UDC 629.735
DOI: 10.18372/2310-5461.38.12840

***Malikov E.*,** Doctoral candidate,
National Aviation Academy, Az 1045, Baku, Bina 25,
orcid.org/0000-0001-5169-4813
e-mail: emin757-200@rambler.ru

FLUCTUATIONS OF AN AIRPLANE WITH AERODYNAMIC WINGTIP «AT WINGLETS»

Introduction

The influence of two-way wing tips (AT winglets) on the lateral oscillatory stability of the aircraft is investigated using the method of equations of steady horizontal flight. A related coordinate system with the center at the center of mass of the aircraft is considered, where the longitudinal axis Ox coincides with the axis of symmetry of the aircraft, the normal axis Oy is directed upwards, and the transverse axis Oz is directed along swing to the right wing.

The lateral oscillatory motion of an airplane is manifested by the action of a perturbation along the slip angle (for example, after a lateral gust of wind) if the aircraft has increased stability with respect to the Ox axis above the stability with respect to the Oy axis [1–3, 7, 8]. In this case, the exit from the roll occurs quickly, and the slip has not yet been eliminated. Under the action of the restoring moment, which still continues to operate, the aircraft will tilt already in the opposite direction, and so on. As a result, the aircraft oscillates about the longitudinal axis. Such a state is called oscillatory instability (the "Dutch step"). Of particular importance here is the requirements for the initial (unperturbed) flight regime.

The case where the initial regime corresponds to a flight in a vertical plane without roll ($\gamma_0 = 0$) and slip ($\beta_0 = 0$) $\omega_{x0} = \omega_{y0} = 0$ is generally accepted is that for a zero index, constants are denoted equal to the values of the corresponding quantities at the beginning of the disturbed motion.

We note that the thesis of this article was published in [5].

Formulation of the problem

The approximate equation of motion of an airplane, with a wing without wingtips, with respect to the slip angle, is written in the form [2]

$$\ddot{\beta} - (\bar{M}_{\psi}^{\Omega_\psi} + \bar{k}\bar{M}_{\psi}^{\Omega_\gamma})\dot{\beta} - \sigma\beta = 0\,, \qquad (1)$$

where the ordinal denotes ordinary derivatives with respect to time from the slip angle, and the following notation is introduced: damping moments in a semi-coupled coordinate system:

$$\bar{M}_{\psi}^{\Omega_\psi} = \bar{M}_{y}^{\omega_y}\cos^2\alpha +$$
$$+ (\bar{M}_{y}^{\omega_x} + \bar{M}_{x}^{\omega_y})\sin\alpha\cos\alpha + \bar{M}_{x}^{\omega_x}\sin^2\alpha,$$
$$\bar{M}_{\psi}^{\Omega_\gamma} = \bar{M}_{y}^{\omega_x}\cos^2\alpha +$$
$$+ (\bar{M}_{x}^{\omega_x} - \bar{M}_{y}^{\omega_y})\sin\alpha\cos\alpha - \bar{M}_{x}^{\omega_y}\sin^2\alpha,$$

angular rotation speed

$$\Omega_\psi = \omega_y\cos\alpha + \omega_x\sin\alpha\,;$$
$$\Omega_\gamma = \omega_x\cos\alpha - \omega_y\sin\alpha\,,$$

coefficient of dynamic stability of the aircraft in yaw $\sigma = \frac{qSl}{I_y}\left(m_y^\beta\cos\alpha + \frac{I_y}{I_x}m_x^\beta\sin\alpha\right)$, I_x and I_y are the moments of inertia, γ is angle of heel, ψ is yaw angle, ω_x and ω_y are angular velocity of rotation around the corresponding axes, $\bar{k} = \Omega_\gamma / \Omega_\psi$, q is dynamic head, S is wing area, l is wingspan, $\bar{M}_{x,y} = M_{x,y} / I_{x,y}$.

