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THE USE OF TWO FREQUENCY VARIABLES IN THE DISTRIBUTED CIRCUITS SYNTHESIS

Introduction

The design of filters from non-homogenous lines (NL) segments can be approached from several positions [1–15]. One of the principles is the use of NLs connected by a four-pole between the generator and the load [1–7]. In this case, the required characteristic of selectivity is achieved by changing the wave resistance of the line according to a certain law. However, the input resistance of the NL loaded with an active resistance cannot be purely reactive at real frequencies. Therefore, the reflection coefficient cannot be equal to one on any of them. Therefore, it is fundamentally impossible to obtain a significant attenuation at least at one point outside the passband of the filter. Only when selecting a certain class of complex loads, it is possible to achieve significant attenuation outside the bandwidth [8, 9]. Another principle of filter design is based on the use of the simplest non-homogeneous lines that perform the functions of resonators. The required selectivity characteristic of the filter is achieved by selecting the parameters of resonators and communication circuits. This way of building filters is more rational. In addition, in the process of synthesis, you can use ideas and techniques that are characteristic of circles from segments of homogenous lines.

Analysis of recent research and publications

Currently, there are two schools of synthesis of circles from segments of homogeneous lines. The first of them is based on Kohn's ideas [2, 4, 9–15] and operates with resonators and immittance inverters, the second uses Richard's transcendental frequency substitution [1, 3]. The extension of the techniques of these schools to circles from segments of non-homogeneous lines is of fundamental importance since it becomes possible to use uniform methods of designing devices on elements with distributed parameters.

Problem statement

The purpose of the article is to describe the segments of nonhomogeneous lines by the "inductance" and "capacitance" elements in the class of functions of two complex frequency variables.

Main material

When using Kohn's method, the procedure for synthesizing filters from segments of homogeneous or non-homogeneous lines is similar. It is based on the determination of inverter parameters based on the known characteristics of the prototype and resonators [2, 4, and 8]:

$$J_{01} = \sqrt{\left(G_a \bar{b}_1 \Delta w\right) / g_0 g_1 \omega_c}; \;\; j_{n,n+1} = \sqrt{\left(G_b \bar{b}_n \Delta w\right) / g_n g_{n+1} \omega_c};$$
$$J_{j,j+1} = \sqrt{\left(\Delta\omega^2 \bar{b}_j \bar{b}_{j+1} \Delta w\right) / \left((\omega_c)^2 g_j g_{j+1}\right)}; \; j = 1, 2, ..., n-1; \quad (1)$$

$$K_{01} = \sqrt{\left(R_a \bar{x}_1 \Delta\omega\right) / \left((g_0 g_1 \omega_c)\right)}; \;\; K_{n,n+1} = \sqrt{\left(R_b \bar{x}_n \Delta\omega\right) / (g_n g_{n+1} \omega_c)};$$
$$K_{j,j+1} = \sqrt{\left(\Delta\omega^2 \bar{x}_j \bar{x}_{j+1}\right) / ((\omega_c)^2 g_j g_{j+1})}; \quad (2)$$

where G_a, G_b, R_a, R_b – conductivities and filter load resistances; $\Delta\omega$ – is the relative bandwidth of the filter; $J_{j,j+1}$ and $K_{j,j+1}$ are parameters of conductivity and resistance inverters; $\bar{b}_j$ and $\bar{x}_j$ – resonator reactance steepness parameters; g_j and ω_c are parameters of the prototype of low frequencies.

Kohn's method is approximate. In addition, it imposes significant restrictions on the circle structure. One of the main tasks facing the developers of frequency devices is the simplification of structures and the creation of calculation methods that will ensure obtaining filters with precise characteristics during their serial production. In particular, this applies to nodes on strip lines, as their design excludes the possibility of any adjustments. The complexity and laboriousness of calculating and designing filters increases as the requirements for their characteristics increase. This leads to the need to create such synthesis methods that require minimal labor costs and allow us to approach the design of various devices from a single point of view. When solving this problem, a significant role is by the transformation of the frequency variable. As for microwave devices, the analytical apparatus of frequency transfor-mations is currently represented by Richard's substitutions and can be used only for the synthesis of circles consisting of proportional segments of homogenous lines. In this regard, another type of frequency transformation is needed, which allows comparing NL of any class with a concentrated element or a circle.

