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ARCHED GEARS TEETH GEOMETRY WITHIN REFERENCE PROFILE SHIFT

Teeth geometry of cylindrical arched gears generated by rack tool which is profiled in normal cross-section and in longitude direction by arbitrary curves taking into account reference profile shift has been researched. Meshing of arched rack's and gear's teeth within reference profile shift considering as an analogue of the teeth generating process by rack tool by means of generating method. The essence of this method is that teeth surfaces of the rack tool envelop the gear's teeth surfaces. The equations of teeth surfaces coordinates were obtained in $X_1Y_1Z_1$ and $X_2Y_2Z_2$ coordinate systems connected with pinion and gear respectively. In common case these equations are equations of helical surfaces of variable height. Using these equations, the surfaces coordinates of pinion convex tooth side and gear concave tooth side can be determined. These equations can be also used for the determination of meshing field borders, appropriate to the tops of the pinion and gear teeth. The equations are applicable both for symmetrical and asymmetrical relatively to XOZ plane arched teeth. Obtained results can be used for the determination of indices of loading ability and other meshing characteristics of arched gears with generalized teeth geometry within reference profile shift.

Key words*: reference profile, generating surface, arched gears, reference profile shift, basic rack.*

Introduction. Modern terms of market economy raise the tasks of quality increase, reliability and durability of machines and mechanisms in front of enterprises of machine-building. Gear transmissions take one of the leading places among the output of machine-building branch because they practically are the constituent part of drives of all machines. That is why the improving of qualitative indices of gears by means of the teeth geometry optimization is an actual scientific and technical task.

Geometrical and kinematic indices of loading ability in dependence of the unknown functions, determining rack type cutting tool geometry, are necessary within the synthesis of gears teeth geometry [1]. General issues of plane meshing geometry are considered in works [2, 3, 4]. Though obtained in it results and relations do not allow to synthesize cylindrical gears' teeth geometry according to loading ability indices. In works [3, 4] geometry of cylindrical arched gears, formed by generalized generating surface, has been researched. Though, research data cannot be applied for arched transmission within the reference profile shift.

1. Generalized surface of arched teeth rack tool within the reference profile shift. It can be considered in coordinate system $X_nY_nZ_n$ (Fig. 1). Curve $\overline{r_0}(\mu)$ determining generating surface's longitude tooth form is given in coordinate system $X_nY_nZ_n$, and rack teeth profile is outlined by generalized reference profile in normal cross-section.

Let's present the equation of rack tool's tooth surface (generating surface) in connected with it coordinate system in the form of vector [1] in order to synthesize cylindrical gears' geometry by given indices of loading ability

$$\bar{r}_n = \bar{r}_0(\mu) + \bar{b}_0 f_1(\lambda) + \bar{n}_0 f_2(\lambda), \tag{1}$$

where $\bar{r}_0(\mu)$ is a vector, determining longitude generating surface tooth form; $\bar{b}_0$, $\bar{n}_0$ are unit binormal and normal vectors of $\bar{r}_0(\mu)$ curve; $f_1(\lambda)$, $f_2(\lambda)$ are functions, determining reference profile geometry of cutting rack tool (generating surface) in normal cross-section ; λ, μ are independent parameters.

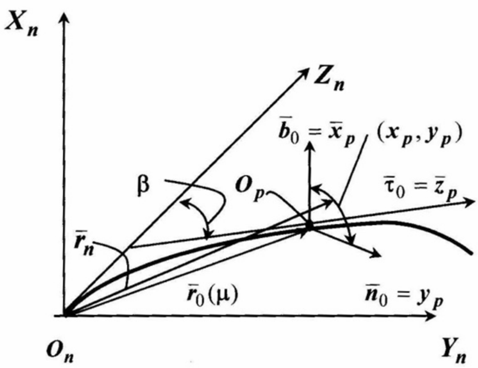

Fig. 1. Parameters of arched teeth generating surface (rack tool teeth surface)

Using the results of the works [3, 4, 5], the equations of arched rack tool teeth surface can be obtained within $m = 1$ mm in the following form (reference profile is placed above pitch line):

- convex side of arched teeth (Fig. 2)

$$x_n = f_1 + \xi, \quad y_n = y_0 + f_2 \cos\beta, \quad z_n = z_0 - f_2 \sin\beta; \tag{2}$$

