UDC 629.735 **DOI:** 10.18372/2310-5461.39.13093

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INFLUENCE OF "AT WINGLETS" WINGTIP TYPE ON THE AERODYNAMIC CHARACTERISTICS OF WINGS

Introduction

Inductive drag of the wing is associated with the finiteness of the wing span and arises from the flow of air through the end sections, perpendicular to the main one - in front of the incoming flow. Being inversely proportional to the wing span, it decreases with increasing span. When designing the constructors in every way try to reduce the drag of the aircraft, which leads to an increase in the aerodynamic quality of the device. In this case, a decrease in the inductive drag of the wing is also one of the first to occur in thought. This, at least, can be achieved by increasing the wing span, or by preventing the flow of air through the end sections of the wing. Thus, the idea of using wing tips (winglets) arose. The wing tips have been used in aviation since the seventies of the last century, and bring tremendous fuel savings for the year. However, theoretical studies in this field began in 2012 in Azerbaijan in the National Aviation Academy [1–3]. In the mentioned studies, the influence of vertical wing tips (VWT) on the inductive drag is investigated. Various mathematical models have been constructed that reflect the influence of the VWT on aerodynamic characteristics. In the present study, similar questions are being solved for a wing with “Advantage Technology winglets” wingtip type.

Formulation of the problem

We take the following coordination system. We place the beginning of the coordinates in the middle of the wing, direct the axis of *Oz* along the span to the right, the axis *Oy's* directed upwards and *Ox* axis on the undisturbed flow. We define all the forces influencing on the aircraft with winglets type “Advantage Technology winglets” at a steady level flight. Due to equilibrium of these forces we determine the influence of aerodynamic forces of wingtip on the inductive drag of the wing.

The solution of the problem

The following forces effect the plane in level flight [4–6]:

– The power of the weight G — always directed vertically down to the center of the earth;

– Lift of the aircraft Y — is perpendicular to the direction of the undisturbed flow;

– Drag force of the aircraft Q — aimed in the direction opposite to the movement of aircraft;

– Thrust P — is generally directed towards the aircraft movement motion, along the axis;

– Full aerodynamic force created by the upper wingtips. The force created by the upper left wingtip is symbolized $\vec{R}_л^в$, but the force created by the right wingtip is symbolized $\vec{R}_n^в$;

– Complete aerodynamic forces created by the lower wingtips. They are symbolized respectively, the left $\vec{R}_л^н$ and the right force $\vec{R}_n^в$.

The angle between the true velocity and the free-flow speed for a wing with wingtips α_z equals to the angle between the vector of the total aerodynamic force generated by the upper left wingtip $\vec{R}_л^в$ and the longitudinal axis of the wing, as the sides of these angles are perpendicular to each other.

Then the projection of the full aerodynamic force of the left wingtip will have the form:

$$\vec{R}_l^в = \left\{ R_{lx}^в, R_{ly}^в, R_{lz}^в \right\},$$

Where

$$R_{lx}^в = R_l^в \cos\varphi \sin\alpha_z$$

Longitudinal force created by the upper left wingtip, φ-angle of wingtip camber (the angle between the vertical plane of aircraft symmetry and the tangent plane to the wingtip surface at the point of its center of pressure),

$$R_{ly}^{в} = R_l^{в}\sin\varphi$$

lift force created by the upper left wingtip,

$$R_{lz}^{в} = R_l^{в}\cos\varphi\cos\alpha_z$$

the lateral force generated by the upper left wingtip.

Here, $R_l^{в}$ is the vector unit of $\vec{R}_l^{в}$,

$$R_l^{в} = \sqrt{R_{lx}^{в2} + R_{ly}^{в2} + R_{lz}^{в2}}\ .$$

This force is applied to the center of the wingtip pressure. The total aerodynamic force of right upper wingtip differs from it only with the mark of the third component, so it can be written as

$$\vec{R}_n^{в} = \left\{R_n^{в}\cos\varphi\sin\alpha_z, R_n^{в}\sin\varphi, -R_n^{в}\cos\varphi\cos\alpha_z\right\}.$$

Obviously, $R_n^{в} = \sqrt{R_{nx}^{в2} + R_{ny}^{в2} + R_{nz}^{в2}} = R_{л}^{в}$

therefore, the lower indices that indicate the left and right wingtips will be removed in the future.

