0.5+0.9 f_C$	$0.5+0.75 f_C$	0.3	0.3
	BPF, 2-ord., 10%	f_C	f_C	0.15	0.15
	BPF, 1-ord., 20%	$0.5+ f_C$	$0.5+0.75 f_C$	0.35	0.4
	BPF, 2-ord., 20%	f_C	f_C	0.13	0.18

It was identified that frequency estimation of a noise process, that is formed by 2nd order filters, is almost proportional to f_C or f_{co}.

The noise process formed by the 1st order filters has much high-frequency spectrum.

This leads to estimation shift. It should be noted that dependencies of mean for 1st order filters (see Table) are almost linear and this linearity stays within ±5 % zone in a range of arguments

0.3…3 MHz. It also concerns to the standard deviation that is shown as constant.

Estimation errors by the correlated noise

The results obtained above are useful for much clear perception of further research of the influence of the input process pre-filtering on estimation precision under conditions of different distorting factors (correlated interferences, noises, harmonics).

In this paragraph a mixture of the signal and correlated interference is considered. Simulation has one feature: in all cases desirable $SNR = P_S / \sigma_R^2$ (referred to correlated noise power at the output of the formative filter (σ_R^2)) is ensured. Also, the pre-filter parameters are variable.

Thus, the mean of estimation in the form of two-dimensional surface $\mathrm{m}_f(f_{co}, SNR)$ by using of the 1st order LPF is shown in Fig. 3.

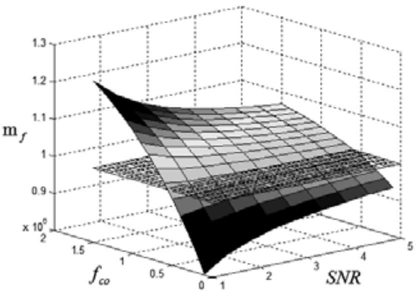

Fig. 3. The mean for the 1st order formative LPF

Estimation errors become low only when cut-off frequency of formative, which will be called optimal f_{co}^{opt}, is near half of signal frequency.

This fact is determined by presence of significant HF-components.

At the same time there is no considerable dependence f_{co}^{opt} on *SNR*. It is also identified that there is minimum of mean-square error (MSE) at 0.5 MHz, which equals approximately $0{,}1 f_S$.

Similar research for wider window relative to the signal period shows that it has little influence on the estimation mean and similar to Fig. 3 surfaces are obtained.

But MSE is decreases with square root dependence. Such effect also was observed in other researches, that will be mentioned further, so results only for the one-period window are presented.

When the 2nd order formative LPF with much steep amplitude-frequency characteristic is used, energetic influence of HF-components is much lesser then when the 1st order filter is used. Therefore, an optimal cut-off frequency is a little higher and equals $f_{co}^{opt} \approx (1{,}3...1{,}4) f_S$ (Fig. 4).

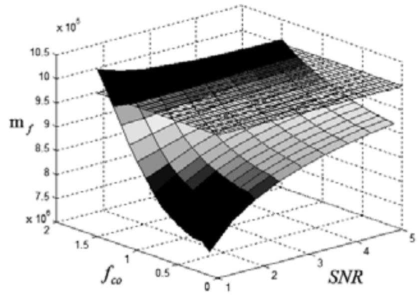

Fig. 4. The mean for the 2nd order formative LPF

This filter is also characterized by significant decreasing of the mean deviation near f_{co}^{opt} and its sharp increasing when $f_{co} < f_S$.

The last fact is caused by significant signal suppression.

We can obtain similar to Fig. 3 and Fig. 4 graphs when the formative filter is a BPF, but in this case the lowest errors are at $f_{co} \approx f_S$.

This fact shows that symmetry of the interference spectrum relative to the signal is one of conditions for errors.

Reasonability of the input mixture pre-filtering

The results of the preceding paragraph have, first of all, a purely cognitive sense. In practice, the correlative noise process is formed by pre-filtering before using the algorithm.

This situation is similar, but has significant difference that the mixture parameter is the $SNR = P_S / \sigma_g^2$ at the filter input and output SNR depends on a filter adjustments and cannot be controlled.

Graph of estimates mean (Fig. 5), when the 1st order LPF is used, shows a sufficiently large working area near nominal frequency.

It can be also confirmed by the MSE surface (Fig. 6) that is characterized by the argument reduction of the MSE minimum while SNR is increasing. It means that more signal suppression by filter is allowed. At the same time a high uncertainty of frequency is allowable.

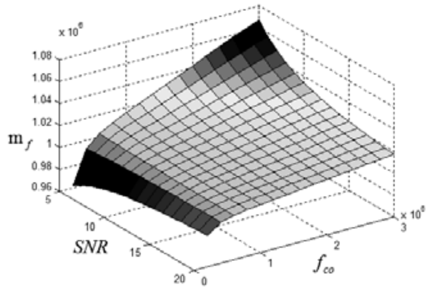

Fig. 5. The mean for the 1st order preliminary LPF

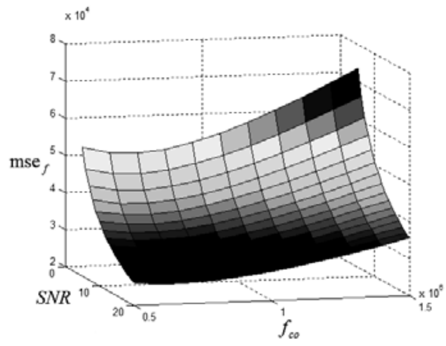

Fig. 6. The MSE for the 1st order preliminary LPF

Some decrease of the error mean can be achieved by reducing the number of samples in the window. This demonstrates by the surface (Fig. 7) for the case of eight samples, where sharp increase of error for small SNR is observed.

