UDC 629.7(045)

VARIATIONAL DIRECT PROBLEM OF TRANSPORT AVIATION SYSTEMS DEVELOPMENT FORECASTING

M. Oleg

National Aviation University

avia_icao@mail.ru

The problem of transport aviation system development forecasting modeling is considered in the article. The Pontryagin's method for variational problems solving is used for this purpose. The problem solution is given in common case, and in particular one, when only two competing aviation systems are considered. For particular case, the optimality condition of aviation groups packaging by one or two competing aviation systems were determined at condition of transport system maximal efficiency achievement during set time period.

Keyword: aviation system, mathematical model, forecasting, transport system

Розглянуто задачу побудови моделі прогнозування розвитку транспортних авіаційних систем. При цьому був використаний метод Понтрягіна для розв'язання варіаційних задач. Наведено розв'язання задачі, як у загальному, так і в окремому випадках, коли розглядається тільки дві конкуруючі авіаційні системи. Для окремого випадку, було визначено умови оптимальності комплектації авіаційних загонів однією або двома конкуруючими авіаційними системи за умови досягнення максимальної ефективності транспортної системи протягом заданого відрізка часу.

Ключові слова: авіаційна система, математична модель, прогнозування, транспортна система.

Introduction

Today the whole world is concerned about the global problem of the integration of unmanned aircraft systems (UAS) in the national air space. This is primarily due to the success of the UAS application for the solution of military tasks in many respects similar to the tasks which are to be solved for peaceful purposes: the observation of the various objects on the earth, communication, delivery of spare parts and supplies, spraying chemicals in the atmosphere or on the earth surface, the mitigation of natural disasters consequence, mapping of the area, the detection of atmospheric phenomena, etc. At the same time, one must consider that the UAS are a threat to our civil liberties. Therefore, the creation of a normative base of the UAS use in the civil aviation is a complex and important work that must be performed in the shortest time, with consideration of the present and future civil society interests. This is a guarantee of many branches effective development of the state economy.

An important issue in the development of the normative-legal base of the UAS integration in the national airspace is the selection of their classification. At the present time there are many of such classifications, but they all basically belong to the military UAS. It should be taken into account that for the UAS which use various types of lift creation, the classification should be fundamentally different. As in the nearest future it is expedient to use the UAS of aircraft type (with fixed wing) for civil purposes, so it is recommended to use the following classification of UAS: ultra-light with a maximum take-off weight up to 20 kg, light — from 20 up to 750 kg, medium — from 750 up to 5 700 kg, heavy — from 5700 up to 20 000 kg, super-heavy — more than 20 000 kg.

Such a classification makes it possible in large measure to use the already developed national and international normative and technical base for manned aircraft systems. First of all it refers to the issues of the UAS type certification and certification of the UAS developer.

According to the international law, the aircraft is any vehicle that is used or intended for use for the purpose of carrying out flights in the air. The UAS corresponded to this definition, and therefore, until recently, submit to the same laws that the aircraft with pilots on board. But many points of the national aviation regulations still do not apply to the UAS. For example, windshields and windows of the aircraft must meet certain criteria for accommodation and durability. These and other similar aviation standards, obviously, are not valid when there are neither pilots nor passengers on-board of the aircraft. So correspondingly, neither windshield nor windows are necessary. At the same time various types of video observation systems are established on the UAS, including the implementation of the UAS management. There are no requirements provided for such systems in existing aviation legislation.

Other norms are still applicable and may prevent the introduction of the UAS into operation. This category includes some of the basic principles of safety, such as, for example, the requirement for each operator of the aircraft, to observe alertness in order to see and avoid collision with another aircraft. Even UAS with the best video observation systems will not be able to "see" air traffic as pilot. Of course, the

UAS pilot being on the earth could see the flying planes with the naked eye, if only the UAS does not go beyond the limits of visibility, if the sky is clear and it happens when there is sufficient daylight. In connection with this, we must admit that the UAS does not correspond to the standard of "see-and-avoid", and therefore may not carry out flights in strict accordance with current legislation. Therefore, in relation to the UAS it is necessary to consider separately the question of the ability to avoid potential collisions with other aircraft and with the obstacles on the ground.

