DOI: 10.18372/2310-5461.65.19927
УДК 621.396.967

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THE METHOD FOR OPTIMIZING SIGNAL PARAMETERS USING LAGRANGE MULTIPLIERS

Introduction

One of the pressing challenges in the scientific and technical tasks of signal reconstruction in complex radio environments is to ensure high accuracy and model stability. Cognitive telecommunication networks require adaptive solutions to guarantee reliable operation in the face of limited spectral resources and dynamic environments characterized by not only interference but also signal fluctuations and delays that can significantly impact data transmission quality.

In practice, when the computational complexity of a telecommunication system is high due to the numerous parameters of Volterra kernels, optimization becomes necessary to enhance reconstruction accuracy, interference resilience, and to account for the orthogonality conditions of the parameters. One effective approach for addressing this challenge is the optimization of parameters using Lagrange multipliers (or the method of undetermined Lagrange multipliers) [1, 6–8, 13].

This method allows for the simultaneous consideration of orthogonality, stability, and interference resilience, ultimately reducing the mean squared error (MSE) of signal reconstruction. This is particularly important in nonlinear modeling problems, where the number of model parameters grows exponentially with increasing order of the Volterra series. Under such conditions, ensuring algorithm convergence, reducing the impact of nonessential components, and achieving high accuracy of signal reconstruction become crucial factors for the effective operation of cognitive networks [4, 9, 12, 14].

In this study, a method for optimizing filtering parameters using Lagrange multipliers is proposed, along with an algorithm that includes an iterative process for finding stationary points of the Lagrange function and updating parameters using the gradient descent method. Experimental calculations confirm the effectiveness of the proposed approach: the use of Lagrange multipliers significantly reduces the mean squared error (MSE) of signal reconstruction, while ensuring stable model operation and reducing the influence of nonessential components. This improves the overall efficiency of telecommunication systems in dynamic and interference-prone environments, which is characteristic of cognitive networks.

Analysis of recent research and publications

Recent studies have made a significant contribution to the development of signal optimization, spectrum balancing, and compensation for nonlinear distortions in telecommunication systems. However, several unresolved issues remain, including the adaptability of methods to rapidly changing conditions and their suitability for real-time signal processsing, particularly in the context of cognitive radio.

Works [1, 2] focus on optimization methods for spectrum balancing and analytical signal modeling. Specifically, [1] proposed the use of Lagrange multipliers for optimal spectrum balancing in DSL systems. However, the method is limited in its applicability to cognitive radio due to insufficient adaptability to dynamic conditions. Study [2] concentrates on the fundamental properties of Volterra equations, laying the theoretical groundwork but failing to address practical aspects of real-time signal processing.

Study [3] analyzes methods for solving second-kind Volterra integral equations. However, the proposed approaches are tailored to theoretical problems and do not consider the specifics of telecommunication systems.

In [4, 5], approaches to compensating for nonlinear distortions and creating schemes that preserve structural properties of systems are explored. While these works show significant potential, they are focused on highly specialized tasks and are not fully adapted for use in cognitive radio.

Studies [6, 7] emphasize the mathematical aspects of the Lagrange multiplier method. Despite the high level of algorithm development, these works do not provide solutions that could be directly integrated into spectral analysis systems.

Papers [8, 9, 11] address theoretical and geometric methods for applying Lagrange multipliers in various problems, including mechanics and regression models. These studies are essential for a broader understanding of the method, but their contribution to practical tasks in cognitive radio remains indirect.

Results in [10, 12, 15] focus on time-frequency analysis and the creation of pseudo-random sequences for interference reduction. These works are promising for improving spectral efficiency but require further research on integration into cognitive radio.

In study [13], the application of the Nelder-Mead method for optimizing the parameters of synthesized signals was proposed, which effectively reduced errors during signal recovery. However, the method has limitations in terms of adaptability to variable spectral conditions and accounting for the impact of interference, which diminishes its efficiency in cognitive networks. Study [14] examined iterative decoding of LDPC codes using differential evolution, achieving a reduction in mean squared error and an increase in decoding speed for short codes. However, this work did not focus on optimization related to spectral reconstruction or adaptation to cognitive radio environment conditions, leaving the question of the method's effectiveness in complex radio-frequency scenarios unresolved.

Thus, despite significant progress in studying optimization methods and signal processing, the problem of adapting to the dynamic conditions of cognitive radio and ensuring real-time processing remains insufficiently addressed.

