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# DOMINANT CONTRIBUTION OF DIPOLES IN THE TURBULENCE-GENERATED NOISE IN A RIGID PIPE

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*A particular case of the theory of noise generation by a limited region of turbulence in an infinite straight rigid pipe of a circular cross-section, which has been developed in work [6], is considered. In this case, the situation is studied in which the generated acoustic field is dominated by the contribution made by surface dipoles. Those flows and shapes of the pipe local narrowings are of concern, which result in occupation of a turbulent flow region by uniformly-distributed large or small eddies. For these cases the corresponding simplified expressions for the generated acoustic power are obtained, and their estimates are carried out for the characteristic scales in the turbulent flow region.*

**Keywords:** noise, turbulence, pipe, dipoles.

*Розглянуто частинний випадок розробленої в праці [6] теорії генерації шуму обмеженою областю турбулентності в нескінченній прямій жорсткостінній трубі кругового поперечного перерізу. Досліджено ситуацію, коли у згенерованому акустичному полі домінує внесок поверхневих диполів. При цьому інтерес становлять такі потоки і форми локальних звужень труб, при яких регіон турбулізованої звуженням течії займають рівномірно розподілені великі або малі вихори. Для цих випадків одержано відповідні спрощені вирази для згенерованої акустичної енергії і проведено їх оцінки для характерних масштабів в області турбулентності.*

**Ключові слова:** шум, турбулентність, труба, диполі.

## Introduction

Study of flows in pipes is an actual problem in car- and aircraft-building industry, gas- and oil-industry, architecture, municipal economy, medicine, etc. Here a significant interest is related to flow turbulization and the acoustic effects appearance due to local pipe narrowings, such as wall deposits, welding joints, stenosis, etc. It is explained by the fact that the generated acoustic field holds information about the pipe and flow parameters in the noise-producing region, and, hence, there is the principal possibility of developing *non-invasive* acoustic diagnostic techniques capable of finding such region and finally the irregularity from an analysis of the indicated field [1–5].

The non-invasive acoustic diagnostic techniques can be developed under the availability of theories describing adequately the fluid rheology and dynamics, as well as the flow acoustics near the narrowing, and, hence, relating quantitatively the generated acoustic field characteristics to the narrowing, pipe and flow parameters.

In reference [6], a theory of noise generation by a limited region of turbulence in an infinite straight rigid-walled pipe of circular cross-section has been developed, and the corresponding quantitative relationships between the generated noise field characteristics and the pipe and flow parameters have been obtained.

A turbulence region was modeled by the distributed quadrupole and dipole noise sources (whose characteristics were assumed to be known), and the cases of *uniform* and *non-uniform* source distribution (i. e., *homogeneous* and *non-homogeneous* turbulence) were considered. In the next work [7] the case of dominant contribution of quadrupoles in the turbulence-generated noise in a pipe has been investigated, and the corresponding simplified expressions for the noise characteristics have been established.

In this paper another particular case of that theory is considered. Here a situation is studied when the above-noted noise field is dominated by the contribution made by surface dipoles. Those flows and shapes of the pipe local narrowing's are of interest, that result in large or small eddies distributed uniformly in the turbulence region immediately behind the narrowing.

The paper consists of two sections, conclusions and a list of references. In its first section, a problem is formulated, the appropriate equations and boundary conditions are written, as well as a general solution to the problem, which has been obtained in work [6], is presented and briefly analyzed. In the second section, the above-noted particular case of the solution is considered, and the corresponding estimates of the acoustic power are carried out. Finally, the conclusions of the investigation are formulated, and a list of references used in this paper is given.

## 1. Formulation of the problem and its general solution

Before proceeding to the above-noted particular case of the theory of noise generation by a compact region of turbulence in a pipe, which has been developed in reference [6], let us remind the physical and corresponding mathematical formulation of the problem, as well as present and briefly analyze its general solution. So, an infinite straight immovable rigid-walled pipe of circular cross-section of radius $a$ is considered. In this pipe, a fluid, of mass density $\rho$, sound speed $c_0$ and kinematic viscosity $\nu$, flows with the mean axial velocity $U$. The flow is characterized by the small Mach number, $M = U/c_0 << 1$. A finite fluid volume, $V_0$, is in the turbulent state, and produces noise in the pipe. It is necessary to find this noise field and establish the quantitative relationships between its characteristics and the pipe and flow parameters.