- concave side of arched teeth (Fig. 3)

$$x_n = f_1 + \xi, \quad y_n = y_0 - f_2 \cos\beta + 0{,}5\pi, \quad z_n = z_0 + f_2 \sin\beta, \tag{3}$$

where y_0, z_0 are the vector's $\bar{r}_0(\mu)$ projections on coordinate axis; β is the corner of teeth incline (the corner between axis $O_n Z_n$ and tangent to curve $\bar{r}_0(\mu)$ (Fig. 1), determined from relations: $\sin\beta = \dot{y}_0 \big/ \sqrt{(\dot{y}_0^2 + \dot{z}_0^2)}$, $\cos\beta = \dot{z}_0 \big/ \sqrt{(\dot{y}_0^2 + \dot{z}_0^2)}$; $\dot{y}_0$, $\dot{z}_0$ are the y_0, z_0 function's derivatives with respect to μ; ξ – reference profile shift.

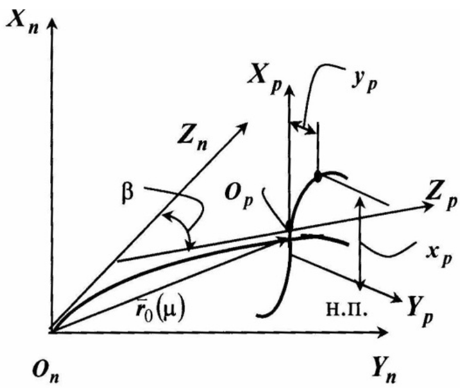

Fig. 2. Convex side of arched rack tool's tooth

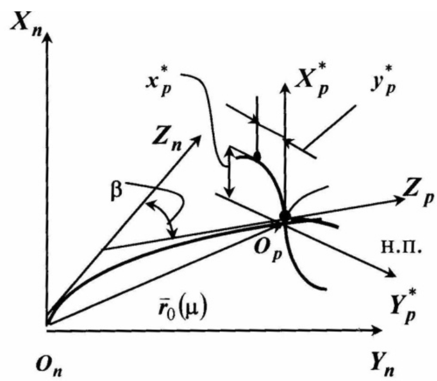

Fig. 3. Concave side of arched rack tool's tooth

2. Meshing of arched teeth rack tool with gears within reference profile shift. It is an analogue of the process of teeth cutting with rack tool by means of generating method. Surfaces of gear's teeth within meshing with rack are enveloping of teeth surfaces of the last one.

The scheme of rack tool meshing with gears is presented at Fig. 4.

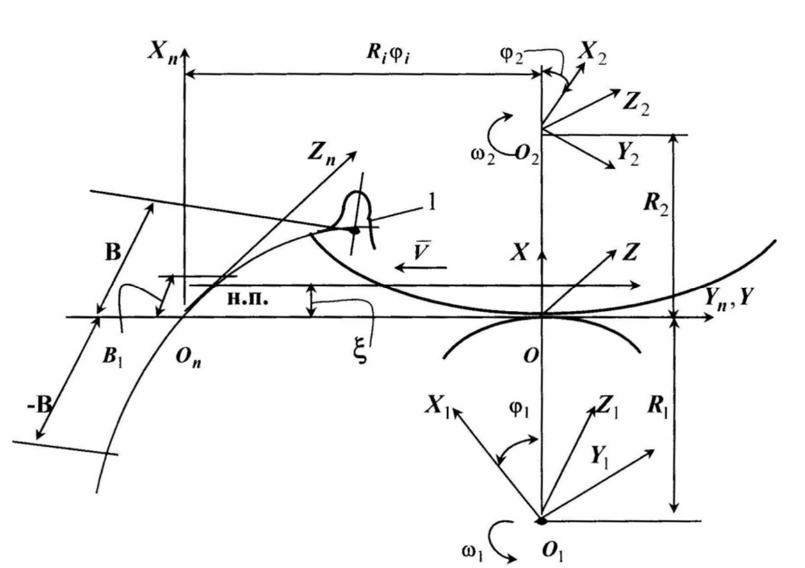

Fig. 4 Scheme of rack tool meshing with pinion and gear (1 is the profile of rack arched tooth in normal cross-section)

Here O_1 and O_2 are axes of pinion and gear, R_1, R_2 are radiuses of initial cylinders of pinion and gear, "н.п." is the pitch plane; $X_nY_nZ_n$ is the coordinate system, connected with rod; $X_1Y_1Z_1$ is the coordinate system, connected with pinion (less gear of teeth pair); $X_2Y_2Z_2$ is the coordinate system, connected with gear (larger gear of teeth pair); coordinate system, connected with gear (large gear of tooth pair); XYZ is fixed coordinate system. Plane $Y_nO_nZ_n$ is the pitch plane of the rack; the YOZ plane coincides with the pitch plane; axes O_1Z_1, O_2Z_2 are directed along gear axes; OZ axis intersects the XOZ plane at the meshing pitch point; φ_1, φ_2 – the pinion's and gear's turning angles ($\varphi_1 = u\varphi_2$, where u – transfer number of tooth transmission); $\bar{V}$ – linear rack speed ($V = \omega_1 R_1 = \omega_2 R_2$); ω_1, ω_2 – corner speeds of pinion and gear.