Problem Statement

One of the key challenges in ensuring the effective operation of cognitive telecommunication networks is adapting models to dynamic spectral conditions, high levels of interference, and nonlinear signal distortions. This is particularly relevant for systems with a large number of parameters, such as those based on Volterra kernels, where computational complexity grows exponentially with the order of the model [2, 3, 6].

Despite advancements in optimization methods, such as the Lagrange multiplier method, limitations remain in their adaptability to real-world cognitive radio conditions. Specifically, the need for rapid algorithm convergence, reducing the impact of nonessential model components, and ensuring interference resilience in rapidly changing environments is insufficiently addressed [1, 4, 7].

Another critical issue is balancing signal reconstruction accuracy and model stability, especially as signal fluctuations and delays in cognitive networks increase. Research indicates that reducing the mean squared error (MSE) while maintaining parameter orthogonality are essential factors for improving system performance under these conditions [5, 8, 12].

Thus, there is a need to develop approaches that optimize model parameters while ensuring adaptability to dynamic conditions, resilience to interference, and minimizing the influence of nonessential components. This will enhance the efficiency of telecommunication systems in cognitive networks [9, 10, 11].

The purpose of the article

The purpose of this article is to develop a method for optimizing the parameters of signal models that ensures high reconstruction accuracy and resilience to interference in cognitive telecommunication networks. The proposed approach accounts for adaptability to dynamic environmental conditions, minimizes the mean squared error (MSE), and reduces the influence of nonessential model components.

Summary of the main material

To achieve this goal, the mathematical optimization problem is formulated as [1, 2]:

$$\min_H L(H,\lambda) = \min_H\left(J(H) + \sum_{k=1}^{m} \lambda_k g_k(H)\right) \quad (1)$$

where $J(H)$ – is a loss function defined based on the mean squared error (MSE) [1,8]:

$$J(H) = \frac{1}{N}\left(\sum_{i=1}^{N} (y_i - \hat{y}_i)^2\right) \quad (2)$$

where N – is the set of Volterra kernel parameters; $y_i, \hat{y}_i$ – are the input and reconstructed signals, respectively; $L\,(H,\lambda)$ – is the Lagrangian function, where λ_k are Lagrange multipliers that account for parameter constraints.

The constraint functions $g_k(H) = 0$ include orthogonnality and interference resilience conditions [4, 10].

To ensure the stable operation of the algorithm, a normalization condition $\|H\|^2 = 1$ is also calculated. This condition prevents excessive growth of the Volterra kernel parameters, which can lead to instability during optimization.

The optimization procedure consists of two main stages:

1 Stage. Finding stationary points of the Lagrangian function by solving a system of equations where the gradient of the function equals zero, which is a

necessary condition for locating the minimum of the function [3, 4,10]:

$$\begin{cases} \dfrac{\partial L(H,\lambda)}{\partial H} = 0 \\ \dfrac{\partial L(H,\lambda)}{\partial \lambda_k} = 0 \\ k = 1, 2, \dots, m \end{cases}, \qquad (3)$$

where $\frac{\partial L(H,\lambda)}{\partial H}$ – is the partial derivative of the Lagrangian function L with respect to the parameter H, used to calculate stationary points of a multi-variable function (ordinary derivatives are used for single-variable functions).

2 Stage. Determining the values of the Volterra kernel parameters H and the Lagrange multipliers λ that minimize the loss function while satisfying all specified constraints.

To update the Volterra kernel parameters and Lagrange multipliers in this study, the gradient descent method is used [13, 14]:

$$H^{(k+1)} = H^{(k)} - \eta \frac{\partial L(H,\lambda)}{\partial H}, \qquad (4)$$

$$\lambda^{(k+1)} = \lambda^{(k)} - \eta \frac{\partial L(H,\lambda)}{\partial \lambda}, \qquad (5)$$

where η – is the optimization step size that determines the rate of parameter updates.

The block diagram of the algorithm for optimizing filtering parameters using Lagrange multipliers to ensure orthogonality and stability of signal reconstruction is presented in Fig. 1.

The main steps of the algorithm include the following:

Stage I. Parameter initialization.