The noise field of interest is governed by the Lighthill's equation, in which the right part contains both quadrupole, $\partial^2 T_{ij}/\partial y_i \partial y_j$, and dipole, $\partial F_i/\partial y_i$, sources due to the pipe wall [6–8], viz.

$$\frac{\partial^2 \rho_a}{\partial t^2} - c_0^2 \nabla^2 \rho_a = \frac{\partial^2 T_{ij}}{\partial y_i \partial y_j} + \frac{\partial F_i}{\partial y_i}, \qquad (1)$$

$$0 < r < a, \quad 0 < \phi < 2\pi, \quad |z| < \infty.$$

The boundary conditions are that the radial component of the acoustic velocity vanishes on the pipe wall, viz.

$$\left.\frac{\partial p_a}{\partial r}\right|_{r=a} = 0 \qquad (2)$$

and that all acoustic waves are outgoing at infinity (i. e., there is no sound reflection at the pipe ends).

In relationships (1) and (2), $\rho_a$ and $p_a$ are the acoustic density and pressure fluctuations, respectively, which are related as [6–8]

$$p_a = c_0^2 \rho_a;$$

$T_{ij} \approx \rho u_i u_j$ and $F_i = n_j(\tau_{ij} + p\delta_{ij})$ the Lighthill's stresses and the $i$-th force component acting on the pipe wall ($T_{ij}$ and $F_i$ vanish outside the volume $V_0$ and the restricting surface $S_0$, respectively); $\tau_{ij} = (2/3)\mu\varepsilon_{kk}\delta_{ij} - 2\mu\varepsilon_{ij}$ the viscous stresses; $\varepsilon_{ij} = (1/2)(\partial u_i/\partial y_j + \partial u_j/\partial y_i)$ the strain rates; $n_j$ the $j$-th component of the outward normal to the pipe wall; $u_i$ the $i$-th component of the fluid velocity; $p$ and $\mu = \rho\nu$ the fluid pressure and dynamic viscosity; $r, \phi, z$ the cylindrical coordinates; $y_1, y_2, y_3$ the other their notations; and $\delta_{ij}$ the Kronecker delta. In addition, hereinafter the summation on repeated indices is assumed.

The boundary problem (1), (2) is solved by the Green's function technique [6, 8, 9], with subsequent application of the normal mode method. After performing the required mathematical operations, a general expression for the acoustic power $\mathrm{P}(\omega)$ generated at the frequency $\omega$ by the quadrupole and dipole sources, distributed *non-uniformly* in the volume $V_0$ and on the surrounding surface $S_0$, respectively, has the following form [6]

$$\mathrm{P}(\omega) = \sum_{q=1}^{2} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \mathrm{P}_{nm}^{(q)}(\omega) =$$

$$= \sum_{q=1}^{2} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \left[ \mathrm{P}_{nm}^{(q)T}(\omega) + \mathrm{P}_{nm}^{(q)F}(\omega) + \mathrm{P}_{nm}^{(q)TF}(\omega) \right];$$

$$\mathrm{P}_{nm}^{(q)T}(\omega) = \frac{1}{4\left\|\Psi_{nm}^{(q)}\right\|^2 k_{nm}\rho_0\omega} \iiint_{V_0} \mathrm{d}V_0(\mathbf{r}_0) \times$$

$$\times \iiint_{V_0} \frac{\partial^4 \mathrm{S}_{ijkl}^T(\mathbf{r}_0, \mathbf{r}_0', \omega)}{\partial y_i \partial y_j \partial y_k' \partial y_l'} \Psi_{nm}^{(q)}(r_0, \phi_0) \times$$

$$\times \Psi_{nm}^{(q)}(r_0', \phi_0') \mathrm{e}^{-\mathrm{sign}(z - z_0)\mathrm{i}k_{nm}(z_0' - z_0)} \mathrm{d}V_0(\mathbf{r}_0');$$

