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INFLUENCE OF THE AT “WINGLETS” TYPE ON THE LONGITUDINAL STATIC STABILITY OF ANGLE OF ATTACK OF THE AIRCRAFT

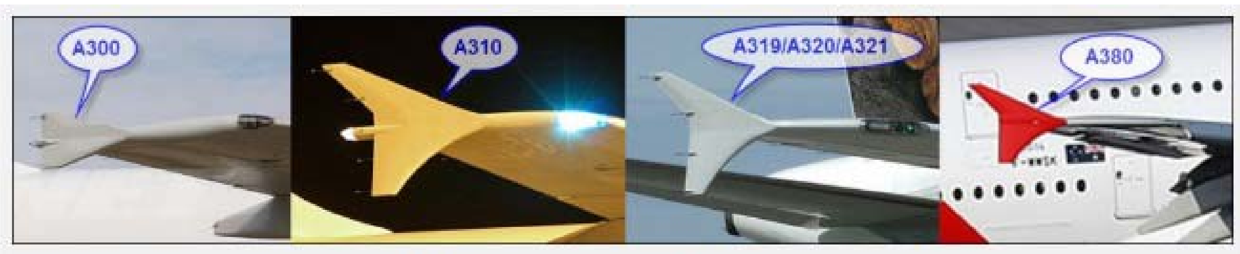
Introduction

Lifting force of the aircraft wing is formed due to the difference in pressure under the wing and above the wing. From the high-pressure region from below, air flows over the edge of the wing into the region of reduced pressure from above. These currents, forming with the main — in front flow, form powerful vortex bundles of air, pulling the aircraft back from the ends of the wing. End vortices lead to a redistribution of the lifting force along the span of the wing, reducing its effective area and elongation, form an inductive resistance and reduce the aerodynamic quality [6–10]. To reduce these negative factors on the ends of the wing of many modern aircraft, aerodynamic wingtips are installed (end washers or Witecomb washers, AT Winglets, sharklets) — small additional elements at the ends of the aircraft wing planes in the form of wings or flat washers. It is proved that these designs increase the effective span of the wing, reduce the inductive resistance, and, as a result, increase the lifting force at the end of the wing, increase the aerodynamic quality and lengthening of the wing, almost without changing its swing [1–5]. They also increase the fuel efficiency of aircraft or the range of flight in gliders. Currently, the same types of aircraft can have different versions of the wingtips. For example, sharklets — lift up the end of the wing. The ends of the keel and stabilizer have a similar design.

Figure shows four different types of Witecomb winglets available to AIRBUS.

In this article, we determine the aerodynamic forces acting on the aircraft in a steady horizontal flight and the longitudinal moment of these forces relative to the transverse axis passing through the center of gravity for a wing with aerodynamic tips such as AT Winglets.

The coefficients of these forces and moments are recorded. Using the expressions for the coefficient of the longitudinal moment for an aircraft with a wing without wingtips, the longitudinal moment coefficient for the aircraft with a wing with aerodynamic ends was recorded, and its derivatives were calculated.



Four different types of winglets

Formulation of the problem

During the flight, various external perturbations of the turbulence type, updraughts and downdraughts, gusty wind, etc. can act on the aircraft.

At the same time, overload, angle of attack, flight speed and other parameters can be changed.

Aircraft is required possess the property of longitudinal static stability for overload, i.e. restore the violated preset flight mode without pilot intervention, provided that the flight speed is practically unchanged.

To determine the characteristics of the stability of the aircraft, it is necessary to know the mathematical model of its movement, which occurs under the action of forces and moments that lead to its rotation and translational motion. The mathematical model of the aircraft, which makes it possible to investigate the stability of its disturbed motion, describes the process of aircraft reaction to the deviation of control organs or the action of external perturbations. This model is nonlinear, which makes it difficult to obtain important results for practice. In a number of problems, in order to simplify the calculations, it is possible to regard the motion as steady and to regard the perturbation as small. This approach will be used in this work. As the studies below show, even in steady traffic, taking into account the influence of wing tips on aircraft stability issues is extremely difficult, which is the main reason for the lack of publications in this field.

Longitudinal wing with aerodynamic tips

We place the origin of coordinates in the middle of the wing, we direct the axis *Oz* along the span to the right, the axis *Oy* upwards and the axis *Ox* along the unperturbed flow. We determine the longitudinal moments (the pitch moment) relative to the axis of all the forces acting on the aircraft. As is known, in a horizontal flight the following forces act on the airplane:

– the force of weight G — is applied to the center of gravity and is always directed vertically down to the center of the earth;

– the lift of the aircraft Y — is applied to the center of pressure and is directed perpendicular to the direction of the undisturbed flow;

– the drag force of the aircraft Q — is located on the longitudinal axis and is directed in the direction opposite to the movement of the aircraft;

– tractive force P — in general, directed towards the aircraft, along the axis of the engine;

– full aerodynamic forces created by the upper tips.