We note that the coefficients of the differential equations in the flight dynamics problems are strictly constant if the rectilinear steady-state motion of the aircraft is considered as the initial motion. In addition, the moments of inertia of the aircraft, as a whole as a solid body, can be assumed constant when analyzing the angular motion of an aircraft. This approach greatly simplifies the solution of problems of the dynamics of the angular motion of the aircraft and the analysis of the solutions obtained.

To obtain an approximate equation for the lateral oscillatory motion of an aircraft with a wing with wingtips, we shall use the expressions of the roll and yaw moment coefficients derived by us [4]

$$\hat{m}_x = m_x - \beta(C^{\text{в}}\cos\varphi + C^{\text{н}}\cos\varphi)\tan\chi,$$

$$\hat{m}_y = m_y - \beta\Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z - 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+\left(C^{\text{н}}\cos\varphi\cos\delta - 2\bar{x}^{\text{н}}\sin\delta\right)\Big],$$

for the front location of the pressure centers of the tips

$$\hat{m}_y = m_y - \beta\Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z + 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+ C^{\text{н}}\cos\varphi\left(\cos\delta + 2\bar{x}^{н}\sin\delta\right)\Big],$$

for the rear location of the centers of the wingtips pressures.

The partial derivatives of these quantities with respect to the slip angle are easily determined

$$\hat{m}_x^{\beta} = m_x^{\beta} - (C^{\text{в}}\cos\varphi + C^{\text{н}}\cos\varphi)\tan\chi;$$

$$\hat{m}_y^{\beta} = m_y^{\beta} - \Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z - 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+ C^{\text{н}}\cos\varphi\left(\cos\delta - 2\bar{x}^{\text{н}}\sin\delta\right)\Big],$$

(for the front location of pressure centers),

$$\hat{m}_y^{\beta} = m_y^{\beta} - \Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z + 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+ C^{\text{н}}\cos\varphi\left(\cos\delta + 2\bar{x}^{\text{н}}\sin\delta\right)\Big],$$

(for the rear location of pressure centers).

For brevity, the expressions relating to the wingtips are denoted by the following symbols

$$\hat{m}_{xz}^{\beta} = -\left(C^{\text{в}}\cos\varphi + C^{\text{н}}\cos\varphi\right)\tan\chi;$$

$$\hat{m}_{yz1}^{\beta} = -\Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z - 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+ C^{\text{н}}\cos\varphi\left(\cos\delta - 2\bar{x}^{\text{н}}\sin\delta\right)\Big];$$

$$\hat{m}_{yz2}^{\beta} = -\Big[C^{\text{в}}\cos\varphi\left(\cos\alpha_z + 2\bar{x}^{\text{в}}\sin\alpha_z\right) + $$
$$+ C^{\text{н}}\cos\varphi\left(\cos\delta + 2\bar{x}^{\text{н}}\sin\delta\right)\Big].$$

Then the derivatives of the moments of roll and yaw in the case of a wing with the wingtips "AT winglets" will be written in the following form

$$\hat{m}_x^{\beta} = m_x^{\beta} - \hat{m}_{xz}^{\beta};$$

$$\hat{m}_y^{\beta} = m_y^{\beta} - \hat{m}_{yz1}^{\beta};$$

$$\hat{m}_y^{\beta} = m_y^{\beta} - \hat{m}_{yz2}^{\beta}.$$

We will assume that the moments of inertia are calculated for an aircraft with a wing with tips. Then, if in all the above expressions we change the moments and their derivatives with the same quantities supplied with the cap, we obtain the equation of the lateral oscillation of the aircraft along the bank for the aircraft with the wing with the wingtips. Suppose that this is done and all of the above equations refer to an aircraft with a wing with wingtips.

Equation (1) is an ordinary differential equation of the second order with constant coefficients $\tau = \frac{1}{2}(\overline{M}_{\psi}^{\Omega_{\psi}} + \bar{k}\overline{M}_{\psi}^{\Omega_{\gamma}})$. As we shall see later, here the coefficient determines the decay of the solution, and the coefficient of dynamic stability of the aircraft along yaw is related to the frequency of oscillations.