Then, the right and left upper part of the wingtips together create a force with components

$$2\vec{R}^{в} = \vec{R}_n^{в} + \vec{R}_l^{в} = \left\{2R^{в}\cos\varphi\sin\alpha_z, 2R^{в}\sin\varphi, 0\right\}. \quad (1)$$

Now we define the forces created by the lower part of wingtips (projections) (Fig 1).

Fig. 1. The AT winglet wingtip of the Airbus A319

Since, under the wing the air pressure is much higher than in the environment, it can be assumed that the lower left wingtip pressure force is applied to the center of pressure of the wingtip, normal to its surface. Lower wingtip camber is indicated by the letter φ, and the twist angle of the center of pressure is indicated by the letter β.

Then, with the same above mentioned argumentation, we can write

$$2\vec{R}^{н} = \vec{R}_n^{н} + \vec{R}_l^{н} = \left\{2R^{н}\cos\varphi\sin\beta, 2R^{н}\sin\varphi, 0\right\}. \quad (2)$$

Thus

$$2R_{lx}^{в} = 2R_l^{в}\cos\varphi\sin\alpha_z$$

the longitudinal is force generated by the lower parts of wingtip, but

$$2R_y^{н} = 2R^{н}\sin\varphi$$

lift force is created by them.

Because of symmetry, the lateral forces created by the left and right wingtips, balance each other.

The amount of power generated by all four parts of the wingtips "Advantage Technology winglets", is indicated by the vector $\vec{R}_z$. Thus

$$\vec{R}_z = \left\{R_{zx}, R_{zy}, R_{zz}\right\},$$

Where

$$R_{zx} = 2R^{в}\cos\varphi\sin\alpha_z + 2R^{н}\cos\varphi\sin\beta \quad (3)$$

the longitudinal component of the vector of total aerodynamic wingtip force, which obviously reduces any drag force, and, of course, is added to the force of traction motors,

$$R_{zy} = 2R^{в}\sin\varphi + 2R^{н}\sin\varphi\ , \quad (4)$$

the vertical component of the vector of total aerodynamic wingtip force, which is added to the lift of the wing, and the lateral component of the vector of the total aerodynamic force due to the symmetry of wingtips equals to zero $R_{zz} = 0$.

Now, considering the formulas (3) and (4), we can write the equilibrium equations of steady motion of the aircraft with wingtips type "AT winglets" in level flight in the form of

$$\left.\begin{aligned} P + R_{zx} - Q = 0 \\ Y + R_{zy} - G = 0 \end{aligned}\right\}. \quad (5)$$

From the first equation of the system (5) it follows that the thrust of the engine P balances the force $Q - R_{zx}$. Since the force $Q = Q_{pr} + Q_i$ consists of the sum of the profile and inductive drag and the profile resistance is almost unchanged, the force R_{zx} reduces the inductive drag of the wing of the finite span without wingtips, i.e.

$$\Delta Q_i = R_{zx}$$

or, the reduced part of the inductive drag force equals to

$$\Delta Q_i = 2R^{в}\cos\varphi\sin\alpha_z + 2R^{н}\cos\varphi\sin\beta\ .$$

Heading in the direction of flight, this force increases the thrust of the engines and reduces the drag of the wing. In the case when the angles of collapse $\varphi = 0$ and $\varphi = 0$ it has the greatest value, and when $\varphi = \varphi = \pi/2$ — it turns to zero. Angles α and β are usually small enough. The latter case is a consequence of the fact that when $\varphi = \varphi = \pi/2$ the tip turns into a wing extension with large sweep angles, in comparison with the wing itself, it also disappears. It turns out a longer wing without wingtips.