Certainly, just because of the fact that the UAS may not fully correspond to the national aviation legislation, it is not necessary to think that they cannot safely be used. In particular, at the moment this question is being solved through special permission of the UAS operation once in a while, i.e., the consideration of individual needs in order to exclude the UAS from the influence of restraining aviation standards. In particular the state aviation authorities require from UAS pilots strictly comply with the aviation rules, which relate to the airspace. The UAS must also be capable of ensuring safety in the event of communication loss between the pilot and the UAS.

The aviation system (AS) development forecasting model should characterize its primary development aim - creation of the system capable to give the best result for whole future time period $0 \le t \le T$ of its operation [1]. The first step is to concretize the functional

$$Э^{c} = \int_0^T f_0 dt .$$

As the functional function f_0 characterizing total effectiveness of AS functioning per time unit will be

$$f_0 = \sum_{i=1}^{n} \beta_i a_i x_i ,$$

where β_i is importance index of AS$_i$;

$$a_i = v_i \frac{m_{H_i} L_i}{t_{\text{p}_i}} P$$

is functioning intensiveness of AS$_i$; v_i is flight day part; m_{H_i} is mass of AS$_\text{i}$ transporting load; L_i is flight range;

$$t_{\text{p}_i} = t_{\text{пол}_i} + t_{\text{пасс}_i}$$

is trip time;

$$t_{\text{пол}_i} = L_i / V_i$$

is flight time with average cruising speed V_i; $t_{\text{пасс}_i}$ is passive time; P_i is probability of flight task performance of AS$_i$.

There is a need in determining the optimal control vector function

$$u_{\text{opt}}(t) = \left\{ u_{i_{\text{opt}}}(t) \right\}$$

and the correspondent optimal forecasting

$$x_{\text{opt}}(t) = \left\{ x_{i_{\text{opt}}} \right\}, i = \overline{1,n} ,$$

which are provide the maximal value of the functional:

$$u_{i_{\text{opt}}}(t) = \arg\max_{u(t)} \int_0^T \sum_{i=1}^{n} \beta_i a_i x_i dt$$

in case of

$$\frac{dx_i}{dt} = q_i(x,t) u_i(t) - \omega_i(t) x_i(t) = Q_i(x, u_i) ,$$

$$i = \overline{1,n} ,$$

where

$$q_i = \frac{f(t) - \sum_{i=1}^{n} C_{0_i}^{э}(x_i, t) x_i(t)}{C_i(x_i, t)}$$

is maximal intensiveness of AS$_\text{i}$ getting into aviation group ($u_i(t) = 1$);

$$\omega_i(t) = \omega_i^{\text{п}}(t) + \omega_i^{\text{г}}(t)$$

is summary intensiveness of AS$_i$ losses.

There are following limits on control functions:

$$0 \le u_i(t) \le 1, \ \sum_{i=1}^{n} u_i(t) = 1$$

and coordinate variables

$$x_i(t) \ge 0, \ g(x(t)) \le 0.$$

Actually, we have transfered our model design problem into Lagrange' variation task with set time $t = T$ and free right end $x(t)$.

Problem solving

To solve this problem by Pontryagin' method will be more rational [2]. First of all it is necessary to consider the task solution in common case.

The restriction

$$g(x(t)) \le 0$$

isn't taken into account for solution simplification.

To add one more equation to differential equations of AS development:

$$\frac{dx_0}{dt} = \frac{dЭ^{c}}{dt} = f_0 = \sum_{i=1}^{n} \beta_i a_i x_i = Q_0(x)$$

with initial condition

$$x_0 = x_0^0(0) = Э_0^{c}\left(x_0^0\right).$$

As a result we will obtain:

$$\frac{dx_i}{dt} = Q_i(x, u), \ i = \overline{0,n} , \qquad (1)$$

with correspondent initial conditions.