The force created by the left upper end is denoted $\vec{R}^{\text{в}}_{\text{л}}$, and the force created by the right upper end like $\vec{R}^{\text{в}}_{n}$. The projections of these forces on the axis of the selected coupled coordinate system are expressed by the formulas [4; 5]

$$\begin{aligned}\vec{R}^{\text{в}}_{\text{л}} &= \left\{R^{\text{в}}_{\text{л}}\cos\varphi\sin\alpha_z,\, R^{\text{в}}_{\text{л}}\sin\varphi, -R^{\text{в}}_{\text{л}}\cos\varphi\cos\alpha_z\right\};\\ \vec{R}^{\text{в}}_{n} &= \left\{R^{\text{в}}_{n}\cos\varphi\sin\alpha_z,\, R^{\text{в}}_{n}\sin\varphi, -R^{\text{в}}_{n}\cos\varphi\cos\alpha_z\right\},\end{aligned} \tag{1}$$

the total aerodynamic forces created by the lower tips, denoted, respectively, the left $\vec{R}^{\text{н}}_{\text{л}}$ and right $\vec{R}^{\text{н}}_{n}$. The projections of these forces on the coordinate axes are expressed by the formulas

$$\begin{aligned}\vec{R}^{\text{н}}_{\text{л}} &= \left\{R^{\text{н}}_{\text{л}}\cos\varphi\sin\beta,\, R^{\text{н}}_{\text{л}}\sin\varphi, R^{\text{н}}_{\text{л}}\cos\varphi\cos\beta\right\};\\ \vec{R}^{\text{н}}_{n} &= \left\{R^{\text{н}}_{n}\cos\varphi\sin\beta,\, R^{\text{н}}_{n}\sin\varphi, -R^{\text{н}}_{n}\cos\varphi\cos\beta\right\}.\end{aligned} \tag{2}$$

Modules of forces defined by formulas (1), which create the upper left and upper right wingtips, are equal. Therefore, the indices denoting “left” and “right” for the modules of these forces are not used in the future. This also applies to forces determined by formulas (2).

So, the modules of the aerodynamic forces created by one upper and one lower wingtips, respectively, will be indicated in the form

$$\left|\vec{R}^{\text{в}}_{\text{л}}\right| = \left|\vec{R}^{\text{в}}_{n}\right| = R^{\text{в}}, \quad \left|\vec{R}^{\text{н}}_{\text{л}}\right| = \left|\vec{R}^{\text{н}}_{n}\right| = R^{\text{н}}.$$

The coefficients of forces $R^{\text{в}}$ and $R^{\text{н}}$ denote, respectively, the letters $C^{\text{в}}$ and $C^{\text{н}}$ i.e.

$$R^{\text{в}} = C^{\text{в}}\frac{\rho V_\infty^2}{2}S, \quad R^{\text{н}} = C^{\text{н}}\frac{\rho V_\infty^2}{2}S, .$$

The forces created by the wingtips are small compared to other forces acting on the aircraft. We will assume that their changes in the angle of attack are also small.

We will assume that the center of gravity of the aircraft is on the axis Ox, at a distance x_T from the nose of the aircraft. The distance of the center of pressure from the nozzle aircraft will be denoted x_D, and the distance of the focus point by the letter x_F. If necessary, we will assume that these points are on the average aerodynamic chord (MAR) and are calculated from the beginning of the MAR. The position of the focus is significantly influenced by the Mach number of the flight, in the transonic zone of Mach numbers there is a sharp shift of the focus back. With a further increase in the Mach number, the position of the focus stabilizes.

Suppose that the vertical projections of the centers of pressures of all parts of the wingtips intersect with the axis Oz. The distance of the centers of pressure of the upper halves of the wingtips from the axis Oz is denoted by the letter $y^{\text{в}}$, and the centers of pressure of the lower halves by the letter $y^{\text{н}}$.

Then, the tops of the wingtips together will create a dive moment relative to the axis Oz:

$$M_z^{\text{в}} = -2y^{\text{в}}R^{\text{в}}\cos\varphi\sin\alpha_z,$$

and the lower parts of the wingtips will create a tuning moment

$$M_z^{\text{н}} = -2y^{\text{н}}R^{\text{н}}\cos\varphi\sin\beta.$$

Taking into account the expressions of the aerodynamic forces of the wingtips through their coefficients and the dynamic head, we can write

$$M_z^{в} = -2y^{в}C^{в}\frac{\rho V_\infty^2}{2}S\cos\varphi\sin\alpha_z,$$
$$M_z^{н} = -2y^{н}C^{н}\frac{\rho V_\infty^2}{2}S\cos\phi\sin\beta. \quad (3)$$

From these formulas it is evident that if the angles of the collapse of the lower wingtips $\phi = \pi/2$, then only the dive moment of the upper wingtips occurs, and if $\varphi = \pi/2$ then only the kicking moment of the lower wingtips occurs. In addition, if some angle of collapse is zero, then the corresponding moment gets the greatest value. Note that for wings with wingtips, the moments (3) are associated with the lifting force of the wing and always arise if a lifting force occurs. In addition, the angles of twisting of the aerodynamic wingtips are sufficiently small.