Consider an arbitrary quadripole characterized by the transfer matrix [A]. The four-pole circuit is connected to other circuits by segments of homogeneous lines with wave resistances W_{01} and W_{02} (Fig. 1). For the certainty of further transfor-mations of the transmission matrix, we will assume that $W_{01} > W_{02}$. Let's find the representation of the matrix [A] based on the fact that the quadripole is invertible:

$$[A]=\begin{bmatrix} A & B \\ C & D \end{bmatrix}=\frac{1}{\sqrt{1-BC/AD}}\times \begin{bmatrix} \sqrt{A/D} & \left(B\sqrt{D/A}\right)/D \\ \left(\left(C\sqrt{A/D}\right)/A\right)/D & \sqrt{D/A} \end{bmatrix}. \quad (3)$$

Since $Y_{11}=D/B,\ Z_{11}=A/C$, the ratio (3) can be written in the following form:

$$[A]=\frac{1}{\sqrt{1-Z_{11}^{-1}}}\begin{bmatrix} \sqrt{A/D} & Y_{11}^{-1}\sqrt{D/A} \\ Z_{11}^{-1}\sqrt{D} & \sqrt{D/A} \end{bmatrix}.$$

If the quadripole is symmetrical, then $A = B$. In this case, the transfer matrix of the quadripole is:

$$[A]_c=\frac{1}{\sqrt{1-Z_{11}^{-1}Y_{11}^{-1}}}\begin{bmatrix} 1 & 1/Y_{11} \\ 1/Z_{11} & 1 \end{bmatrix}. \quad (4)$$

We use the normalized values $Z_{11}^{'}=Z_{11}/W_{01}$, $Y_{11}^{'}=Y_{11}/W_{01}$ and assume that the four-pole is loaded symmetrically: $W_{01}=W_{02}$. Then you can enter the frequency variables:

$$S_1=1/Z_{11}^{'},\ S_2=1/Z_{11}^{'}, \quad (5)$$

and bring formula (4) to the following form:

$$[A]_c=\frac{1}{\sqrt{1-S_1/S_2}}\begin{bmatrix} 1 & W_{01}S_2 \\ S_1/W_{01} & 1 \end{bmatrix}. \quad (6)$$

Fig. 1. Quadripole with communication lines

Ratio (6) is a mapping of the properties of a symmetric quadripole into a complex two-dimensional space.

Let's use another representation of the matrix [A]:

$$[A]=\frac{1}{\sqrt{Z_{11}Y_{11}-1}}\begin{bmatrix} Z_{11}\sqrt{C/B} & \sqrt{B/C} \\ \sqrt{C/B} & Y_{11}\sqrt{B/C} \end{bmatrix}. \quad (7)$$

Then, taking into account dependence (5), we get:

$$[A]=\frac{1}{\sqrt{1/(S_1S_2)-1}}\times \begin{bmatrix} \left(W_{01}\sqrt{C/B}\right)/S_1 & \sqrt{B/C} \\ \sqrt{C/B} & \sqrt{B/C}/\left(W_{01}S_2\right) \end{bmatrix} \quad (8)$$

We number the elements C and B of the matrix [A] in accordance with the existing rules [3, 4]:

$$B'=B/\sqrt{W_{01}W_{02}}\,\frac{-b\pm\sqrt{b^2-4ac}}{2a},$$
$$C'=C\sqrt{W_{01}W_{02}}.$$

In an antimetric quadripole, the condition $C'=B'$ is fulfilled. Therefore, relation (8) is written as follows:

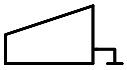
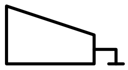
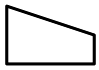
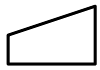
$$[A]_a=\frac{1}{\sqrt{1-S_1S_2}}\times \begin{bmatrix} \sqrt{(W_{01}S_2)/(W_{02}S_1)} & \sqrt{W_{01}W_{02}S_1S_2} \\ \sqrt{(S_1S_2)/(W_{01}W_{02})} & \sqrt{(W_{02}S_1)/(W_{01}S_2)} \end{bmatrix}. \quad (9)$$

Thus, the image of the properties of the antimetric quadripole in a complex two-dimensional space is obtained. The antimetric four-pole is interesting in that the normalized no-load resistance on one pair of clamps is equal to the normalized short-circuit conductivity on the second pair of clamps: $Z_{11}^{'}=Y_{22}^{'},\ Z_{22}^{'}=Y_{11}^{'}$.