The adjoint system of equations should be added to this system of equations:

$$\frac{d\varphi_i}{dt} = -\sum_{\alpha=0}^{n} \frac{\partial Q_\alpha}{\partial x_i} \varphi_\alpha, \; i = \overline{0,n}, \quad (2)$$

with conditions: at

$$t = T\ \varphi_1(T) = \varphi_2(T) = \ldots = \varphi_n(T) = 0;$$

$$\varphi_0 = 1 \text{ for } 0 \le t \le T.$$

To equate the Hamiltonian function:

$$H = \sum_{\alpha=0}^{n} \varphi_\alpha Q_\alpha = \varphi_0 Q_0 + \sum_{i=1}^{n} \varphi_i Q_i = = Q_0 + \sum_{i=1}^{n} \varphi_i Q_i. \quad (3)$$

At substitution into (3) the right part of equation (1), we obtain

$$H = \sum_{i=1}^{n} (\beta_i \alpha_i - \omega_i \varphi_i) x_i + \sum_{i=1}^{n} \varphi_i q_i u_i.$$

It follows from Pontryagin'maximum principle, that

$$u_{\text{opt}}(t) = \left\{ u_{i_{\text{opt}}}(t) \right\} = \arg \max_{u_i(t) \in U} H = = \arg \max_{u_i(t) \in U} \sum_{i=1}^{n} \varphi_i q_i u_i,$$

where area

$$U \text{ is } 0 \le u_i(t) \le 1, \; \sum_{i=1}^{n} u_i(t) = 1.$$

As H and ratio U are linear control functions $u_i(t)$, that:

$$u_{i_{\text{opt}}}(t) = u_i(t) = = \begin{cases} 1, & if \quad \varepsilon_i = \max, \\ 0, & if \quad \varepsilon_i \ne \max, \text{ for } 0 \le t \le T, \end{cases} \quad (4)$$

where

$$\varepsilon_i(t) = \frac{\varphi_i(t)}{C_i} \quad (5)$$

is importance index of AS_i (dynamical comparison criterion of aviation system).

The adjoint functions $\varphi_i(t)$ are determined by integration (2).

So, the optimal control u_{opt} of AS development consists in the direction of all resources $C^c(t)$ on creation of valuable AS, i.e. systems that have the great influence on aviation group final development – the functional value

$$Э^c = \int_0^T f_0 dt.$$

We can make the conclusion, if on some time period

$$\Delta t_j \in [0, T]\ \varepsilon_j(t) > \varepsilon_i(t), \; j, i = \overline{1, n} j, j \ne i,$$

so AS_j is more preferable than AS_i. If on another time period

$$\Delta t_i \in [0, T]\ \varepsilon_i(t) > \varepsilon_j(t),$$

then AS_i is more preferable than AS_j and aviation group is formed from mixture of AS_j and AS_i.

Let's show how to solve this problem in particular case.

Let's assume, that:

Aviation group can consist of two aviation systems: AS_1 and AS_2 ($n = 2$; $i = 1, 2$). At initial time period $t = 0$ $x_1(0) = x_1^0$; $x_2^0 = 0$;

$$a_i = \text{const}, \; C_i = \text{const}, \; C_i^{п} = \text{const},$$

$$C_i^{э} = \text{const}, \; \omega_i^{п} = \text{const}, \; \omega_i^{г} = \text{const},$$

$$\omega_i = \text{const}, \; f = \frac{dC^c}{dt} = \text{const}.$$

It is necessary to determine the optimal equations

$$u_{i_{\text{opt}}}(t) = u_i(t)$$

and AS development trajectory

$$x_{i_{\text{opt}}}(t) = x_i(t).$$

From (4) follows, that optimal equations have a view:

$$u_1(t) = \begin{cases} 1, & 0 \le t \le t_{\text{пер}}, \quad \varepsilon_1(t) > \varepsilon_2(t), \\ 0, & t_{\text{пер}} \le t \le T, \quad \varepsilon_1(t) < \varepsilon_2(t); \end{cases}$$

$$u_2(t) = \begin{cases} 0, & 0 \le t \le t_{\text{пер}}, \quad \varepsilon_2(t) < \varepsilon_1(t), \\ 1, & t_{\text{пер}} \le t \le T, \quad \varepsilon_2(t) > \varepsilon_1(t). \end{cases}$$