Dividing the expression of the moments into an expression $\frac{\rho V_\infty^2}{2}Sb$ where b is the characteristic linear dimension, for example, the length of the average aerodynamic chord, and denoting

$$m_z^{в} = \frac{M_z^{в}}{\frac{\rho V_\infty^2}{2}Sb}, \quad m_z^{н} = \frac{M_z^{н}}{\frac{\rho V_\infty^2}{2}Sb}, \quad (4)$$

we obtain the expressions for the coefficients of the moments created by the wingtips

$$m_z^{в} = -2\bar{y}^{в}C^{в}\cos\varphi\sin\alpha_z;$$
$$m_z^{н} = -2\bar{y}^{н}C^{н}\cos\phi\sin\beta,$$

where $\bar{y}^{в} = \frac{y^{в}}{b}$, $\bar{y}^{н} = \frac{y^{н}}{b}$ are denoted.

The sum of expressions (4) is the coefficient of the longitudinal moment created by the wingtips

$$m_{zz} = -2\bar{y}^{в}C^{в}\cos\varphi\sin\alpha_z + 2\bar{y}^{н}C^{н}\cos\phi\sin\beta. \quad (5)$$

Where m_{zz} is the coefficient of longitudinal moment of the tips; Here the first letter of the index indicates the coordinate axis along which the moment is calculated, and the second indicates the belonging of the magnitude of the moment to the aerodynamic wingtip.

The coefficient of the longitudinal moment of the aircraft consists of the coefficients of the longitudinal moments of the wing, horizontal tail, fuselage, nacelle, power plants, etc:

$$m_z = m_{z,kp} + m_{z,го} + m_{z,ф} + m_{z,mг} + m_{z,cy} + \cdots$$

When the coefficients of each aircraft element are added [6–10]

$$m_z = m_{zo} + \left(\bar{x}_T - \bar{x}_F\right)C_y + m_z^{\delta}\delta_{рв} + ... \quad (6)$$

where $\delta_{рв}$ is the angle of the elevator deflection,

$$m_{zo} = m_{zo,kp} + m_{zo,го} + m_{zo,ф} + \cdots$$

The coefficient of the longitudinal moment of the aircraft with zero lifting force and with steady RS,

$$\bar{x}_F = \bar{x}_{F,kp} + \bar{x}_{F,го} + \bar{x}_{F,ф} + \cdots$$

The relative coordinate (related to the chord b) of the aerodynamic focus of the aircraft, taking into account its displacement, due to the influence of other parts of the aircraft, $\bar{x}_T = \frac{x_T}{b}$ — the alignment of the aircraft (the relative coordinate of the center of gravity of the aircraft), $(\bar{x}_T - \bar{x}_F)$ — the centering reserve. Note that the relative location of the center of gravity and focus, i.e. The reserve of centering is decisive in the formation of the moment ensuring the stability of the aircraft. Here, the linear dependence of the pitch moment coefficient on the transverse Oz axis on the lift coefficient

$$m_z = m_{z0} + \frac{\partial m_z}{\partial C_y}C_y = m_{z0} + \left(\bar{x}_T - \bar{x}_F\right)C_y.$$

This formula holds for approximately angles of attack $\alpha \le 12^{o}$, and the coefficients of the formula are independent of C_y. Basically, it is valid for the aerodynamic moment of the pitch of the wing of a rectangular shape with respect to the center of mass. The focus and the center of pressure in the general case, when $m_{z0} \ne 0$, do not coincide with each other. The position of the focus corresponding to the linear dependence, approximately in the range of angles of attack, $-10^{o} \le \alpha \le +10^{o}$ can be assumed constant.

As is known, the focus is the point of application of the increment of the aerodynamic force, which arises from the perturbations of the motion of the aircraft, and the moments of these forces act relative to the center of gravity of the aircraft. The focus position affects the magnitude of the longitudinal moment coefficient, since when $M > M_{kr}$ the focus moves backwards, here M_{kr} is the critical Mach number.