$$\mathrm{P}_{nm}^{(q)F}(\omega) = \frac{1}{4\left\|\Psi_{nm}^{(q)}\right\|^2 k_{nm}\rho_0\omega} \iint_{S_0} \mathrm{d}S_0(\mathbf{r}_{0a}) \times$$

$$\times \iint_{S_0} \frac{\partial^2 \mathrm{S}_{ik}^F(\mathbf{r}_{0a}, \mathbf{r}_{0a}', \omega)}{\partial y_i \partial y_k'} \Psi_{nm}^{(q)}(a, \phi_0) \times$$

$$\times \Psi_{nm}^{(q)}(a, \phi_0') \mathrm{e}^{-\mathrm{sign}(z - z_0)\mathrm{i}k_{nm}(z_0' - z_0)} \mathrm{d}S_0(\mathbf{r}_{0a}');$$

$$\mathrm{P}_{nm}^{(q)TF}(\omega)=\frac{1}{4\left\|\Psi_{nm}^{(q)}\right\|^{2}k_{nm}\rho_0\omega}2\,\mathrm{Re}\Bigg(\iiint_{V_0}\mathrm{d}V_0(\mathbf{r}_0)\times$$

$$\times\iint_{S_0}\frac{\partial^3\mathrm{S}_{ijk}^{TF}(\mathbf{r}_0,\mathbf{r}_{0a}^{/},\omega)}{\partial y_i\partial y_j\partial y_k^{/}}\Psi_{nm}^{(q)}(r_0,\phi_0)\times$$

$$\times\Psi_{nm}^{(q)}(a,\phi_0^{/})\mathrm{e}^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}(z_0^{/}-z_0)}\mathrm{d}S_0(\mathbf{r}_{0a}^{/}) \quad . \quad (3)$$

Here $\mathbf{r}=(r,\phi,z)$ is the field-point vector; $\mathbf{r}_0=(r_0,\phi_0,z_0)\in V_0$ and $\mathbf{r}_0^{/}=(r_0^{/},\phi_0^{/},z_0^{/})\in V_0$ the position vectors of quadrupoles in the region $V_0$; $\mathbf{y}=(y_i)_{i=1}^{3}$ and $\mathbf{y}^{/}=(y_i^{/})_{i=1}^{3}$ the other notations of $\mathbf{r}_0$ and $\mathbf{r}_0^{/}$, respectively; $\mathbf{r}_{0a}=\mathbf{r}_0\big|_{r_0=a}=(a,\phi_0,z_0)\in S_0$ and $\mathbf{r}_{0a}^{/}=\mathbf{r}_0^{/}\big|_{r_0^{/}=a}=(a,\phi_0^{/},z_0^{/})\in S_0$ the position vectors of dipoles on the surface $S_0$; $\mathrm{d}V_0(\mathbf{r}_0)=r_0dr_0d\phi_0dz_0$ and $\mathrm{d}S_0(\mathbf{r}_{0a})=ad\phi_0dz_0$ the volume and area elements, respectively;

$$\Psi_{nm}^{(1)}=\mathrm{J}_n(\alpha_{nm}r)\cos(n\phi),$$

$$\Psi_{nm}^{(2)}=\mathrm{J}_n(\alpha_{nm}r)\sin(n\phi)$$

the pipe acoustic modes whose squared norms, $\left\|\Psi_{nm}^{(q)}\right\|^{2}$, are written as

$$\left\|\Psi_{nm}^{(1)}\right\|^{2}=\begin{cases}\pi a^2\mathrm{J}_0^2(\alpha_{0m}a),n=0,\\ \dfrac{\pi a^2}{2}\mathrm{J}_n^2(\alpha_{nm}a)\left[1-\dfrac{n^2}{\alpha_{nm}^2a^2}\right],n\ge1,\end{cases}$$

$$\left\|\Psi_{nm}^{(2)}\right\|^{2}=\begin{cases}0,n=0,\\ \left\|\Psi_{nm}^{(1)}\right\|^{2},n\ge1,\end{cases}$$

$\mathrm{J}_n$ cylindrical Bessel functions of $n$-th order; $\alpha_{nm}=\zeta_{nm}/a$ the radial wavenumbers; $\zeta_{nm}$ the tabular roots of equation

$$\mathrm{J}_n^{/}(\zeta_{nm})=0\,,\ m=1,2,...\,;$$

$k_{nm}=\sqrt{k_0^2-\alpha_{nm}^2}$ the axial wavenumbers; $k_0=\omega/c_0$ the acoustic wavenumber; and

$$\mathrm{sign}(z-z_0)=\begin{cases}1,z\ge z_0\\ -1,z<z_0\end{cases}$$

the sign-function.