The optimal AS development trajectory $x_1(t)$ and $x_2(t)$ is described by formulas for $0 \le t < t_{\text{пер}}$

$$x_1(t) = \frac{k_{11}}{k_{12}} - \frac{k_{11} - k_{12} x_1^0}{k_{12}} e^{-k_{12} t};$$

$$x_2(t) = 0,$$

where $k_{11} = \dfrac{f}{C_1}$, $k_{12} = \dfrac{C_{01}^{э}}{C_1} + \omega_1$ are indexes taking into account dynamical factors of development f, $C_1^{э}$ and ω_1;

for $t_{\text{пер}} \le t \le T$

$$x_1(t) = x_1^{\text{пер}} e^{-\omega_1 (t - t_{\text{пер}})};$$

$$x_2(t) = \frac{k_{21}}{k_{22}} - \frac{C_{01}^{э} x_1^{\text{пер}}}{C_2 (k_{22} - \omega_1)} e^{-\omega_1 (t - t_{\text{пер}})} - - \left[\frac{k_{21}}{k_{22}} - \frac{C_{01}^{э} x_1^{\text{пер}}}{C_2 (k_{22} - \omega_1)} \right] e^{-k_{22} (t - t_{\text{пер}})}$$

where

$$k_{21} = \frac{f}{C_2}; \; k_{22} = \frac{C_{02}^{э}}{C_2} + \omega_2;$$

$x_1^{\text{пер}}$ is determined at $t = t_{\text{пер}}$.

The time $t_{\text{пер}}$ is determined as crossing point of functions $\varepsilon_1(t)$ and $\varepsilon_2(t)$, i.e. from equation

$$\varepsilon_1(t_{\text{пер}}) = \varepsilon_2(t_{\text{пер}}).$$

The integration of adjoint equations (2) allows to find $\varphi_1(t)$ and $\varphi_2(t)$ and further

$$\varepsilon_1(t) = \varphi_1(t)/C_1$$

$$\text{and } \varepsilon_2(t) = \varphi_2(t)/C_2,$$

$$\varepsilon_1 = k_1 + k_2 e^{-\omega_1(T-t)} - k_3 e^{-b_2(T-t)}, \tag{6}$$

where

$$k_1 = \frac{1}{\omega_1}\left(\frac{\beta_1\alpha_1}{C_1} - \frac{\beta_2\alpha_2 C_{01}^{э}}{b_2 C_2 C_1}\right);\ b_2 = \frac{C_{02}^{э}}{C_{02}} + \omega_2;$$

$$k_2 = \frac{C_{01}^{э}\beta_2\alpha_2}{C_1 b_2 C_2}\left(\frac{1}{\omega_1} + \frac{1}{b_2 - \omega_1}\right) - \frac{\beta_1\alpha_1}{\omega_1 C_1};$$

$$k_3 = \frac{C_{01}^{э}\beta_2\alpha_2}{C_1 b_2 C_2 (b_2 - \omega_1)},$$

where

$$C_{01}^{э} = C_1^{э} - \alpha\omega_1^{п} C_1^{п};\ C_{02}^{э} = C_2^{э} - \alpha\omega_2^{п} C_2^{п},$$

$$\varepsilon_2 = \frac{\beta_2\alpha_2}{b_2 C_2}\left(1 - e^{-b_2(T-t)}\right). \tag{7}$$

We can consider the indirect conditions of transport AS (TAS) competitiveness at pairwise comparison using the results of obtained model.

Let on time period adjoint to $t = T$ the TAS$_i$ is more preferable than TAS$_j$. Then if in point $t = T$ the AS significance coefficients are the same:

$$\varepsilon_i(T) = \varepsilon_j(T) \text{ (as } \varphi_i(T) = \varphi_j(T) = 0),$$

than in point, which is near to T ($t = T - \Delta t$, Δt is small value) $\varepsilon_i(t = T - \Delta t) > \varepsilon_j(t = T - \Delta t)$, i.e.

$$K = \frac{\varepsilon_i(t = T - \Delta t)}{\varepsilon_j(t = T - \Delta t)} > 1,$$

where K is AS preference coefficient. Lets determine K for $\Delta t \to 0$.

For this purpose we use (6) and (7), substituting indexes «1» and «2» by indexes «j» and «i».