For simplicity, we shall only take into account the written terms of the longitudinal moment coefficient aircraft in formula (6). Then the coefficient of the longitudinal moment aircraft in the presence of aerodynamic wing tips is obtained by summing formulas (5) and (6)

$$\tilde{m}_z = m_{zo} + \left(\bar{x}_T - \bar{x}_F\right)C_y - 2\bar{y}^{в}C^{в}\cos\varphi\sin\alpha_z + $$
$$+2\bar{y}^{н}C^{н}\cos\phi\sin\beta + m_z^{\delta}\delta_{рв}, \quad (7)$$

where the coefficient of the longitudinal moment for an aircraft with a wing having wingtips is indicated by the same symbol for a wing without an end, but is equipped with a cap from above $\tilde{m}_z$.

As can be seen, the coefficient of the longitudinal moment for an aircraft with a wing without a tip depends linearly on the lift coefficient, and, as we shall see below, for a aircraft with a wing with wingtips, this dependence is nonlinear.

Analysis of the stability of the aircraft in the case of quadratic dependence of the coefficients of the aerodynamic forces of the winglets on the coefficient of lift of the wing

Let us consider some properties of the coefficients of the aerodynamic forces of the tips, taking into account that the wing tips have the main purpose — to reduce the inductive drag of the wing. Calculations show that the coefficient of inductive resistance is proportional to the square of the lift coefficient. Bearing this in mind, we accept the assumptions that the coefficients of the aerodynamic forces created by the tips, having a structure similar to the structure of the drag coefficient of the wing, can also be proportional to the square of the lift coefficient of the wing. To prove this, the only assumption we will have is the independence of the effective elongation from the lift coefficient. Taking into account the following formula

$$\tilde{\lambda} = \lambda\left[1 - \frac{\pi\lambda}{C_y^2\left(1+\delta\right)} C_{zx}\right]^{-1}$$

and combining formulas

$$C_{zxi} = \frac{C_y^2}{\pi\tilde{\lambda}}\left(1+\delta\right), \quad C_{zx} = \frac{\Delta\lambda}{\tilde{\lambda}} C_{xi},$$

$$C_{zx} = 2C^{\text{в}}\cos\varphi\sin\alpha_z + 2C^{\text{н}}\cos\phi\sin\beta,$$

with the expression of the coefficient of inductive resistance

$$C_{xi} = \frac{C_y^2}{\pi\lambda}\left(1+\delta\right)$$

as a result, we get

$$2C^{\text{в}}\cos\varphi\sin\alpha_z + 2C^{\text{н}}\cos\phi\sin\beta = \frac{\Delta\lambda}{\tilde{\lambda}}\frac{C_y^2}{\pi\lambda}\left(1+\delta\right).$$

The longitudinal component of aerodynamic forces created by the tip has the structure of inductive drag of the wing and reduces it. From the last equality, for a constant value of the effective wing extension, it can be assumed that for a given aircraft the coefficients of the aerodynamic forces of the upper $C^{\text{в}}$ and lower $C^{\text{н}}$ parts of the wing tips are proportional to the square of the lift coefficient C_y^2. Comparing the expression for the coefficient of the longitudinal moment of the wingtips (5) with the last equality, we conclude that the coefficient of the longitudinal moment m_{zz} is also proportional C_y^2. In this case, the values of $C^{\text{в}}$ and $C^{\text{н}}$ can be written in the form of equalities $C^{\text{в}} = B_1 C_y^2$ and $C^{\text{н}} = B_1 C_y^2$, where the quantities B_1 and B_2 are considered positive constants for small changes in the angle of attack α. From the system of equations (5) and the last equation it is not difficult to determine

$$B_1 = \frac{\overline{y}^{\text{н}} C_{zx} - m_{zz}}{2\left(\overline{y}^{\text{в}} + \overline{y}^{\text{н}}\right) C_y^2 \cos\varphi\sin\alpha_z};$$

$$B_2 = \frac{\overline{y}^{\text{в}} C_{zx} + m_{zz}}{2\left(\overline{y}^{\text{в}} + \overline{y}^{\text{н}}\right) C_y^2 \cos\phi\sin\beta}.$$

Here the values of B_1 and B_2 do not depend on the lift coefficient, with small changes in the angle of attack, since, as noted above, the numerators of the right-hand parts, having a structure of the coefficient of inductive drag of the wing, are proportional C_y^2.

We immediately note the following simple formulas

$$\frac{\partial C^{\text{в}}}{\partial C_y} = 2B_1 C_y, \qquad \frac{\partial C^{\text{н}}}{\partial C_y} = 2B_1 C_y.$$

We expand the functions $C^{\text{в}} = B_1 C_y$ and $C^{\text{н}} = B_1 C_y$ in a Taylor series with respect to the angle of attack in a neighborhood of some angle of attack α_0, provided that $\alpha - \alpha_0 \ll 1$ as a result we obtain

$$C^{\text{в}} = 2B_1 C_y \frac{\partial C_y}{\partial\alpha}\left(\alpha-\alpha_0\right) = 2B_1 C_y C_y^{\alpha}\left(\alpha-\alpha_0\right),$$

$$C^{\text{н}} = 2B_1 C_y \frac{\partial C_y}{\partial\alpha}\left(\alpha-\alpha_0\right) = 2B_1 C_y C_y^{\alpha}\left(\alpha-\alpha_0\right).$$

Here the coefficients of the difference $\alpha - \alpha_0$ depend on the angle α_0. In these formulas only the second (linear) terms of the Taylor series are written, since the first terms are zero at the point α_0, and the next ones are infinitely small of higher order and therefore are not written down. On the right-hand side of this equality the coefficients of the difference $\alpha - \alpha_0$ depend on α_0, but do not depend on the current angle of attack α.