In addition, in relationship (3) $\mathrm{S}_{ijkl}^{T}$ and $\mathrm{S}_{ik}^{F}$ are the cross-spectra of the Fourier images of, respectively, the Lighthill's stresses $T_{ij}$, viz.

$$\mathrm{S}_{ijkl}^{T}(\mathbf{r}_0,\mathbf{r}_0^{/},\omega)\delta(\omega-\omega^{/})=<\breve{T}_{ij}^{*}(\mathbf{r}_0,\omega)\breve{T}_{kl}(\mathbf{r}_0^{/},\omega^{/})>,$$

and the forces $F_k$, viz.

$$\mathrm{S}_{ik}^{F}(\mathbf{r}_{0a},\mathbf{r}_{0a}^{/},\omega)\delta(\omega-\omega^{/})=<\breve{F}_i^{*}(\mathbf{r}_{0a},\omega)\breve{F}_k(\mathbf{r}_{0a}^{/},\omega^{/})>$$

$\mathrm{S}_{ijk}^{TF}$ is the cross-spectrum of the Fourier images of the stresses $T_{ij}$ and the forces $F_k$, viz.

$$\mathrm{S}_{ijk}^{TF}(\mathbf{r}_0,\mathbf{r}_{0a}^{/},\omega)\delta(\omega-\omega^{/})=<\breve{T}_{ij}^{*}(\mathbf{r}_0,\omega)\breve{F}_k(\mathbf{r}_{0a}^{/},\omega^{/})>$$

$\delta(...)$ the Dirac delta-function, $\mathrm{Re}(...)$ denotes a real part of the complex quantity indicated in the parenthesis, and the location of frequency $\omega$ relative to the pipe cut-off frequencies

$$\omega_{nm}=c_0\alpha_{nm} \qquad (4)$$

specifies (via the wavenumbers $k_{nm}$ in the exponent $\exp(-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}(z_0^{/}-z_0))$ the cases of propagating (homogeneous), viz.

$$\omega\ge\omega_{nm}$$

and non-propagating (evanescent), viz.

$$0<\omega<\omega_{nm}$$

waves.

When the quadrupole and dipole noise sources are distributed *uniformly* in their domains, formula (3) is simplified due to simplification of expressions for the spectra $\mathrm{S}_{ijkl}^{T}$, $\mathrm{S}_{ik}^{F}$ and $\mathrm{S}_{ijk}^{TF}$, which, in that case, become the functions of the source separation distance and the frequency only, viz.

$$\mathrm{S}_{ijkl}^{T}(\mathbf{r}_0,\mathbf{r}_0^{/},\omega)=\mathrm{S}_{ijkl}^{T}(\xi,\omega)\,,\quad \xi=\mathbf{r}_0^{/}-\mathbf{r}_0\,;$$

$$\mathrm{S}_{ik}^{F}(\mathbf{r}_{0a},\mathbf{r}_{0a}^{/},\omega)=\mathrm{S}_{ik}^{F}(\xi_{aa},\omega)\,,\quad \xi_{aa}=\mathbf{r}_{0a}^{/}-\mathbf{r}_{0a}\,;\quad (5)$$

$$\mathrm{S}_{ijk}^{TF}(\mathbf{r}_0,\mathbf{r}_{0a}^{/},\omega)=\mathrm{S}_{ijk}^{TF}(\xi_a,\omega)\,,\quad \xi_a=\mathbf{r}_{0a}^{/}-\mathbf{r}_0\,.$$