The solution of the problem

We seek the solution of (1) by the Euler method, according to which the solution is sought in this form $\beta = e^{kt}$. Substituting this into the equation, we obtain

$$(k^2 - 2\hat{\tau}k - \hat{\sigma})e^{kt} = 0.$$

To satisfy this equality, there must be

$$k^2 - 2\hat{\tau}k - \hat{\sigma} = 0.$$

This is the characteristic quadratic equation of the differential equation (1). The solutions of the quadratic equation are the quantities $k_{1,2} = \hat{\tau} \pm \sqrt{\hat{\tau}^2 + \hat{\sigma}}$.

Then the general solution of the differential equation (1) can be written in the form

$$\beta(t) = C_1 e^{(\hat{\tau}+\sqrt{\hat{\tau}^2+\hat{\sigma}})t} + C_2 e^{(\hat{\tau}-\sqrt{\hat{\tau}^2+\hat{\sigma}})t}.$$

Here C_1 and C_2 arbitrary constants of integration, which are determined from the initial conditions.

We analyze this solution: if at least one of the numbers $\hat{\tau}$ is either positive $\sqrt{\tau^2 + \sigma}$, then the solution is unstable, it tends exponentially to infinity. Hence we immediately conclude: that for a stable motion the number $\hat{\tau}$ must be negative. If the root $\sqrt{\tau^2 + \sigma}$ is positive or negative, then the solution is also unstable, either the first or the second exponential tends to infinity. It follows that this root can not be a real number.

However, if we take the root $\sqrt{\tau^2 + \sigma}$ as a purely imaginary number, i.e. $\hat{\tau}^2 + \hat{\sigma} < 0$, then it is possible to choose a stable solution describing the oscillatory motion of the aircraft. So, let's say a negative number $\sigma < -\tau^2$. Then we represent the root in the form

$$\sqrt{\tau^2 + \sigma} = i\sqrt{|\sigma| - \tau^2},$$

and write the general solution again $\beta(t)=e^{-|\tau|t}[C_1\cos(t\sqrt{|\sigma|-\tau^2})+C_2\sin(t\sqrt{|\sigma|-\tau^2})]$.

This solution can be presented as follows $\beta(t)=Ce^{-|\tau|t}\cos(t\sqrt{|\sigma|-\tau^2}+\vartheta)$, which describes a damped oscillatory motion. Here $Ce^{-|\tau|t}$ is the amplitude of the oscillations, $|\tau|$ — is the attenuation coefficient, $\sqrt{|\sigma|-\tau^2}$ is the oscillation frequency, and ϑ is the initial phase. In solving non-stationary problems, the constants of integration u are determined from the initial conditions.

If the amplitude of the oscillation decreases all the time, the oscillations are called damped, if the amplitude of the oscillations is constant, then they are harmonic, and with an increasing amplitude, they are increasing (divergent). If the damping forces prevail over the forces causing the oscillations, then the oscillations decay, and otherwise the amplitude of the oscillations increases, and may lead to the destruction of the structure.

Thus, the following two conditions

$$\hat{\tau}<0 \quad \text{and} \quad \hat{\sigma}<-\hat{\tau}^2$$

provide a stable lateral oscillation of the aircraft, with aerodynamic wingtips, along the roll around the unperturbed balanced flight regime. These conditions are more stringent than those for lateral static stability, which require the fulfillment of the inequalities $\hat{m}_x^{\beta}<0$ and $\hat{m}_y^{\beta}<0$ the positivity of the relation $\varepsilon=\dfrac{\hat{m}_x^{\beta}}{\hat{m}_y^{\beta}}>0$, since the condition $\hat{\sigma}<-\hat{\tau}^2$ requires not only the negativity of these derivatives, but even that the linear combination of these quantities $\hat{\sigma}$ must be less than some negative number $-\hat{\tau}^2$.