From the last formulas we obtain:

$$\frac{\partial C^{\text{в}}}{\partial\alpha} = 2B_1 C_y C_y^{\alpha}, \qquad \frac{\partial C^{\text{н}}}{\partial\alpha} = 2B_2 C_y C_y^{\alpha}.$$

Taking into account the dependence of the quantities m_{zo}, m_z^{δ}, B_1 and B_2 from C_y and α,for small changes in the angle of attack α, we calculate the following derivatives of formula (7):

$$\frac{\partial\tilde{m}_z}{\partial C_y} = \tilde{m}_z^{C_y} = \left(\overline{x}_T - \overline{x}_F\right) = 4\overline{y}^{\text{в}} B_1 C_y \cos\varphi\sin\alpha_z +$$

$$+ 4\overline{y}^{\text{н}} B_2 C_y \cos\phi\sin\beta,$$

$$\frac{\partial \tilde{m}_z}{\partial \alpha} = \tilde{m}_z^{C_y} = \Big[\left(\bar{x}_T - \bar{x}_F\right) = 4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z + $$
$$+ 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta\Big] C_y^{\alpha},$$

We also note the dependence of the lift coefficient on the angle of attack

$$C_y = \frac{\partial C_y}{\partial \alpha}\left(\alpha - \alpha_0\right) = C_y^{\alpha}\left(\alpha - \alpha_0\right),$$

where the derivatives are written at a point.

Suppose that under the influence of a short-term disturbance, the angle of attack of the flight increased. Then, if the aircraft is stable in the angle of attack, then, without pilot intervention, the aircraft creates a dive moment and eliminates the increments of the angle of attack. Conversely, by reducing the angle of attack under the influence of perturbations, the stable aircraft creates a tuning moment and eliminates increments in the angle of attack. As can be seen, in both cases the increments of the angle of attack and the longitudinal moment have opposite signs, i.e. their ratio is negative for a stable angle of attack aircraft. At sufficiently small angles of attack, the ratio of these increments can be replaced by the corresponding partial derivative. Thus, we obtain the following results for the static longitudinal stability of an aircraft with a wing without wingtips (we will not consider changes in the parameters of the disturbed motion with time, which relate to an extremely complex and interesting region, the dynamic stability of the aircraft):

If $m_z^{\alpha} < 0$ then the aircraft is stable;

If $m_z^{\alpha} = 0$, then the aircraft is neutral (indifferent) to stability;

If $m_z^{\alpha} > 0$, then the aircraft is not stable.

These conditions are written down with the help of derivatives with respect to the lift coefficient:

If $m_z^{C_y} < 0$, then the aircraft is stable;

If $m_z^{C_y} = 0$, then the aircraft is neutral (indifferent) to stability;

If $m_z^{C_y} > 0$, then the aircraft is not stable.

In these formulas, the derivatives are calculated for the balancing angle of attack $\alpha = \alpha_{\text{бал}}$, where $m_x\left(\alpha_{\text{бал}}\right) = 0$. Derivatives $m_z^{C_y}$ and $m_z^{C_y}$ are called coefficients, or degrees, of longitudinal static stability. They determine the speed at which the perturbed flight parameters return to their initial values prior to the action of perturbations. The faster the perturbations of the parameters decay (in time), the higher is the degree of stability of the original unperturbed motion of the aircraft.

These conditions are reduced to the following equivalent conditions for an aircraft with a wing without wingtips:

If $\bar{x}_T - \bar{x}_F < 0$, then the aircraft is stable;

If $\bar{x}_T - \bar{x}_F = 0$, then the aircraft is neutral (indifferent) to stability;

If $\bar{x}_T - \bar{x}_F > 0$, then the aircraft is not stable.

Thus, if the center of gravity is located in front of the focus, then the aircraft is longitudinally statically stable, if on the contrary, the aircraft is statically longitudinally stable, and if these points coincide, then the VS is indifferent to stability. Obviously, the greater the distance between the center of gravity and focus, the greater is the recovery torque. The quantity $\bar{x}_T - \bar{x}_F$ is called the degree of longitudinal static stability at constant speed.