At this stage, the initial set of Volterra kernel parameters H_0 is selected, and initial values for the Lagrange multipliers λ_0 and the convergence criteria $\epsilon 0$ are set. Initialization enables the optimization process to start with initial approximations. Setting the initial values of the Volterra kernel parameters H_0 allows the algorithm to iteratively refine these parameters to minimize the signal reconstruction error. The initial values of the Lagrange multipliers λ_0 are necessary to account for orthogonality and interference resilience conditions from the very first iteration. The convergence criteria ϵ_0 define the accuracy threshold of the algorithm, upon reaching which the iterations are terminated as convergence (the optimal solution) is achieved.

Stage II. Computation of the Lagrangian function (formulas 1–2).

At this stage, the Lagrangian function is evaluated to determine its current value. The function integrates two critical aspects of the optimization process. First, it considers the accuracy of signal reconstruction by minimizing the mean squared error (MSE), ensuring that the reconstructed signal closely approximates the original input signal. Second, it incorporates additional constraints, such as maintaining the orthogonality of the Volterra kernel parameters, which helps prevent redundancy and ensures better separation of frequency components. Additionally, the function enforces resilience to interference, ensuring that the reconstructed signal remains stable and accurate even in the presence of external noise.

Stage III. Computation of partial derivatives (formula 3).

The goal of the third stage is to identify stationary points where the gradient of the function equals zero, which is a necessary condition for finding the minimum of the function. While gradient-based methods are often employed for parameter optimization tasks, alternative methods may also be used, depending on the complexity and nature of the problem.

1. Newton's method. This approach involves computing the Hessian matrix (a matrix of second derivatives) to obtain more precise information about the curvature of the function, thereby facilitating the search for its minimum. Newton's method is particularly effective for achieving convergence in problems where the computation of second derivatives is feasible and beneficial [1, 7, 9, 13].

2. Levenberg–Marquardt method. This hybrid approach combines gradient descent with second-order approximations, similar to Newton's method. It is particularly well-suited for nonlinear problems, balancing convergence speed with robustness to local minima [5, 11, 12, 13].

3. Optimization methods without derivative computation. Techniques such as the Nelder–Mead method or differential evolution are applied in cases where calculating derivatives is difficult or impossible. These methods optimize the function through iterative approximations and do not rely on gradient information [6, 13].