From the second equation of system (5) it follows that

$$Y + R_{zy} = G.$$

As follows from this equality, the lift Y is less than the weight of the aircraft, and the weight of the aircraft is balanced by the lifting forces created by the wing and the tips together. The quantities on the left side of this equation for wings without wingtips, with upper endings and with double endings, will be indicated, respectively, by the indices 0, 1 and 2 from above. Since the weight of the aircraft is constant and $R_{zy}^0 = 0$, then it can be written as follows:

$$Y^0 = Y^1 + R_{zy}^1 = Y^2 + R_{zy}^2 = G.$$

Following from formula (4)

$$0 < R_{zy}^1 < R_{zy}^2.$$

Then from the preceding chain of equations, it follows that

$$Y^0 > Y^1 > Y^2.$$

Thus, provided the weight of the aircraft is constant, the lift of the wing with the tips is reduced, and the smallest is obtained for the wing with double wingtips of the “AT winglets” type. The remaining part of the lifting force creates aerodynamic wingtips. In this regard, the distribution of the lifting force over the entire wing span with the “AT winglets” wingtip is more even. And this leads to a decrease in the amplitude of wing oscillations and noise during flight. Note that the tip itself is also a profiled winglet and from its end there is a flow of air towards the fuselage at the upper wingtips and from the fuselage at the lower wingtips. These currents, forming with the main flow of air against motion, create vortices of relatively low intensity, flowing from the ends of the tips. The inductive drag of the wing is affected only by the longitudinal component of the total aerodynamic force of the tips, defined by formula (3).

Power R_{zx} can be represented through the high-speed head,

$$R_{zx} = \overline{C}_{zx} q_\infty S_z = C_{zx} q_\infty S$$

where

$$q_\infty = \frac{\rho V_\infty^2}{2}$$

is velocity head, S wing area without wingtips, and C_{zx} the coefficient of longitudinal force of the wingtips. The coefficient of this force is

$$\overline{C}_{zx} = C_{zx} \frac{S}{S_z}$$

where S_z is the sum of the areas of the wingtips. Then the force ΔQ_i can be written in the form

$$\Delta Q_i = C_{zx} q_\infty S.$$

The coefficient of inductive drag of the wing with the tips is written in the form of the difference in the inductive drag of the wing without the tip and the force factor ΔQ_i:

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta) - C_{zx}.$$

We express this expression in the following form

$$C_{zxi} = \frac{C_y^2}{\pi\lambda}(1+\delta)\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)} C_{zx}\right]$$

or

$$C_{zxi} = \frac{C_y^2}{\pi\tilde{\lambda}}(1+\delta)$$

where

$$\tilde{\lambda} = \lambda\left[1 - \frac{\pi\lambda}{C_y^2(1+\delta)} C_{zx}\right]^{-1}.$$

Aspect ratio of the wing. As can be seen the value of the aspect ratio makes it possible to record the inductive drag of the wing with the tips in the usual form. It is easy to see the inequality, which shows a decrease in the inductive drag of the wing under the influence of the wingtips.

Since

$$\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} < 1$$

then a square bracket with negative degree is the sum of a geometric progression

$$1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)} + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^2 + \left(\frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right)^3 + \cdots$$

Substituting this into the expressions for the effective elongation and leaving only the first two terms, we have

$$\tilde{\lambda} = \lambda\left[1 + \frac{\pi\lambda C_{zx}}{C_y^2(1+\delta)}\right]$$

or, taking into account the coefficient of inductive drag of the wing without wingtips,

$$\tilde{\lambda} = \lambda\left(1 + \frac{C_{zx}}{C_{xi}}\right).$$

Taking into account the following formula

$$C_{zx} = 2C_{zx}^{в} \cos\varphi \sin\alpha_z + 2C_{zx}^{н} \cos\varphi \sin\beta$$

we get the following

$$\tilde{\lambda}=\lambda\left(1+\frac{2C_{zx}^{в}\cos\varphi\sin\alpha_z+2C_{zx}^{н}\cos\varphi\sin\beta}{C_{xi}}\right).$$

This is an approximate expression of the wing aspect ratio with double tips.