Therefore, it can be stated that: $S_1=1/Y_{22}^{'}$, $S_2=1/Z_{22}^{'}$. Regarding the properties of the quadripole, this means the following:

Table 1

Elements of distributed circles with antimetric NL

Element:	designation	S_1 - plane	S_2 - plane
Inductance	$\bar{L}$	 W_{02} $\bar{L} = W_{02}$	 W_{01} $\bar{L} = W_{01}$
Capacitor	$\bar{C}$	 W_{01} $\bar{C} = 1/W_{01}$	 W_{02} $\bar{C} = 1/W_{02}$

– for an antimetric quadripole, the input resistance when the output terminals are open and the output conductivity when the input terminals are short-circuited are described by the same frequency variable S_1;

– for an antimetric quadripole, the input conductivity when the output terminals are short-circuited and the output resistance when the input terminals are open are described by the same frequency variable S_2;

In the future, based on the strip options for implementing circles, it is convenient to introduce the concepts of convergent and divergent lines (*CL* and *DL*, respectively). The input resistance of the short-circuited *CL* is based on the condition:

$$Z_{sc} = 1/Y_{11} = W_{01}S_2 \qquad (10)$$

and the input conductivity of the open DL is determined by the formula:

$$Z_0 = Z_{22} = W_{02}/S_2. \qquad (11)$$

Comparison of dependencies (10) and (11) shows that the short-circuited *CL* is equivalent to the inductance $\bar{L} = W_{01}$, and the open DL is equivalent to the capacity $\bar{C} = 1/W_{02}$ in the scale of the variable S_2.

For the reverse situation, we get the following result:

$$Z_{кз} = 1/Y_{22} = W_{02}S_1,\ Z_{x.x} = Z_{11} = W_{01}/S_1.$$

Therefore, the short-circuited DL is equivalent to the inductance $\bar{L} = W_{02}$ and the open *CL* is equivalent to the capacity $\bar{C} = 1/W_{01}$ in the scale of the frequency variable S_1 (Table 1).

A characteristic feature of circles with distributed parameters is that direct connection of $\bar{L}$ and $\bar{C}$ is impossible. Quadripoles (single elements) are necessary, which ensure their spatial separation. As follows from formula (9), a single element is described by two frequency variables. Therefore, a real circle consisting of antimetric distributed elements is described by a function of two frequency variables.

Let the transfer matrix of some antimetric quadripole have the following form:

$$[A]_1 = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix}. \qquad (12)$$

We number the elements of the matrix (12) in accordance with the current rules:

$$\begin{aligned} A_1' &= A_1\sqrt{W_{02}/W_{01}}, \\ B_1' &= B_1/\sqrt{W_{01}W_{02}}, \\ C_1' &= C_1\sqrt{W_{01}W_{02}}, \\ D_1' &= D_1\sqrt{W_{01}/W_{02}}. \end{aligned} \qquad (13)$$

Let's assemble a pair of quadripoles taking into account condition (13).

$$[A']_{c1} = \begin{bmatrix} A_1' & B_1' \\ B_1' & D_1' \end{bmatrix}\begin{bmatrix} D_1' & B_1' \\ B_1' & A_1' \end{bmatrix} = \begin{bmatrix} A_1'D_1' + (B_1')^2 & 2A_1'B_1' \\ 2B_1'D_1' & A_1'D_1' + (B_1')^2 \end{bmatrix} = \frac{1}{\sqrt{1 - S_1S_2}}\begin{bmatrix} 1 & S_2 \\ S_1 & 1 \end{bmatrix}, \qquad (14)$$

$$[A']_{c2} = \begin{bmatrix} D_1' & B_1' \\ B_1' & A_1' \end{bmatrix}\begin{bmatrix} A_1' & B_1' \\ B_1' & D_1' \end{bmatrix} = \begin{bmatrix} A_1'D_1' + (B_1')^2 & 2B_1'D_1' \\ 2A_1'B_1' & A_1'D_1' + (B_1')^2 \end{bmatrix} = \frac{1}{\sqrt{1 - S_1S_2}}\begin{bmatrix} 1 & S_1 \\ S_2 & 1 \end{bmatrix} \qquad (15)$$

Where $S_1 = (2B_1'D_1')/(A_1'D_1' + (B_1')^2)$, $S_2 = (2A_1'B_1')/(A_1'D_1' + (B_1')^2)$.

Then, on the basis of formulas (14) and (15), we obtain the frequency variables for the formed symmetrical quadripoles:

$$S_1 = 1/\left(Y_{11}^{'}\right)_2 = 1/\left(Z_{11}^{'}\right)_1,$$
$$S_2 = 1/\left(Z_{11}^{'}\right)_2 = 1/\left(Y_{11}^{'}\right)_1. \quad (16)$$

Renormalizing the elements of the transfer matrix and taking into account the dependence (16), we find:

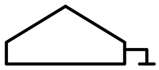
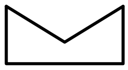
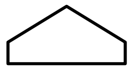
$$(Z_{\text{кз}})_1 = W_{01}S_2,\ (Z_{x.x})_2 = W_{02}/S_2;$$
$$(Z_{x.x})_1 = W_{01}S_1,\ (Z_{\text{кз}})_2 = W_{02}/S_1.$$

Thus, the considered pairs of lines implement in the frequency scale S_2 the inductance $\overline{L} = 1/W_{02}$ and the capacity $\overline{C} = 1/W_{02}$, and in the frequency scale S_1 – the inductance $\overline{L} = W_{02}$ and the capacity $\overline{C} = 1/W_{01}$.