The derivative $\tilde{m}_z^{C_y}$ for an aircraft with a wing with wingtips, calculated above, is written in the following form

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T + 4\bar{y}^{\text{í}} B_2 C_y \cos\phi \sin\beta\right) - $$
$$- \left(\bar{x}_F + 4\bar{y}^{\text{â}} B_1 C_y \cos\varphi \sin\alpha_z\right). \quad (8)$$

The first bracket on the right side of this equality is called the effective centering, and the second is the effective focus of the aircraft, the wing that has two-sided wingtips:

$$\bar{x}_{TЭ} = x_T + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta;$$

$$\bar{x}_{FЭ} = x_A + 4\bar{y}^{\text{в}} B_2 C_y \cos\varphi \sin\alpha_z.$$

As can be seen, the upper parts of the wingtips, creating a dive moment, contribute to an increase in the effective focus of the aircraft, and the lower end of the end pieces, creating a kicking moment; increase the effective centering of the aircraft. Effective values, being proportional to the lift coefficient, begin to differ from the corresponding wing values without the wingtips, once the wing creates a lift.

Wingtips shift back degrees of longitudinal static stability of the aircraft at a constant speed. In this case, if

$$4\bar{y}^{\text{н}} B_2 C_y \cos\varphi \sin\beta = 4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z,$$

then the expansion of the degree of longitudinal static stability of the aircraft at a constant speed does not occur. As is well known, the degree of stability determines the speed of return of the perturbed motion parameters to their original, unperturbed values. When the last equality is fulfilled, the wingtips do not affect the longitudinal static stability of the wing, which does not have a tip.

If the angle of collapse of the lower wingtips $\phi = 0$, then the elongation of the effective centering of the aircraft is greatest at a constant twist angle of the lower wingtip β, and if $\phi = \pi/2$, under the same condition, the displacement of the aircraft alignment does not occur. Similarly, if the angle of cambering of the upper wingtip $\varphi = 0$, then the extension of the effective focus of the aircraft is greatest with the unchanged twist angle of the upper wingtip α_z, and if $\varphi = \pi/2$, under the same condition, the shift of the effective focus of the aircraft does not occur. Obviously, in the presence of only vertical (upper) wing tips, the effective focus of the aircraft is shifted as much as possible, and the alignment does not change. At the same time, the module of the centering reserve $(\bar{x}_{TЭ} - \bar{x}_{FЭ})$ increases and the stability properties of the aircraft are still being strengthened.

Thus, for an aircraft with a wing with wingtips, the derivative of the coefficient of the longitudinal moment with respect to the lift coefficient can be written in the following form

$$\tilde{m}_z^{C_y} = \bar{x}_{TЭ} - \bar{x}_{FЭ}.$$

Then the analysis of the longitudinal static stability of an aircraft with a wing with tips leads to the following conditions:

If $\bar{x}_{TЭ} - \vec{x}_{FЭ} < 0$, then the aircraft is stable;

If $\bar{x}_{TЭ} - \vec{x}_{FЭ} = 0$, then the aircraft is neutral (indifferent) to stability;

If $\bar{x}_{TЭ} - \vec{x}_{FЭ} > 0$, then the aircraft is not stable.

Let us analyze these conditions. Suppose that the wing has only the upper wingtips, then, according to formula (8), we have the following

$$\tilde{m}_z^{C_y} = \bar{x}_T - \left(\bar{x}_F + 4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z\right).$$

If aircraft without wingtips is neutral to stability, i.e. $\bar{x}_T - \bar{x}_F = 0$, then the wingtip turns the aircraft to stable

$$\tilde{m}_z^{C_y} = -4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z < 0.$$

If aircraft without wingtips is not stable, i.e. $\bar{x}_T - \bar{x}_F > 0$, then, with

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) - \left(4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z\right) < 0.$$

The aircraft with the wingtips will be stable, with

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) - \left(4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z\right) = 0.$$

It will be neutral to stability, and when

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) - \left(4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z\right) > 0,$$

it is not stable.

If aircraft without wingtips is stable, i.e. $\bar{x}_T - \bar{x}_F < 0$, then the wing tips still strengthen this property

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) - \left(4\bar{y}^{\text{в}} B_1 C_y \cos\varphi \sin\alpha_z\right) < \bar{x}_T - \bar{x}_F < 0.$$

Suppose now that the wing has only the lower wingtips; then from (8) we have

$$\tilde{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) - \left(4\bar{y}^{\text{і}} B_2 C_y \cos\phi \sin\beta\right).$$

If aircraft without wingtips is not stable, i.e. $\bar{x}_T - \bar{x}_F > 0$, then the wingtips enhance this property aircraft:

$$\hat{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta > \bar{x}_T - \bar{x}_F > 0.$$

If aircraft without wingtips is neutral to stability, i.e. $\bar{x}_T - \bar{x}_F = 0$, the lower wing tips make the aircraft non-stable

$$\hat{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta = = 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta > 0.$$

I aircraft without wingtips is stable, i.e. $\bar{x}_T - \bar{x}_F < 0$, the lower wing tips can turn it to neutral to stability provided:

$$\hat{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta = 0;$$

to the unstable — under the condition

$$\hat{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta > 0$$

and rest stable, but reduce the reserve of alignment provided

$$\hat{m}_z^{C_y} = \left(\bar{x}_T - \bar{x}_F\right) + 4\bar{y}^{\text{н}} B_2 C_y \cos\phi \sin\beta < 0.$$

As follows from this analysis, the upper and lower parts of the wingtips have the opposite effect on the stability of the aircraft. Let us consider the issues of their joint influence on the stability of the aircraft.