The analysis of expression (3) shows that the acoustic power $\mathrm{P}$ is a sum of powers $\mathrm{P}_{nm}^{(q)}$ of the pipe acoustic modes, $\psi_{nm}^{(q)}$, the individual mode power, $\mathrm{P}_{nm}^{(q)}$, consisting of the three parts. The first part, $\mathrm{P}_{nm}^{(q)T}$, is the acoustic power generated by the quadrupoles $\partial^2T_{ij}/\partial y_i\partial y_j$ in the mode $\Psi_{nm}^{(q)}$, the second one, $\mathrm{P}_{nm}^{(q)F}$, results from the dipoles $\partial F_i/\partial y_i$, and the third one, $\mathrm{P}_{nm}^{(q)TF}$, is due to interaction of the quadrupoles and dipoles in the same duct mode.

Further analysis of formula (3) shows that the relative contribution of each part to the mode power $\mathrm{P}_{nm}^{(q)}$ (and, hence, to the acoustic power $\mathrm{P}$) is different for the different Mach number values. In fact, when the Mach number $M$ is such that the noise field is dominated by the contribution from volume quadrupoles, only the first part, $\mathrm{P}_{nm}^{(q)T}$, remains in the expression for $\mathrm{P}_{nm}^{(q)}$, and then relationship (3) takes the following form

$$\mathrm{P}(\omega)=\sum_{q=1}^{2}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{1}{4\left\|\Psi_{nm}^{(q)}\right\|^{2}k_{nm}\rho_0\omega}\iiint_{V_0}\mathrm{d}V_0(\mathbf{r}_0)\times$$
$$\times\iiint_{V_0}\frac{\partial^4\mathrm{S}_{ijkl}^{T}(\mathbf{r}_0,\mathbf{r}_0^{/},\omega)}{\partial y_i\partial y_j\partial y_k^{/}\partial y_l^{/}}\Psi_{nm}^{(q)}(r_0,\phi_0)\times$$
$$\times\Psi_{nm}^{(q)}(r_0^{/},\phi_0^{/})\mathrm{e}^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}(z_0^{/}-z_0)}\mathrm{d}V_0(\mathbf{r}_0^{/})\,. \quad (6)$$

When the Mach number falls in a range where surface dipoles dominate, the second part, $\mathrm{P}_{nm}^{(q)F}$, dominates in $\mathrm{P}_{nm}^{(q)}$, and one has instead of expression (6)

$$\mathrm{P}(\omega)=\sum_{q=1}^{2}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{1}{4\left\|\Psi_{nm}^{(q)}\right\|^{2}k_{nm}\rho_0\omega}\iint_{S_0}\mathrm{d}S_0(\mathbf{r}_{0a})\times$$
$$\times\iint_{S_0}\frac{\partial^2\mathrm{S}_{ik}^{F}(\mathbf{r}_{oa},\mathbf{r}_{oa}^{/},\omega)}{\partial y_i\partial y_k^{/}}\Psi_{nm}^{(q)}(a,\phi_0)\times$$
$$\times\Psi_{nm}^{(q)}(a,\phi_0^{/})\mathrm{e}^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}(z_0^{/}-z_0)}\mathrm{d}S_0(\mathbf{r}_{0a}^{/})\,. \quad (7)$$

## 2. Dominant contribution of dipoles

Let us consider the situation when the acoustic field in a pipe is dominated by the contribution of surface dipoles. The dipoles are assumed to be distributed uniformly over the surface $S_0$ surrounding the turbulence region $V_0$. The first of these conditions can be realized in practice when the Reynolds number $\mathrm{Re}$ in the turbulent flow region $V_0$ behind a local pipe narrowing is either close to the critical value $\mathrm{Re}_{cr}$ (i.e., $\mathrm{Re}\sim\mathrm{Re}_{cr}$) or slightly higher than $\mathrm{Re}_{cr}$ (i.e., $\mathrm{Re}>\mathrm{Re}_{cr}$, $M<<1$). The second condition (as in the case of uniform distribution of quadrupoles [7]) can be ensured when

- the basic flow upstream of the local pipe narrowing is characterized by axial symmetry and has a parabolic velocity profile;
- the pipe narrowing has an axisymmetric and smooth geometry.