Using the above expressions $\hat{m}_x^{\beta}$ and $\hat{m}_y^{\beta}$ calculate the magnitude modulus $\hat{\sigma}$:

$$|\hat{\sigma}|=\frac{qSl}{I_y}\left(\sqrt{\left(m_y^{\beta}\right)^2+\left(\hat{m}_{yz1}^{\beta}\right)^2}\cos\alpha+\right.$$
$$\left.+\frac{I_y}{I_x}\sqrt{\left(m_x^{\beta}\right)^2+\left(\hat{m}_{xz}^{\beta}\right)^2}\sin\alpha\right),$$

for the front location of the centers of the pressure of the tips, and

$$|\hat{\sigma}|=\frac{qSl}{I_y}\left(\sqrt{\left(m_y^{\beta}\right)^2+\left(\hat{m}_{yz2}^{\beta}\right)^2}\cos\alpha+\right.$$
$$\left.+\frac{I_y}{I_x}\sqrt{\left(m_x^{\beta}\right)^2+\left(\hat{m}_{xz}^{\beta}\right)^2}\sin\alpha\right),$$

for the rear location of the centers of the wingtips pressures.

In the most simplified case for the damping coefficient we obtain

$$\hat{\tau}=\overline{M}_y^{\omega_y}\cos^2\alpha=\hat{m}_y^{\omega_y}\frac{qSl}{I_y}\cos^2\alpha$$

where

$$\hat{m}_y^{\omega_y}=m_y^{\omega_y}-\hat{m}_{yz}^{\omega_y},$$

$m_y^{\omega_y}$ — is the derivative of the yaw moment coefficient for the component of the ω_y angular velocity vector of the aircraft as a rigid body around the center of mass, for an aircraft with a wing without wingtips; for the front location of the centers of the pressure of the tips

$$\hat{m}_{yz1}^{\omega_y}=-\beta\left[\frac{\partial C^{\text{в}}}{\partial\omega_y}\cos\varphi\left(\cos\alpha_z-2\overline{x}^{\text{в}}\sin\alpha_z\right)+\right.$$
$$\left.+\frac{\partial C^{\text{н}}}{\partial\omega_y}C^{\text{н}}\cos\varphi\left(\cos\delta-2\overline{x}^{\text{н}}\sin\delta\right)\right],$$

for the rear location of the centers of the pressure of the tips

$$\hat{m}_{yz2}^{\omega_y}=-\beta\left[\frac{\partial C^{\text{в}}}{\partial\omega_y}\cos\varphi\left(\cos\alpha_z+2\overline{x}^{\text{в}}\sin\alpha_z\right)+\right.$$
$$\left.+\frac{\partial C^{\text{н}}}{\partial\omega_y}C^{\text{н}}\cos\varphi\left(\cos\delta+2\overline{x}^{\text{н}}\sin\delta\right)\right].$$

In these relations, the derivatives and sufficiently small numbers. Since the damping property of an airplane is sufficiently small in comparison with the oscillatory one, we can write:

$$|\hat{\sigma}|>>\hat{\tau}^2.$$

From the last two formulas it follows that the wing tips increase the frequency of free lateral oscillations of the aircraft along the slip angle. Even in the case of an aircraft with a wing without winglets $m_x^{\beta}=m_y^{\beta}=0$, due to wing tips, the aircraft makes lateral oscillations with a frequency, the square of which is defined by formula:

$$\omega^2=\frac{qSl}{I_y}\left(\left|\hat{m}_{yzi}^{\beta}\right|\cos\alpha+\frac{I_y}{I_x}\left|\hat{m}_{xz}^{\beta}\right|\sin\alpha\right)-\hat{\tau}^2.$$

where either $i=1$ or 2 wingtip on the front or rear position of the pressure centers of the wingtips relative to the coordinate plane *Oyz*.