The relative ratio of the wing is in the following form

$$\frac{\Delta\lambda}{\lambda}=\frac{C_{zx}}{C_{xi}}. \quad (6)$$

This result can be obtained in another way. We transform the expression for the aspect ratio

$$\tilde{\lambda}=\lambda\left[1-\frac{\pi\lambda}{C_y^2(1+\delta)}C_{zx}\right]^{-1}=\lambda\frac{1}{1-\frac{C_{zx}}{C_{xi}}}=$$

$$=\lambda\frac{C_{xi}}{C_{xi}-C_{zx}}=\lambda\frac{C_{xi}}{C_{zxi}}.$$

Thus

$$\tilde{\lambda}=\lambda\frac{C_{xi}}{C_{zxi}}$$

or

$$\tilde{\lambda}C_{zxi}=\lambda C_{xi}.$$

With the following formula $C_{zx}=C_{zx}^{в}+C_{zx}^{н}$, the last equation can be extended

$$\tilde{\lambda}C_{zxi}=\tilde{\lambda}^{в}C_{zxi}^{в}=\lambda C_{xi}. \quad (7)$$

Here both $\tilde{\lambda}^{в}$ and $C_{zxi}^{в}$ are aspect ratio, and the inductive drag of the wing with some upper wingtips. As can be seen, the product of effective lengthening of the wings with double and only upper tips on their coefficients of inductive drag is equal to the product of the wing extension without the wingtip to its coefficient of inductive drag. Thus, with increasing effective lengthening of the wing, its inductive drag decreases and vice versa.

This fundamental result can also be said as follows: when the weight of the aircraft is constant, the product of the effective wing aspect ratio with its wingtips to its coefficient of inductive drag is constant.

Alternatively, provided the weight of the aircraft is constant, the ratio of effective wing extensions is equal to the inverse ratio of their inductive drag coefficients.

We also must note the following proof of this result. From the expression for the coefficient of inductive drag for all wings

$$C_{zxi}=\frac{C_y^2}{\pi\tilde{\lambda}}(1+\delta)$$

we get the following

$$\tilde{\lambda}C_{zxi}=\frac{C_y^2}{\pi}(1+\delta)$$

which is constant with a constant weight of the aircraft.

Assuming the effective wing aspect ratio is known, from formulas (1.7) it is possible to determine the loss of inductive drag

$$\Delta C_{xi}=C_{xi}-C_{zxi}=C_{xi}-\frac{\lambda}{\tilde{\lambda}}C_{xi}=\frac{\tilde{\lambda}-\lambda}{\tilde{\lambda}}C_{xi}=\frac{\Delta\lambda}{\lambda}C_{xi}.$$

Conclusion

1. The aerodynamic forces created by the AT winglet wingtips during the flight are determined.
2. A system of algebraic equations containing all the forces acting on an airplane with a steady horizontal flight, which constitutes the mathematical model of the problem under investigation, is recorded.
3. It is shown that the wing tips reduce the inductive drag.
4. The aspect ratio of the wing is determined.
5. It is shown that, when the weight of the aircraft is constant, the product of the aspect ratio of the wing with its ends to its coefficient of inductive drag is constant.

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INFLUENCE OF "AT WINGLETS" WINGTIP TYPE ON THE AERODYNAMIC CHARACTERISTICS OF WINGS

In this article by the method of equations of steady horizontal flight the impact «AT winglets» wingtips on the wing aerodynamic quality is explored. The total aerodynamic force, created by all four parts of the wingtips is determined. The vector of the total aerodynamic force of the wingtips is represented in the form of components of a linked coordinate system. Equilibrium equations for the steady motion of an aircraft with «AT winglets type wing tips in a horizontal flight are recorded. From these equations it is obtained that, in the direction of motion, the longitudinal component of the vector of the total aerodynamic force of the tips reduces the inductive drag of the wing (increases the thrust of the engines), the vertical component is added to the lifting force of the wing and increases it, and the lateral components, due to the symmetry of the wing, are in balancing state. It is shown that the lifting force created by the «AT winglets» type wingtips is larger compared to the same ones created by the upper wingtips. An important consequence is that the distribution of the lifting force over the wing span with double tips is more uniform in comparison with the wing without a wingtip or with a wing with the upper wingtip. The expression for the effective aspect ratio of the wing with the tips is defined, which is greater than the inductive drag of the wing without the tip. An important consequence is obtained: at the constant weight of the aircraft , the product of effective wing aspect ratio with the «AT winglets» wingtips and only the upper wingtips by their coefficients of inductive resistance is equal to the product of the wing aspect ratio without a tip by its coefficient of inductive drag.