The block diagram of the algorithm is presented in Fig. 1

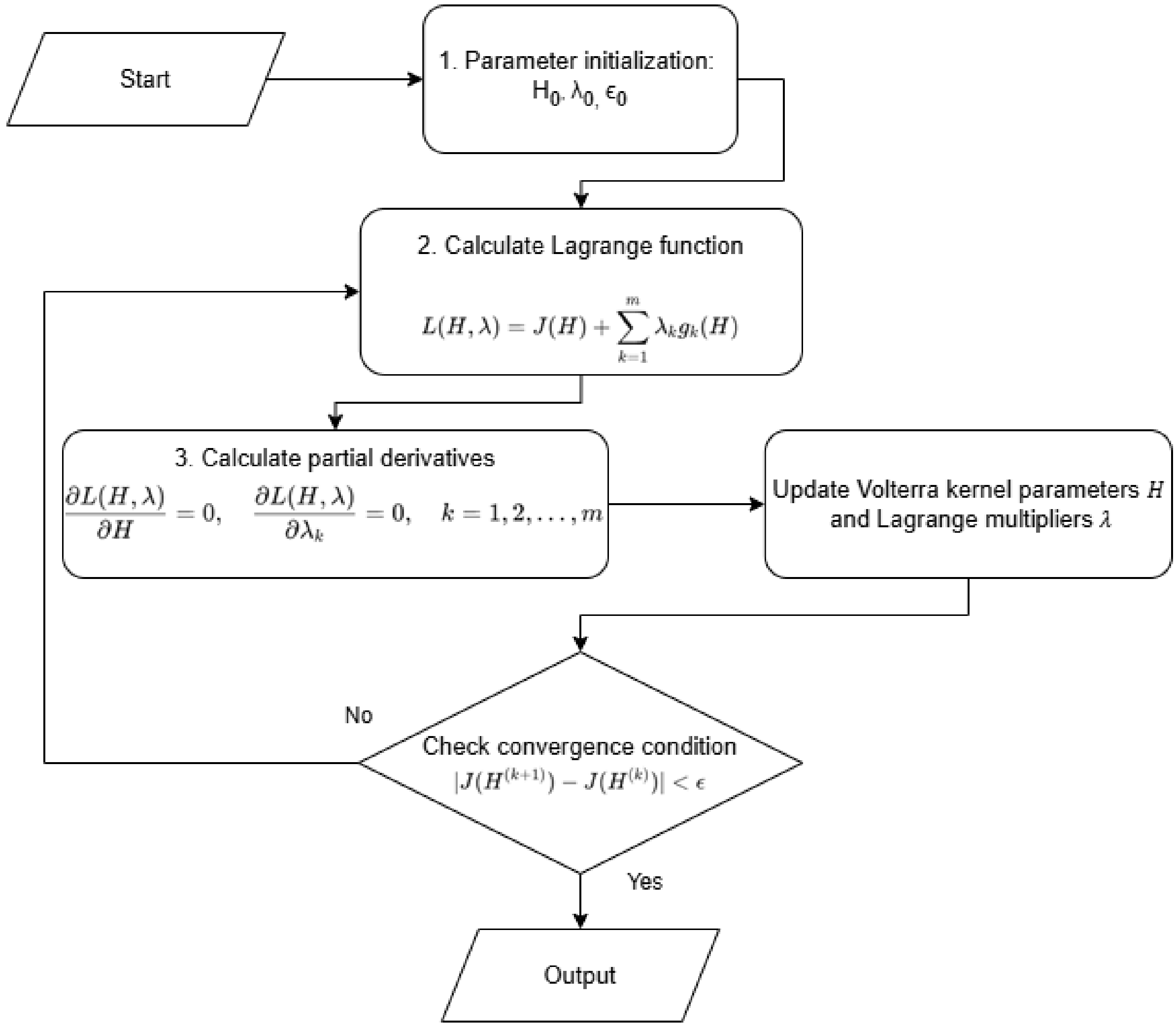

Fig. 1. Block diagram of the optimization algorithm using Lagrange multipliers

Stage IV. Updating the Volterra kernel parameters H and the Lagrange multipliers λ. At this stage, the parameters are updated based on the previously computed partial derivatives. The updates are performed using an optimization method (e.g., gradient descent) to minimize the loss function $J(H)$. This iterative process allows the error in signal reconstruction to be reduced progressively with each iteration while ensuring the constraints on the kernel parameters are satisfied [1, 7, 13].

Stage V. Convergence condition check.

The convergence condition is verified using the formula [6, 11]:

$$\left|J\left(H^{k+1}\right) - J\left(H^{k}\right)\right| < \epsilon \quad (6)$$

where $J\left(H^{k}\right)$ – is the loss function value at the k-th iteration, ϵ – is the convergence criterion (accuracy threshold).

This check evaluates the reduction in the loss function value between consecutive iterations relative to the set threshold. If the convergence condition is satisfied, the iterations terminate as the algorithm has reached its optimal solution. If the condition is not met, the process returns to Stage II to recompute the Lagrangian function.

The optimization of parameters using Lagrange multipliers enables minimizing the reconstruction error by incorporating the orthogonality of the Volterra kernel parameters. This reduces mutual interference among model components, thus impro-ving model stability and resilience to interference [4, 10].

To verify the effectiveness of the proposed optimization algorithm using Lagrange multipliers, calculations were performed, and dependency analyses were conducted. The purpose of the experiment is to justify the efficiency of reducing the mean squared error (MSE) during the optimization process for different signal-to-noise ratio (SNR) values using various optimization methods (Fig. 1). Additionally, the calculations accounted for the fulfillment of conditions ensuring stability and orthogonality of the Volterra kernel parameters under conditions of radio interference.

The experiment was conducted for 4G LTE and 5G NR technologies. For each technology, the efficiency of error reduction was evaluated at various

SNR values (ranging from -4 dB to 10 dB), demonstrating the adaptability and robustness of the proposed optimization algorithm (tabl. 1, 2; figs. 3, 4). Each graph illustrates the dependence of the loss function $J(H)$, defined based on the mean squared error (MSE), on SNR values when using the following optimization methods: Lagrange multipliers, Newton's method, Levenberg-Marquardt algorithm, and Nelder-Mead method [13].