Conclusion

The lateral oscillatory stability of a plane with a wing with wingtips of the "AT winglets" type is investigated in the article. The total aerodynamic forces and moments created by the wingtips correspond to the established horizontal flight. The coefficients of roll and yaw moments, as well as their slip angle derivatives for an airplane with a wing with tips, are determined. Replacing these quantities with the corresponding values for an aircraft with a wing without wingtips, the equation of lateral oscillation of the aircraft along the slip angle was obtained. The general solution of this equation is written. In the case of damped oscillatory motion, the attenuation coefficient and the oscillation frequency are determined, which are increased by the influence of wing tips.

REFERENCE

1. **Byshgens G. S**. Dynamics of longitudinal and lateral movement / G. S. Byshgens, / R.V.M : Mechanical Engineering, 1979.
2. **Efremov A. V.** Flight dynamics / A. V. Efremov, V. F. Zakharchenko, V. N, Ovcharenko, V. L. Sukhanov, U. F. Shelyukhin, A. S. Ustinov. — M., Mechanical Engineering, 2011. — 776 p.
3. **Dobrolensky U. P.** The dynamics of flight in a troubled atmosphere/ U. P. Dobrolensky. — M. : Mechanical Engineering, 1969.
4. **Iliyasov M.** Influence of AT winglets wingtips on the inductive reactance of the wing / M. Iliyasov, E. Malikov. Bulletin of Science and Practice, 2017(8), 157–166 p.
5. **Ilyasov M.** Lateral airplane oscillations with wing tips of the “at winglet” type / M. Ilyasov, E. Malikov / The 15th International Conference “Aviation and Cosmonautics – 2016”. — M. : MAI, November 14–18, 2016. Abstracts. — P. 56–57. Type-I “Luxor”, 2016. — 738 p.
6. **Jafarzade R. M.** Calculation of the effect of the wingtips of a rectangular wing of a finite span on its inductive resistance, by the method of distributed vortices / R. M. Jafarzade, M. Kh. Ilyasov, J. N. Guseinli. Scientific Proceedings of the National Aviation Academy, No. 1, 2015. — P. 63–77.
7. **Arzhanov N. S.**, **Maltsev V. N.** Aerodynamics, Moscow: EKOLIT, 2011. – 483 p.
8. **Krasnov N. F**. Aerodynamics. Part 1. Moscow: Book house “LIBROKOM”, 2010. — 496 p.

Malikov E.

FLUCTUATIONS OF AN AIRPLANE WITH AERODYNAMIC WINGTIP “AT WINGLETS”

By the method of equations of steady rectangular horizontal flight the effect of the aerodynamic wingtips "AT winglets" on the lateral oscillating stability of the aircraft is investigated. This method calculates the expressions for the coefficients of the roll and yaw moments for a wing with wingtips, which are used here to obtain an approximate equation for the lateral oscillatory motion of the aircraft. The forward and backward arrangements of the centers of pressure of the tips with respect to the vertical plane passing through the lateral coordinate axis are considered. The derivatives of the coefficients of the roll and yaw moment coefficients for the slip angle for the wing with the tips that enter the equation for the lateral oscillation of the aircraft are determined. The correspondence between the derivatives of the coefficients of the moments of roll and yaw for the wings without the tips and with the tips for the slip angle is established. Using this correspondence, from the known differential equation of the lateral oscillation of an aircraft with a wing without an wingtip, the corresponding equation of lateral oscillation of the airplane is obtained for the aircraft for aircraft with a wing with wingtips. A study is made of the solution of this equation in terms of stability, depwingtip on the ratio of the attenuation coefficient τ *and the coefficient of dynamic stability of the aircraft in yaw* σ*. The study showed that for a stable lateral oscillation of an airplane, the numbers* τ *and* σ *must be negative and, in addition, still should be less than* $-\tau^2$ *a negative number, i.e* $\sigma < -\tau^2$*. It is shown that, in the case of steady lateral oscillations, the presence of wing tips leads to an increase in the oscillation frequency and amplitude decay factor as compared to the wing without the wingtips.*

Keywords: lateral oscillation, AT winglets, mathematical model, frequency, attenuation coefficient.