But we must remember that reducing the number of samples generally causes the increase of the MSE.

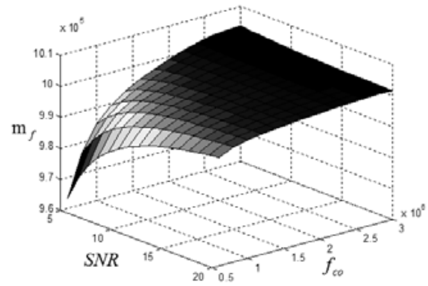

Fig. 7. The mean for the 1st order preliminary LPF

The 2nd order LPF can be considered as more efficient in use. The MSE surface for such filter is shown in Fig. 8.

Thus, in comparison with Fig. 6 MSE values at the points of optimum decreases 2 ... 3 times, but if the cut-off frequency is lesser than $f_S < 0{,}7 f_S$, MSE increases sharp.

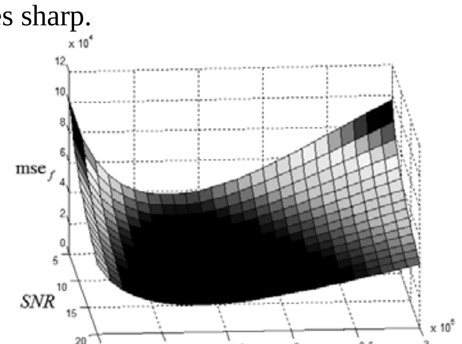

Fig. 8. The MSE for the 2nd order preliminary LPF

The usage of BPF allows us to further reduce MSE at optimal points, but requires a precise coordination of frequency tuning to a range of possible signal frequencies.

According to mean (Fig. 9) obtained for the 1st order BPF with a 20% bandwidth, a range of prior uncertainty should not exceed the value 0.8 ... 1.2 relative to the true value. When the 2nd order filter is used or bandwidth is narrower, requirements for a priori knowledge become stricter.

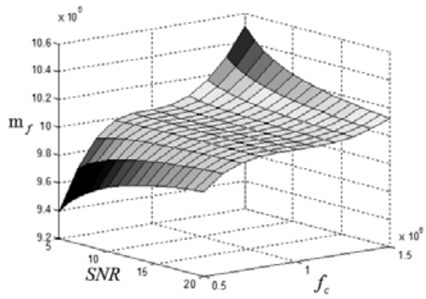

Fig. 9. The mean for the 1st order preliminary BPF

Improvement of non-harmonic signals estimation

Identified positive effect of mixture pre-filtering is also useful for estimation of frequency of periodic non-harmonic signals, the main feature of which is presence of higher harmonics. For example, the MSE surface of estimation of square wave frequency after the 1st order LPF is shown in Fig. 10.

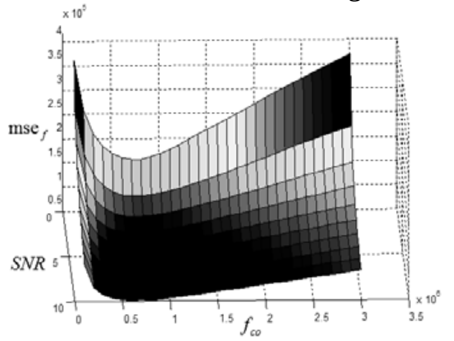

Fig. 10. The MSE for the 1st order preliminary LPF

A specific feature is the gradual shift of f_{co}^{opt} towards zero, due to the negative value of the second derivative of response of the LPF in the HF range. Feasibility of pre-filtering follows from the fact that without it errors of square wave frequency estimation (even without noise) exceed 50 % .

It was found that the value of MSE for trapezoidal signals, for which harmonics are much smaller, decreases 4...6 times in comparison to the meander.

Detection of the signal frequency trend

A great feature of the algorithm is a sufficiently accurate estimation at intervals (windows) equal to a period of the signal [2] and even at half period with the small noise, which brings us to the actual IF and provides the opportunity to observe its trend over time. The practical application of this feature can be single linear frequency-modulated (LFM) signal processing with pre-filtering by the 1st order LPF. As an example, the results of measuring the IF of such signal with frequency that varies from 0.9 to 1.8 MHz in 10 ms with the sampling frequency 16 MHz are shown in Fig. 11. MSE values are obtained by averaging by all signal counts.

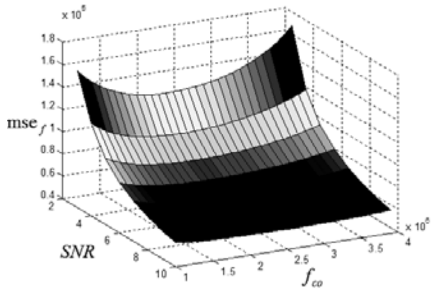

Fig. 11. The MSE for the LFM signal

The task of pre-filtering of the signal, when the frequency changes abruptly, is related to the previous problem. The form of the average MSE coincides with the surface in Fig. 11 with 15 % precision, but there is some shift towards zero.

Conclusions

1. Distorted factors cause significant worsening of frequency estimation precision. It can be improved by pre-filtering.

2. The 2nd order filter using is reasonable only in case of weak a priori ambiguity, in other cases the 1st order filter is more preferable.

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