Table 1 – Comparison of optimization methods based on MSE performance (4G LTE)

SNR (dB)	(MSE) Method			
	Lagrange Multipliers	Newton's	Levenberg-Marquardt	Nelder-Mead
-10	0,721	0,686	0,753	0,772
-6	0,585	0,552	0,612	0,645
-2	0,462	0,443	0,485	0,512
2	0,365	0,359	0,383	0,413
6	0,274	0,264	0,291	0,321
10	0,211	0,203	0,226	0,256
14	0,161	0,154	0,172	0,203

Table 2 – Comparison of optimization methods based on MSE performance MSE (5G NR)

SNR (dB)	(MSE) Method			
	Lagrange Multipliers	Newton's	Levenberg-Marquardt	Nelder-Mead
-10	0,975	0,988	1,005	1,116
-6	0,783	0,795	0,812	0,935
-2	0,512	0,538	0,563	0,684
2	0,334	0,354	0,367	0,482
6	0,224	0,238	0,247	0,353
10	0,178	0,189	0,195	0,286
14	0,161	0,154	0,172	0,203

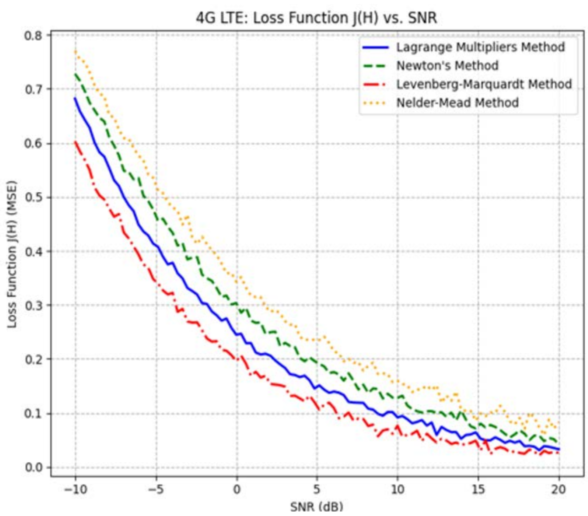

Fig. 2. Comparison of Optimization Methods for 4G LTE Technologies

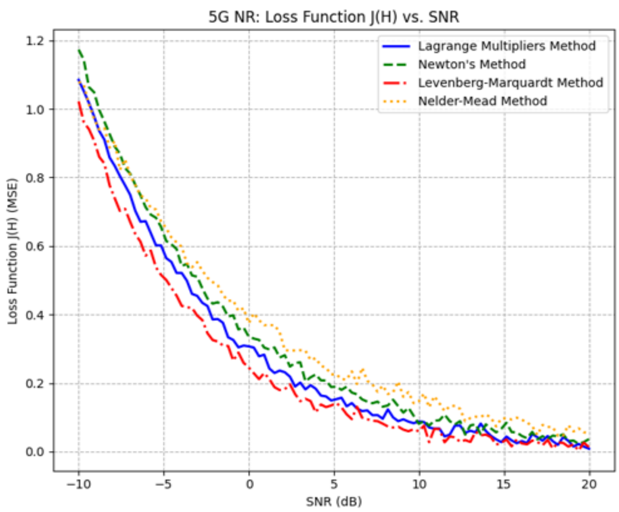

Fig. 3. Comparison of Optimization Methods for 5G NR Technologies

The analysis of Fig. 2 and Table 1 demonstrates that the Lagrange multipliers method delivers the best results for high SNR values (above 5 dB). The MSE for the Lagrange method decreases more consistently and achieves the lowest values compared to other methods. At SNR = 20 dB, the mean squared error is approximately 0,05, which is 15–20 % lower than that of Newton's method and 30 % lower than that of the Nelder-Mead method. This indicates that for 4G LTE, the Lagrange multipliers method is an effective tool for ensuring high signal recovery accuracy, particularly in conditions with a high signal-to-noise ratio.

Newton's method, however, outperforms the Lagrange method in the SNR range of -5 to 5 dB. At SNR = 0 dB, Newton's method achieves a mean squared error of approximately 0,25, which is 10% lower than that of the Lagrange method. This suggests that Newton's method is more efficient in scenarios with low signal-to-noise ratios for 4G LTE technology.

The Levenberg-Marquardt method demonstrates similar results to the Lagrange method but shows slightly lower stability at high SNR values. On average, the efficiency of this method is 5–10 % worse than the Lagrange method when SNR exceeds 10 dB.