Under these conditions, relationship (3) is reduced to formula (7), which, due to simplification of expressions for the functions $\mathrm{S}_{ik}^{F}$ (see formulas (5)), is simplified to the form

$$\mathrm{P}(\omega)=\sum_{q=1}^{2}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{a^2\mathrm{J}_n^2(\alpha_{nm}a)}{4\left\|\Psi_{nm}^{(q)}\right\|^{2}k_{nm}\rho_0\omega}\int_{0}^{2\pi}\int_{z_{0i}}^{z_{0e}}\mathrm{b}_q(n\phi_0)\mathrm{d}\phi_0\times$$
$$\times\,\mathrm{d}z_0\int_{-\phi_0}^{2\pi-\phi_0}\int_{z_{0i}-z_0}^{z_{0e}-z_0}\frac{\partial^2\mathrm{S}_{ik}^{F}(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}\mathrm{b}_q(n(\phi_0+\xi_\phi))\times$$
$$\times e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}\xi_z}\mathrm{d}\xi_\phi\mathrm{d}\xi_z \quad (8)$$

(here $\mathrm{b}_1(n\phi_0)=\cos(n\phi_0)$, $\mathrm{b}_2(n\phi_0)=\sin(n\phi_0)$, $\partial\mathrm{S}_{ik}^{F}/\partial\xi_r=0$).

The double integral over $\xi_\phi$ and $\xi_z$ in relationship (8) depends on $\phi_0$ and $z_0$. Therefore, it is evident that in general the expression for the spectrum $\mathrm{P}$ cannot be simplified significantly.

However, such simplification becomes possible when the turbulence region $V_0$ is occupied primarily by large-scale or small-scale vortex structures (the situations, when it is possible in practice, are described in [7]).

Let us consider these cases.

### 2.1. Large eddies

Let us consider the case when the region $V_0$ is occupied primarilly by so large vortex structures that surface dipoles are completely correlated around the circle $r_0=a$.

In such a situation, the cross-spectra $\mathrm{S}_{ik}^{F}$ will not depend on the azimuthal coordinate $\xi_\phi$, viz.

$$\frac{\partial\mathrm{S}_{ik}^{F}}{\partial\xi_\phi}=0\,,\qquad \mathrm{S}_{ik}^{F}=\mathrm{S}_{ik}^{F}(\xi_z,\omega)\,,$$

and relationship (8) is reduced to the following expression

$$\mathrm{P}(\omega)=\sum_{m=1}^{\infty}\frac{|S_0|}{2ak_{0m}\rho_0\omega}\int_{-\infty}^{\infty}\frac{\partial^2\mathrm{S}_{zz}^{F}(\xi_z,\omega)}{\partial\xi_z^2}\times$$
$$\times e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{0m}\xi_z}\mathrm{d}\xi_z\,, \quad (9)$$

where $|S_0|$ is the area of the surface $S_0$.

One can see that only the axial dipoles $\partial F_z/\partial z_0$ contribute to the acoustic field in the pipe when the region $V_0$ is occupied primarily by large eddies. The main part of that contribution is made by the first acoustic mode of the pipe, $\Psi_{01}^{(1)}=1$.

It is corresponded to by a plane acoustic wave propagating in the axial direction at the speed $c_0$.

The total acoustic power,

$$\Pi=\int_{-\infty}^{\infty}\mathrm{P}(\omega)\mathrm{d}\omega\,,$$

generated by the axial dipoles $\partial F_z/\partial z_0$ has the following form (here only the contribution of the mode $\Psi_{01}^{(1)}$ has been allowed for)

$$\Pi\approx\frac{|S_0|}{2a\rho_0c_0}\int_{-\infty}^{\infty}\mathrm{K}_{zz}^{F}(\xi_z,\tau)\mathrm{d}\xi_z\,, \quad (10)$$

where $\mathrm{K}_{zz}^{F}(\xi_z,\tau)$ is the correlation of the forces $F_z$, viz.

$$\mathrm{K}_{zz}^{F}(\xi_z,\tau)=\int_{-\infty}^{\infty}\mathrm{S}_{zz}^{F}(\xi_z,\omega)\mathrm{e}^{-\mathrm{i}\omega\tau}\mathrm{d}\omega\,,$$

and $\tau=\xi_z/c_0$ the time needed for the acoustic wave to pass the axial distance $\xi_z$ between the dipole sources.