Меліков Э. Т.

КОЛИВАННЯ ЛІТАКА З АЕРОДИНАМІЧНИМИ ЗАКІНЦІВКАМИ «AT WINGLETS»

Методом рівнянь встановленого горизонтального польоту досліджується вплив аеродинамічних закінцівок крила «AT winglets» на бічну стійкість літака коливанням. Цим методом визначені вирази коефіцієнтів моментів крену і рискання для крила із закінцівками, які використовуються тут для отримання наближеного рівняння бічного коливання літака. Розглянуті переднє і заднє розташування центрів тисків закінцівок відносно вертикальної площини, що проходить через бічну координатну вісь. Визначені похідні коефіцієнтів моментів крену і рискання по куту ковзання для крила із закінцівками, які входять в рівняння бічного коливання літака. Встановлена залежність похідних від коефіцієнтів моментів крену і рискання для крил без закінцівок та із закінцівками по куту ковзання. Використовуючи цю залежність, із відомого диференційного рівняння бічного коливання літака з крилом без закінцівок, отримано відповідне рівняння бічного коливання літака за креном для літака з крилом із закінцівками. Досліджено рішення цього рівняння за стійкістю залежно від відношення коефіцієнта загасання і коефіцієнта динамічної стійкості літака за рисканням. У результаті *отримано, що для стійкого бічного коливання літака числа* τ *і* σ *мають бути від'ємними і, крім того,* σ *ще має бути менше від'ємного числа* $-\tau^2$ *тобто* $\sigma < -\tau^2$. *Показано, що у разі бічного встановленого коливання, наявність закінцівок крила призводить до збільшення частоти коливань і коефіцієнта загасання амплітуди в порівнянні з крилом без закінцівок.*

Ключові слова: бічне коливання, AT winglets, математична модель, частота, коефіцієнт загасання.

Меликов Э.Т.

КОЛЕБАНИЯ САМОЛЕТА С АЭРОДИНАМИЧЕСКИМИ ЗАКОНЦОВКАМИ «AT WINGLETS»

Методом уравнений установившегося прямоугольного горизонтального полета исследуется влияние аэродинамических законцовок крыла "AT winglets" на боковую устойчивость самолета колебаниям. Этим методом вычислены выражения коэффициентов моментов крена и рыскания для крыла с законцовками, которые используются здесь для получения приближенного уравнения бокового колебания самолета. Рассмотрены переднее и заднее расположения центров давлений законцовок относительно вертикальной плоскости, проходящей через боковую координатную ось. Определены производные коэффициентов моментов крена и рыскания по углу скольжения для крыла с законцовками, которые входят в уравнение бокового колебания самолета. Установлена зависимость производных от коэффициентов моментов крена и рыскания для крыльев без законцовок и с законцовками по углу скольжения. Используя эту зависимость, из известного дифференциального уравнения бокового колебания самолета с крылом без законцовок получено соответствующее уравнение бокового колебания самолета по крену для самолета с крылом с законцовками. Исследовано решение этого уравнения по устойчивости в зависимости от отношения коэффициента затухания τ *и коэффициента динамической устойчивости самолета по рысканию* σ. *В итоге получено, что для устойчивого бокового колебания самолета числа* τ *и* σ *должны быть отрицательными и, кроме того,* σ *еще должно быть меньше отрицательного числа* $-\tau^2$, *т.е.* $\sigma < -\tau^2$. *Показано, что в случае установившегося бокового колебания наличие законцовок крыла приводит к увеличению частоты колебаний и коэффициента затухания амплитуды по сравнению с крылом без законцовок.*

Ключевые слова: боковое колебание, AT winglets, математическая модель, частота, коэффициент затухания.

Стаття надійшла до редакції 16.05.2018 р.
Прийнято до друку 04.06.2018 р.
Рецензент — д-р фіз.-мат. наук, проф. Талибли Л. Х.