The Nelder-Mead method shows the poorest performance among the methods presented in the experiment. It exhibits high mean squared error, particularly at high SNR values. At SNR = 20 dB, the error for the Nelder-Mead method is approximately 0,1, which is about 40–50 % higher than that of the Lagrange method. This proves that the Nelder-Mead method is inefficient under 4G LTE conditions.

Thus, for 4G LTE, the Lagrange multipliers method is the most effective in ensuring signal stability and reducing mean squared error at high SNR values. The Levenberg-Marquardt method can be used as an alternative when rapid convergence is needed. However, due to its high error rates, the Nelder-Mead method is suitable only for limited cases where signal quality requirements are less stringent.

Experimental calculations for 5G NR (Figs. 3 and Table 2) confirm that the Lagrange multipliers method achieves the best performance for high SNR values exceeding 10 dB. Specifically, the mean squared error (MSE) for the Lagrange method is 5,3–10,5 % lower than that of Newton's method and 10,3–15,7 % lower than that of the Levenberg-Marquardt method. This demonstrates the high robustness of the Lagrange method in noisy conditions, which is critical for data transmission in 5G NR networks.

The Levenberg-Marquardt method produces results close to those of the Lagrange method but exhibits lower stability at high SNR values, particularly above 15 dB. On average, its efficiency is 5,5–10,5 % worse than that of the Lagrange method under high SNR conditions.

Newton's method also performs reasonably well but shows higher error rates at high SNR values (above 10 dB), with MSE being 5,4–15,7 % greater than that of the Lagrange method.

The Nelder-Mead method yields the worst results, with significantly higher error rates, especially at high SNR values. At SNR = 20 dB, the mean squared error is approximately 0,25, which is 40–50 % higher than that of the Lagrange method. This indicates that the Nelder-Mead method is insufficiently effective for 5G NR networks under conditions of high noise levels.

Conclusions

The conducted calculations and comparative analysis confirmed that optimization using the Lagrange multiplier method is effective for both mobile communication standards, 4G LTE and 5G NR. Specifically, at high SNR values, this method achieves a mean squared error (MSE) that is 10–15 % lower than that of Newton’s and Levenberg-Marquardt methods and 40–50 % lower than that of the Nelder-Mead method.

For 4G LTE technology, the Lagrange method provides high stability and optimization accuracy even under significant interference conditions. It effectively minimizes the mean squared error, particularly at SNR > 10 dB, and demonstrates consistent error reduction even in adverse conditions. In such scenarios, the error for the Lagrange method is 15–20 % lower than for other methods, making it a reliable tool for ensuring data transmission accuracy in 4G LTE.

For 5G NR, the conditions are significantly more challenging due to higher demands for adaptability and algorithm robustness in complex radio-frequency environments. The Lagrange method exhibits effectiveness at high SNR values; however, achieving stability comparable to 4G LTE requires further adaptation. For instance, at SNR > 10 dB, the error of the Lagrange method is 5,3–15 % lower than that of Newton’s and Levenberg-Marquardt methods. This demonstrates its efficiency, yet the complexity of 5G NR networks necessitates additional approaches to account for the variable characteristics of the radio environment.

Future research should focus on improving the Lagrange method to adapt it to the challenges of 5G NR networks, particularly addressing the higher data transmission speed requirements and the increased variability of radio-frequency characteristics. Studies could also explore integrating the Lagrange method with hybrid approaches that combine the strengths of classical algorithms and modern optimization methods, such as machine learning algorithms. Such integration could ensure even greater efficiency and accuracy in future telecommunication systems.

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Комар О. М., Перець К. Г.

МЕТОД ОПТИМІЗАЦІЇ ПАРАМЕТРІВ СИГНАЛІВ З ВИКОРИСТАННЯМ МНОЖНИКІВ ЛАГРАНЖА

У статті розроблено метод оптимізації параметрів сигналів за допомогою методу множників Лагранжа для забезпечення високої точності реконструкції сигналів та завадостійкості у когнітивних телекомунікаційних мережах. Розглянуто основні проблеми, пов'язані з адаптацією до динамічних спектральних умов, високим рівнем інтерференції та нелінійними спотвореннями сигналів. На основі аналізу сучасних досліджень обґрунтовано необхідність впровадження запропонованого методу, який враховує умови ортогональності параметрів ядра Вольтерра та забезпечує стійкість алгоритму в умовах динамічного радіосередовища.