As $t = T - \Delta t$, than:

$$\varepsilon_i = \frac{\beta_i\alpha_i}{b_i C_i}\left(1 - e^{-b_i\Delta t}\right) = \frac{\beta_i\alpha_i}{C_i}\Delta t,$$

$$\varepsilon_j = k_1 + k_2(1 - \omega_j\Delta t) - k_3(1 - b_i\Delta t).$$

Taking into account, that in point $t = T$ $\varepsilon_j(T) = 0$ and it means $k_1 + k_2 - k_3 = 0$, and substituting k_2 and k_3 by its connection with parameters we obtain

$$\varepsilon_j = k_2\omega_j\Delta t + k b_i\Delta t = \frac{\beta_j\alpha_j}{C_j}\Delta t.$$

Therefore, the preference condition of AS$_i$ over AS$_j$ in points $t \to T$ has the view

$$K = \frac{\beta_i\alpha_i C_j}{\beta_j\alpha_j C_i} > 1. \tag{8}$$

In order that TAS$_j$ was competitive with TAS$_i$, the transport system should need it (in such case aviation group is formed consequently from TAS$_j$ and TAS$_i$), i.e. in order that significance coefficient TAS$_j$ ε_j was more than significance coefficient TAS$_i$ ε_i in time points adjointing to $t = 0$, in particular, and in point $t = 0$: $\varepsilon_j(0) > \varepsilon_i(0)$.

Using this inequality, and also formulas (6) and (7), substituting indexes «1» and «2» by indexes «j» and «i», the competitiveness condition of TAS$_j$ with TAS$_i$ can be represented in view

$$\Delta = \left(1 - e^{-\omega_j T}\right) - \delta K > 0, \tag{9}$$

where

$$\delta = \frac{C_{0j}^{э}}{C_j b_i} - \frac{C_{0j}^{э}}{C_j(b_i - \omega_j)}e^{-\omega_j T} +$$

$$+\frac{C_{0j}^{э}\omega_j}{C_j b_i(b_i - \omega_j)}e^{-b_i T} + \frac{\omega_j}{b_i}\left(1 - e^{-b_i T}\right).$$

At sufficiently big T the TAS competitiveness condition is transformed into view:

$$\Delta = 1 - \delta K > 0, \tag{10}$$

where

$$\delta = \frac{b_j}{b_i};$$

$$b_j = \frac{C_{0j}^{э}}{C_j} + \omega_j;$$

$$b_i = \frac{C_{0i}^{э}}{C_i} + \omega_i;$$

$$C_{0j}^{э} = C_j^{э} - \alpha C_j^{п}\omega_j^{п};$$

$$C_{0i}^{э} = C_i^{э} - \alpha C_i^{п}\omega_i^{п}. \tag{11}$$

So, in order to form aviation group on time period $0 \le t \le T$ from combination TAS$_i$ and TAS$_j$, it is necessary to fulfill the conditions:

$$K > 1 \text{ and } K < \frac{b_i}{b_j} \tag{12}$$

(assume that time of TAS$_i$ development does not exceed the transfer time from production of TAS$_j$ to production of TAS$_i$ and $C_{0j}^{э}$, $C_{0i}^{э} > 0$).

Conclusions

So, in order to form aviation group only from TAS$_i$ (TAS$_j$ is not competitive with TAC$_i$), the observation of unequality (8), and equality (10) with variable sign is necessary: $\Delta < 0$ (at these conditions $\varepsilon_i(t) > \varepsilon_j(t)$ for whole period $0 \le t \le T$), i.e.

$$\mathrm{K} > 1,\ K > \frac{b_i}{b_j}. \tag{13}$$

These ratios are essence of TAS_i monopolity condition. The TAS_j monopolity conditions include (10), and also (8) with variation sign:

$$K < 1$$

(at such conditions $\varepsilon_j(t) > \varepsilon_i(t)$
for whole period $0 \le t \le T$),

i.e.

$$\mathrm{K} > 1,\ K < \frac{b_i}{b_j}. \tag{14}$$

The competitiveness conditions of TAS_i and TAS_j (12), and also monopolity of TAS_i (13) and TAS_j (14) are valid at independent from time intensiveness of development appropriation of aviation group $f = dC^{c}/dt$ and TAS losses from system ω, and also from time and TAS quantity, functioning intensiveness a, creation cost C and sale abroad $C^{п}$ and year operational expenses $C_{э}$. The values of TAS competitiveness indexes (12) and monopolity (13) and (14) depend on a, C, $C^{п}$, $C^{о}$, $\omega^{п}$, $\omega^{г}$, α.

REFERENCES

1. *Smith D.* Dynamic Programming: A Practical Introduction, Ellis Horwood/ D. Smith.– London, 1991

2. *Boyce William E.* Elementary differential equations and boundary value problems / William E. Boyce, Richard C. DiPrima – 7th ed.

Article received 20.02.13.