Introducing in the domain $V_0$ the length scale, viz.

$$L_t=\alpha a \tag{11}$$

and the frequency scale, viz.

$$f_t=\beta\frac{U}{a} \tag{12}$$

(where $\alpha$ and $\beta$ are the respective scale coefficients), as well as the ratio of the characteristic turbulent flow velocity, $u_t$, to the undisturbed basic flow velocity, $U$, viz.

$$\gamma_t=\frac{u_t}{U} \tag{13}$$

allows one to obtain an estimate for the energy (10), viz

$$\Pi\sim\frac{|S_0|}{2}\rho_0U^3M^3\alpha\gamma_t^4,\quad \alpha\sim 1. \tag{14}$$

One has the classical cubic dependence of the intensity of acoustic radiation of dipoles on the Mach number [8, 10].

**2.2. Small eddies**

Now let the turbulence region $V_0$ be occupied primarilly by vortex structures being small compared to the pipe radius, $a$. In such a situation, the correlation lengths in the radial, $\lambda_r$, azimuthal, $\lambda_\phi$, and axial, $\lambda_z$, directions, as well as the length scale $L_t$ in the region $V_0$ will be small compared to $a$, viz.

$$\lambda_r\sim\lambda_\phi\sim\lambda_z\sim L_t=\alpha a<<a,\qquad \alpha<<1.$$

Then the integration ranges over $\xi_\phi$ and $\xi_z$ in formula (8) can be extended from $-\infty$ to $\infty$, viz.

$$\mathrm{P}(\omega)=\sum_{q=1}^{2}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{a^2\mathrm{J}_n^2(\alpha_{nm}a)}{4\left\|\Psi_{nm}^{(q)}\right\|^2k_{nm}\rho_0\omega}\int_0^{2\pi}\int_{z_{0i}}^{z_{0e}}\mathrm{b}_q(n\phi_0)\mathrm{d}\phi_0\mathrm{d}z_0\times$$
$$\times\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial^2\mathrm{S}_{ik}^F(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}\mathrm{b}_q(n(\phi_0+\xi_\phi))\times$$
$$\times e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}\xi_z}\mathrm{d}\xi_\phi\mathrm{d}\xi_z. \tag{15}$$

Let us consider the cases of low and high frequencies in the obtained relationship.

***2.2.1. Low frequencies***

The low frequencies are assumed to be those satisfying the condition

$$0<\omega<\omega_{nm},\quad (n,m)\neq(0,1).$$

Under this condition the contribution to the noise field in the pipe will only be made by its first acoustic mode, $\Psi_{01}^{(1)}$, and relationship (15) becomes as follows

$$\mathrm{P}(\omega)=\mathrm{P}_{01}^{(1)}(\omega)=\frac{|S_0|}{4\pi ak_0\rho_0\omega}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial^2\mathrm{S}_{ik}^F(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}\times$$
$$\times e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_0\xi_z}\mathrm{d}\xi_\phi\mathrm{d}\xi_z. \tag{16}$$

An analysis of this expression shows that all the dipoles contribute to the low-frequency domain of the spectrum P when the turbulence region $V_0$ is occupied primarily with small eddies. Herewith the generated acoustic power propagates in the axial direction in the form of a plane wave at a speed $c_0$.

Application of the integral-mean-theorem [7], viz.

$$\iint\frac{\partial^2\mathrm{S}_{ik}^F(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_0\xi_z}\mathrm{d}\xi_\phi\mathrm{d}\xi_z=$$
$$=\frac{2\lambda_\phi}{a}\int\frac{\partial^2\mathrm{S}_{ik}^F(\xi_{\phi*},\xi_z,\omega)}{\partial\xi_i\partial\xi_k}e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_0\xi_z}\mathrm{d}\xi_z$$

(where $\xi_{\phi*}$ is the point of the segment $\left[0,\lambda_\phi\right]$, and $\lambda_\phi\propto L_t$) to relationship (16), viz.