Запропонований метод оптимізації дозволяє мінімізувати середньоквадратичну похибку (MSE) реконструкції сигналів, зменшити вплив несуттєвих компонентів моделі та підвищити стабільність алгоритму. На відміну від традиційних методів, таких як методи Ньютона, Левенберга-Марквардта та Нелдера-Міда, метод Лагранжа забезпечує ефективне досягнення низьких значень MSE, особливо за високих значень відношення сигнал-шум (SNR).

Доведено, що впровадження запропонованого методу оптимізації суттєво підвищує ефективність телекомунікаційних систем як для стандарту 4G LTE, так і для 5G NR. Для 4G LTE метод забезпечує стабільну реконструкцію сигналу навіть за умов значної інтерференції. Експерименти показали, що середньоквадратична похибка (MSE) знижується на 15–20 % у порівнянні з методами Ньютона та Левенберга-Марквардта і на 40–50 % у порівнянні з методом Нелдера-Міда.

Для 5G NR, де умови є значно складнішими через динамічні спектральні зміни та високий рівень інтерференції, метод показує високу ефективність при високому рівні SNR, середньоквадратична похибка зменшується на 10–15 % у порівнянні з методами Ньютона та Левенберга-Марквардта. При низьких значеннях SNR ефективність зниження похибки зменшується через складні умови радіосередовища, характерні для мереж нового покоління.

Експериментальна оцінка і порівняльний аналіз показали, що метод Лагранжа є найбільш ефективним для досягнення стабільної реконструкції сигналів у когнітивних мережах за високих рівнів SNR. Проте для мереж 5G NR, враховуючи їхні підвищені вимоги до адаптивності, необхідно подальше вдосконалення методу для забезпечення стабільності, аналогічної досягнутій у 4G LTE.

Ключові слова: когнітивне радіосередовище, підвищення завадостійкості, методи оптимізації, відношення сигнал-шум (SNR), реконструкція сигналів, ортогональність, 4G LTE, 5G NR, метод множників Лагранжа, середньоквадратична похибка (MSE).

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THE METHOD FOR OPTIMIZING SIGNAL PARAMETERS USING LAGRANGE MULTIPLIERS

This article presents a method for optimizing signal parameters using the Lagrange multiplier method to ensure high accuracy of signal reconstruction and interference resilience in cognitive telecommunication networks. The study addresses key challenges related to adapting to dynamic spectral conditions, high levels of interference, and nonlinear signal distortions. Based on an analysis of recent research, the necessity of implementing the proposed method is substantiated. This method considers the orthogonality conditions of Volterra kernel parameters and ensures algorithm stability in dynamic radio environments.

The proposed optimization method minimizes the mean squared error (MSE) of signal reconstruction, reduces the influence of nonessential model components, and enhances algorithm stability. Unlike traditional methods, such as Newton's, Levenberg-Marquardt, and Nelder-Mead methods, the Lagrange multiplier method effectively achieves lower MSE values, particularly at high signal-to-noise ratio (SNR) levels.

It has been demonstrated that the implementation of the proposed optimization method significantly improves the efficiency of telecommunication systems for both 4G LTE and 5G NR standards. For 4G LTE, the method ensures stable signal reconstruction even under significant interference. Experiments have shown that the MSE is reduced by 15–20 % compared to Newton's and Levenberg-Marquardt methods and by 40–50% compared to the Nelder-Mead method.

For 5G NR, where conditions are significantly more challenging due to dynamic spectral changes and high interference levels, the method demonstrates high efficiency at high SNR values, reducing MSE by 10–15 % compared to Newton's and Levenberg-Marquardt methods. At lower SNR levels, the efficiency of error reduction decreases due to the complex radio environment characteristic of next-generation networks.

Experimental evaluation and comparative analysis have confirmed that the Lagrange multiplier method is the most effective for achieving stable signal reconstruction in cognitive networks under high SNR levels. However, further refinement of the method is necessary for 5G NR networks to meet their heightened adaptability requirements and achieve stability comparable to that in 4G LTE.

Keywords: cognitive radio environment, interference resilience enhancement, optimization methods, signal-to-noise ratio (SNR), signal reconstruction, orthogonality, 4G LTE, 5G NR, Lagrange multiplier method, mean squared error (MSE).

Стаття надійшла до редакції 09.12.2024 р.
Прийнято до друку 28.02.2025 р.