Variants of using symmetrical segments of non-homogeneous lines are given in the table. 2. Since no restrictions were imposed when constructing symmetric quadripoles from antimetric ones, the procedure is valid for any class of non-homogeneous lines.

Table 2

Elements of distributed circles with symmetrical NLs

Element:	designation	S_1-plane	S_2-plane
Inductance	$\overline{L}$	 W_{02} $\overline{L} = W_{02}$	W_{01} $\overline{L} = W_{01}$
Capacitor	$\overline{C}$	 W_{01} $\overline{C} = 1/W_{01}$	 W_{02} $\overline{C} = 1/W_{02}$

Conclusions

The obtained relations allow synthesizing frequency domain circles of functions of two variables S_1, S_2. At the same time, a direct connection of $\overline{L}$ and $\overline{C}$ elements is impossible. For their spatial distribution, it is necessary to use a single element with a transfer matrix (6). According to the accepted classification [3, 16], the lines are given in the table 2 are convergent-divergent (CDL) and divergent-convergent (DCL). To switch to DCL, it is sufficient to replace formulas S_1 with S_2 and W_{01} with W_{02}.

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THE USE OF TWO FREQUENCY VARIABLES IN THE DISTRIBUTED CIRCUITS SYNTHESIS

One of the approaches to the design of filters from segments of non-uniform lines is the use of NLs connected by a quadrupole between the generator and the load. In this case, the required characteristic of selectivity is achieved by changing the wave resistance of the line according to a certain law. However, the input op of the NL loaded with an active resistance cannot be purely reactive at real frequencies, so the reflection coefficient cannot be equal to unity at any of them. Therefore, it is fundamentally impossible to obtain a significant attenuation at least at one point outside the passband of the filter. Only by choosing a certain class of complex loads can significant attenuation be achieved outside the passband. Another principle of filter construction is based on the use of the simplest heterogeneous lines that perform the functions of resonators. The necessary selective characteristic of the filter is achieved by selecting the parameters of resonators and communication circuits. This method of building filters is more rational. In addition, in the process of synthesis, you can use ideas and techniques characteristic of circles from segments of uniform lines.

It is shown that in the synthesis of distributed circuits based on transmission lines, complex functions of two frequency variables should be used, which allows the construction arbitrary circuit functions and fully cover the classes of physically realized transfer functions. A description of the elements "inductance" and "capacitance" in the class of functions of two complex frequency variables was obtained, which allows synthesizing circuits using the ideas and methods of Richards's circuit theory.

Keywords: quadripole, transmission matrix, wave resistance, conductivity, convergent-divergent line (CDL), divergent-convergent line (DCL)

Баланюк Ю. В.

ВИКОРИСТАННЯ ДВОХ ЧАСТОТНИХ ЗМІННИХ ПРИ СИНТЕЗІ РОЗПОДІЛЕНИХ КАНАЛІВ

Одним із підходів до проектування фільтрів із сегментів неоднорідних ліній є використання НЛ, з'єднаних чотириполюсником між генератором і навантаженням. У цьому випадку необхідна характеристика вибірковості досягається зміною хвильового опору лінії за певним законом. Однак вхідний опі НЛ, навантаженого активним опором, не може бути чисто реактивним на реальних частотах, отже на жодному з них коефіцієнт відбиття не може дорівнювати одиниці. Тому принципово неможливо отримати значне загасання хоча б в одній точці за межами смуги пропускання фільтра. Тільки при виборі певного класу комплексних навантажень можна досягти значного загасання за межами смуги пропускання. Інший принцип побудови фільтрів заснований на використанні найпростіших неоднорідних ліній, які виконують функції резонаторів. Необхідна селективна характеристика фільтра досягається підбором параметрів резонаторів і ланцюгів зв'язку. Такий спосіб побудови фільтрів більш раціональний. Крім того, в процесі синтезу можна використовувати ідеї та прийоми, характерні для кіл із відрізків однорідних ліній.

У статті показано, що при синтезі розподілених схем на основі ліній передачі необхідно використовувати комплексні функції двох частотних змінних, що дозволяє побудувати довільні схемні функції та повністю охопити класи фізично реалізованих передавальних функцій. Отримано опис елементів «індуктивність» і «ємність» в класі функцій двох комплексних частотних змінних, що дозволяє синтезувати схеми з використанням ідей і методів теорії схем Річардса.

Ключові слова: чотириполюсник, матриця пропускання, хвильовий опір, провідність, збіжно-розбіжна лінія (CDL), збіжно-збіжна лінія (DCL).

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