Keywords: AT winglet, a mathematical model, the aerodynamic characteristics, effective aspect ratio, steady motion.

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ВПЛИВ ЗАКІНЦІВОК «AT WINGLETS» НА АЕРОДИНАМІЧНІ ХАРАКТЕРИСТИКИ КРИЛА

У статті методом системи рівнянь установленого горизонтального польоту досліджується вплив закінцівок «AT winglets» на аеродинамічні характеристики крила. Визначається повна аеродинамічна сила, що створюється всіма чотирма частинами крила. Вектор повної аеродинамічної сили закінцівок уявляється у вигляді компонент у зв'язаній системі координат. Наводяться рівняння рівноваги установленого руху літака із закінцівками крила типу «AT winglets» у горизонтальному польоті. З цих рівнянь отримано, що прямуючи у бік руху, поздовжня компонента вектора повної аеродинамічної сили закінцівок зменшує індуктивний опір крила (збільшує силу тяги двигунів), вертикальна компонента додається до підйомної сили крила і збільшує її, а бічні компоненти, за рахунок симетрії крила, знаходяться в балансувальному стані. Показано, що частка підйомної сили, яка створена закінцівками типу «AT winglets», більша у порівнянні з тими ж, які створені верхніми закінцівками. Зроблено важливе слідство про те, що розподіл підйомної сили за розмахом крила з подвійними закінцівками більш рівномірний порівняно з крилом без закінцівок або ж з крилом з верхніми закінцівками. Визначається вираз ефективного подовження крила з закінцівками, яке більше індуктивного опору крила без закінцівок. Показано, що за умови сталості ваги літака, множення ефективного подовження крила з закінцівками і коефіцієнта індуктивного опору крила є величина постійна.

Ключові слова: AT winglets, математична модель, аеродинамічні характеристики, ефективне подовження, індуктивний опір, усталений рух.

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ВЛИЯНИЕ ЗАКОНЦОВОК «AT WINGLETS» НА АЭРОДИНАМИЧЕСКИЕ ХАРАКТЕРИСТИКИ КРЫЛА

В статье методом системы уравнений установившегося горизонтального полета исследуется влияние законцовок «AT winglets» на аэродинамические характеристики крыла. Определяется полная аэродинамическая сила, созданная всеми четырьмя частями законцовок. Вектор полной аэродинамической силы законцовок представляется в виде компонент в связанной системе координат. Приводятся уравнения равновесия установившегося движения самолета с законцовками крыла типа «AT winglets» в горизонтальном полете. Из этих уравнений получено, что направляясь в сторону движения, продольная компонента вектора полной аэродинамической силы законцовок уменьшает индуктивное сопротивления крыла (увеличивает силу тяги двигателей), вертикальная компонента прибавляется к подъемной силе крыла и увеличивает ее, а боковые компоненты, за счет симметрии крыла, находятся в балансировочном состоянии. Показано, что доля подъемной силы, созданная законцовками типа «AT winglets» больше по сравнению с теми же, созданными верхними законцовками. Сделано важное следствие о том, что распределение подъемной силы по размаху крыла с двойными законцовками более равномерное по сравнению с крылом без законцовок или же с крылом с верхними законцовками. Определяется выражение эффективного удлинения крыла с законцовками, которое больше индуктивного сопротивления крыла без законцовок. Показано, что при условии постоянства веса самолета, произведение эффективного удлинения крыла с законцовками и коэффициента индуктивного сопротивления крыла есть величина постоянная.

Ключевые слова: AT winglets, математическая модель, аэродинамические характеристики, эффективное удлинение, индуктивное сопротивление, установившееся движение.

Стаття надійшла до редакції 03.09.2018 р.
Прийнято до друку 10.09.2018 р.
Рецензент – д. т. н., проф. Абдуллаєв П. Ш.