If the aircraft without wing tips is not stable, i.e. $\bar{x}_T - \bar{x}_F > 0$, then the wing tip does not change this property of the aircraft provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} > 0$; provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} = 0$ that the wing tips end up turning aircraft to neutral to stability, and provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} < 0$ — to stable ones.

If the aircraft without wing tips is neutral to stability, i.e. $\bar{x}_T - \bar{x}_F = 0$, then the wing tip does not change this property of the aircraft provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} = 0$; provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} > 0$ that the wing tips turn the aircraft to not stable, and provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} < 0$ — to the stable ones.

If the aircraft without wing tips is longitudinally stable, i.e. $\bar{x}_T - \bar{x}_F < 0$, then the wing tip does not change this property of the aircraft provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} < 0$; provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} > 0$ that the ends of the wing transform the aircraft to non-stable ones, and provided $\bar{x}_{TЭ} - \bar{x}_{FЭ} = 0$ that they are neutral to stable structures. As follows from formula (7), if the elevator is rejected by an angle

$$\delta_{\text{рв}} = \frac{2\bar{y}^{\text{в}} C^{\text{в}} \cos\varphi \sin\alpha_z - 2\bar{y}^{\text{н}} C^{\text{н}} \cos\varphi \sin\beta}{m_z^{\delta}}.$$

Then the stability properties of an aircraft with a wing without a tip will not be disturbed by the influence of the wingtips.

Conclusions

The aerodynamic forces created by the Witecomb-type "winglet" wingtips during flight have been determined and an expression of the longitudinal moment coefficient and its coefficient has been obtained.

The case of the quadratic dependence of the coefficients of the aerodynamic forces of the wingtips on the coefficient of lift of the wing is considered.

With small changes in the angle of attack, the partial derivatives of the coefficient of the longitudinal moment with respect to the lift coefficient and the angle of attack, which is called the degree of longitudinal static stability, are calculated.

The concepts of effective centering and effective focus of the aircraft, a wing that has bilateral wingtips, are introduced. It is shown that the conditions of the longitudinal static stability of an airplane along the angle of attack are recorded by analogous formulas for an airplane without wingtips when the centering and focus are replaced by the corresponding effective values.

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INFLUENCE OF THE AT "WINGLETS" TYPE ON THE LONGITUDINAL STATIC STABILITY OF ANGLE OF ATTACK OF THE AIRCRAFT

The effect of the aerodynamic "AT winglets" wingtips on the longitudinal static stability of the aircraft is investigated by the method of equations of a steady straight-line horizontal flight. The aerodynamic forces created by the upper and lower parts of the wingtip "AT winglets" are defined. Assuming that the vertical projections of the pressure centers of all parts of the wingtips intersect the transverse axis, the dive and kicking moments of these forces are determined. The aerodynamic forces created by all four parts of the wingtip "AT winglets" are determined and their pitch moments are calculated. The vector of the total aerodynamic force of the wingtips is represented in the form of components along the associated coordinate system. Equilibrium equations for the steady motion of an aircraft with wingtips of the "AT winglets" type in a horizontal flight are recorded. This method is used to calculate the expression for the longitudinal moment coefficient of an aircraft in the presence of aerodynamic wingtips. The forward and backward arrangements of the centers of pressure of the tips with respect to the vertical plane passing through the lateral coordinate axis are considered. The derivatives of the coefficients of the roll and yaw moment coefficients for the slip angle for the wing with the tips that enter the equation for the lateral oscillation of the aircraft are determined. The correspondence between the derivatives of the coefficients of the moments of roll and yaw for the wings without the tips and with the tips for the slip angle is established. Using this correspondence, from the known differential equation of the lateral oscillation of an airplane with a wing without a wingtip, the corresponding equation of lateral oscillation of the airplane is obtained for the aircraft for a wing with a wing with wingtips. The expression of the longitudinal moment and its coefficient for an aircraft taking into account the influence of aerodynamic wing tips is derived. With small changes in the angle of attack, the partial derivatives of the longitudinal moment coefficient with respect to the angle of attack and the coefficient of lift are calculated. The conditions for the longitudinal static stability of the aircraft along the angle of attack are recorded, taking into account the influence of wing tips.