$$\mathrm{P}(\omega)\approx\frac{|S_0|\alpha}{2\pi ak_0\rho_0\omega}\int_{-\infty}^{\infty}\left.\frac{\partial^2\mathrm{S}_{ik}^F(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}\right|_{\xi_\phi=\xi_{\phi*}}\times$$
$$\times e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_0\xi_z}\mathrm{d}\xi_z,\quad \alpha<<1$$

allows making its comparative analysis with expression (9). One can see that the acoustic power generated by the dipoles at low frequencies in the case of occupying the region $V_0$ primarily with small eddies is a small value of the order $\alpha/\pi$ ($\alpha<<1$) compared to the acoustic power produced by the dipoles in the same frequency range when the domain $V_0$ is occupied primarily with large eddies.

Accordingly, an expression for the total acoustic power, P, will only differ from expression (10) practically by the additional factor $\alpha/\pi$, and the estimate for P will actually have the form (14) multiplied by $\alpha/\pi$, viz.

$$\Pi\sim\frac{|S_0|}{2\pi}\rho_0U^3M^3\alpha^2\gamma_t^4,\quad \alpha<<1. \tag{17}$$

One has again the classical cubic dependence of the acoustic power generated by dipoles on the Mach number.

***2.2.2. High frequencies***

Let us proceed to the frequencies higher than all the pipe cut-off frequencies, viz.

$$\omega>\omega_{nm},\quad n\geq 0,\ m\geq 1.$$

In this case all the acoustic modes $\Psi_{nm}^{(q)}$ will be propagating and take part in forming the acoustic far-field in the pipe. Consequently, they must be taken into account when making further analysis of relationship (15).

Rewriting the cosine and the sine of the sum of two arguments in that relationship and taking account of the orthogonality properties of the resultant trigonometric functions yields the following expression for the spectrum

$$\mathrm{P}(\omega)=\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{|S_0|}{4\pi a\varepsilon_n k_{nm}\rho_0\omega}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial^2 \mathrm{S}_{ik}^F(\xi_\phi,\xi_z,\omega)}{\partial\xi_i\partial\xi_k}\times$$
$$\times\cos(n\xi_\phi)e^{-\mathrm{sign}(z-z_0)\mathrm{i}k_{nm}\xi_z}\mathrm{d}\xi_\phi\mathrm{d}\xi_z\,, \qquad (18)$$

where

$$\varepsilon_n=\begin{cases}1, n=0\\ \dfrac{1}{2}\left[1-\left(\dfrac{n}{\alpha_{nm}a}\right)^2\right], & n\geq 1\end{cases}.$$

An analysis of formula (18) shows that all the dipoles contribute to the high-frequency domain of the spectrum P when the turbulence region $V_0$ is occupied primarily by small eddies. In this case all the pipe acoustic modes take part in forming the acoustic filed in the pipe.

Substituting relationship (18) into the integral for the total acoustic power P, viz.

$$\Pi=\int_{-\infty}^{\infty}\mathrm{P}(\omega)\mathrm{d}\omega\,,$$

and introducing the turbulence scales (11)–(13) into the resultant expression allows one to obtain the estimate for P, viz.

$$\Pi\sim\frac{|S_0|}{2\pi}\rho_0 U^3 M^3\alpha^3\beta\gamma_t^4\,,\quad \alpha<<1\,. \qquad (19)$$

One can see that in the case of small eddies and high frequencies the acoustic power generated by dipoles is also determined by the third power of the Mach number. However, it is a small value of the order $\alpha\beta$ ($\alpha<<1$) compared to the power produced by dipoles in the case of small eddies and low frequencies, which has been described in subsection 2.2.1 (compare estimates (19) and (17)).

**Conclusion**

In this paper, a particular case of the theory of noise generation by a limited region of turbulence in an infinite straight immovable rigid-walled pipe of circular cross-section, which has been developed in reference [6], has been considered.

In that case the situation is studied when the generated noise field is dominated by the contribution made by surface dipoles. Those flows and shapes of the local pipe narrowing have been of interest, that result in large or small eddies distributed uniformly in the turbulence region behind the narrowing.

The corresponding simplified expressions for the generated acoustic power have been obtained in the considered cases, and their estimates have been carried out for the characteristic scales in the turbulence region.

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