Keywords: AT winglet; mathematical model; centering; focus; longitudinal static stability in the angle of attack.

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ВПЛИВ ЗАКІНЦІВОК КРИЛА ТИПУ "AT WINGLETS" НА ПРОДОЛЬНУ СТАТИЧНУ СТІЙКІСТЬ ЛІТАКА ЗА КУТОМ АТАКИ

Методом рівнянь встановленого прямолінійного горизонтального польоту досліджується вплив аеродинамічних закінцівок крила "AT winglets" на подовжню статичну стійкість літака. Визначені аеродинамічні сили, створені верхніми і нижніми частинами законцовок "AT winglets". Передбачаючи, що вертикальні проекції центрів тиску всіх частин закінцівок перетинаються поперечною віссю, визначені пікіруючі та кабріруючі моменти цих сил. Визначені аеродинамічні сили, створені всіма чотирма частинами закінцівок крила "AT winglets" і обчислені їх моменти тангажу. Вектор повної аеродинамічної сили закінцівок представляється у вигляді компонент в зв'язаній системі координат. Записується рівняння рівноваги сталого руху літака із закінцівками крила типу «AT winglets» в горизонтальному польоті. Цим методом обчислено вираження коефіцієнта подовжнього моменту ПС за наявності аеродинамічних закінцівок крила. Розглянуті переднє і заднє розташування центрів тиску закінцівок відносно вертикальної площини, що проходить через бічну координатну вісь. Визначені похідні від коефіцієнтів моментів крену і рискання по куту ковзання для крила із закінцівками, які входять в рівняння бічного коливання літака. Встановлена відповідність похідних від коефіцієнтів моментів крену і рискання для крил без закінцівок та із закінцівками за кутом ковзання. Використовуючи цю відповідність, з відомого диференційного рівняння бічного вагання літака з крилом без закінцівок отримано відповідне рівняння бічного вагання літака за креном для літака з крилом із закінцівками.

Отримано вираження подовжнього моменту і його коефіцієнта для літака, що враховує вплив аеродинамічних закінцівок крила. При малих змінах кута атаки обчислені приватні похідні від коефіцієнта подовжнього моменту за кутом атаки і за коефіцієнтом підіймальної сили. Приведені умови подовжньої статичної стійкості літака за кутом атаки, що враховують впливи закінцівок крила.

Ключові слова: AT winglet; математична модель; центрівка; фокус; подовжня статична стійкість за кутом атаки.

Меликов Э. Т., Мильцов В. Е.

ВЛИЯНИЕ ЗАКОНЦОВОК КРЫЛА ТИПА "AT WINGLETS" НА ПРОДОЛЬНУЮ СТАТИЧЕСКУЮ УСТОЙЧИВОСТЬ САМОЛЕТА ПО УГЛУ АТАКИ

Методом уравнений установившегося прямолинейного горизонтального полета исследуется влияние аэродинамических законцовок крыла "AT winglets" на продольную статическую устойчивость самолета. Определены аэродинамические силы, созданные верхними и нижними частями законцовок "AT winglets". Предполагая, что вертикальные проекции центров давлений всех частей законцовок пересекаются поперечной осью, определены пикирующий и кабрирующий моменты этих сил. Определены аэродинамические силы, созданные всеми четырьмя частями законцовок крыла "AT winglets" и вычислены их моменты тангажа. Вектор полной аэродинамической силы законцовок представляется в виде компонент в связанной системе координат. Записывается уравнения равновесия установившегося движения самолета с законцовками крыла типа «AT winglets» в горизонтальном полете. Этим методом вычислено выражение коэффициента продольного момента ВС при наличии аэродинамических законцовок крыла. Рассмотрены переднее и заднее расположения центров давлений законцовок относительно вертикальной плоскости, проходящей через боковую координатную ось. Определены производные от коэффициентов моментов крена и рыскания по углу скольжения для крыла с законцовками, которые входят в уравнение бокового колебания самолета. Установлено соответствие производных от коэффициентов моментов крена и рыскания для крыльев без законцовок и с законцовками по углу скольжения. Используя это соответствие, из известного дифференциального уравнения бокового колебания самолета с крылом без законцовок получено соответствующее уравнение бокового колебания самолета по крену для самолета с крылом с законцовками.

Получено выражение продольного момента и его коэффициента для самолета учитывающее влияние аэродинамических законцовок крыла. При малых изменениях угла атаки вычислены частные производные от коэффициента продольного момента по углу атаки и по коэффициенту подъемной силы. Приведены условия продольной статической устойчивости самолета по углу атаки, учитывающие влияния законцовок крыла.

Ключевые слова: AT winglet; математическая модель; центровка; фокус; продольная статическая устойчивость по углу атаки.

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