Within gear rotation (pinion rotation) on the corner φ_i ($i = 1$ for the pinion and $i = 2$ for the gear) rack will move on the distance $R_i\varphi_i$.

Equations of surfaces of arched rack teeth in unmovable coordinate system XYZ have the following form within $m = 1$ mm:

- for left side of reference profile (convex side of arched teeth) using eq. (2)

$$x = f_1 + \xi, \quad y = y_0 + f_2 \cos\beta - R_i\varphi_i, \quad z = z_0 - f_2 \sin\beta; \tag{4}$$

- for right side of reference profile (concave side of arched teeth) using eq. (3)

$$x^* = f_1 + \xi, \quad y^* = y_0 - f_2 \cos\beta + \pi/2 - R_i\varphi_i, \quad z^* = z_0 + f_2 \sin\beta. \tag{5}$$

Equations of rack meshing with gears we can write down in the form [2]:

- for rack and pinion meshing

$$F_1^* = \bar{e} \cdot \bar{V}^{p1}; \tag{6}$$

- for rack and gear meshing

$$F_2^* = \bar{e} \cdot \bar{V}^{p2}, \tag{7}$$

where $\bar{V}^{p1}$, $\bar{V}^{p2}$ are the relative speed vectors for rack meshing with pinion and gear; $\bar{e}$ – unit normal vector with coordinates within $n = \sqrt{(f_1')^2 + (f_2')^2}$:

$$e_{xn} = \pm f_2' n^{-1},\ e_{yn} = -f_1' n^{-1} \cos\beta,\ e_{zn} = f_1' n^{-1} \sin\beta . \tag{8}$$

Relative speed vectors (within $\omega_1 = 1$ rad/s, $\omega_2 = 1$ rad/s) for rack meshing with pinion and gear are determined:

- using eq. (4) (convex side of pinion teeth, concave side of gear teeth)

$$\begin{aligned} \bar{V}^{p1} &= -(y_0 + f_2 \cos\beta - R_1\varphi_1)\bar{i} + (f_1 + \xi)\bar{j} + o \cdot \bar{k}, \\ \bar{V}^{p2} &= (y_0 + f_2 \cos\beta - R_2\varphi_2)i - (f_1 + \xi)\bar{j} + o \cdot \bar{k}; \end{aligned} \tag{9}$$

- using eq. (5) (concave side of pinion teeth, convex side of gear teeth)

$$\begin{aligned} \bar{V}^{p1} &= -(y_0 - f_2 \cos\beta + \pi/2 - R_1\varphi_1)\bar{i} + (f_1 + \xi)\bar{j} + o \cdot \bar{k}, \\ \bar{V}^{p2} &= (y_0 - f_2 \cos\beta + \pi/2 - R_2\varphi_2)\bar{i} - (f_1 + \xi)\bar{j} + o \cdot \bar{k}, \end{aligned} \tag{10}$$

where $\bar{i}$, $\bar{j}$, $\bar{k}$ are unit vectors of fixed coordinate system.

It is necessary to multiply ω_1 and ω_2 appropriately for identifying true values of relative speed vector's projections $\bar{V}^{p1}$ and $\bar{V}^{p2}$.

Let's present the equations of meshing (6) and (7) with the account of meanings of vector $\bar{e}$ projections in the following way:

- within rack meshing with pinion and gear (convex side of pinion teeth, concave side of gear teeth)

$$\begin{aligned} F_1^* &= \left[-(y_0 + f_2 \cos\beta - R_1\varphi_1) f_2' - (f_1 + \xi) f_1' \cos\beta \right] n^{-1} = 0, \\ F_2^* &= \left[(y_0 + f_2 \cos\beta - R_2\varphi_2) f_2' + (f_1 + \xi) f_1' \cos\beta \right] n^{-1} = 0; \end{aligned} \tag{11}$$

- within rack meshing with pinion and gear (concave side of pinion teeth, convex side of gear teeth)

$$\begin{aligned} F_1^* &= \left[-(y_0 - f_2 \cos\beta + \frac{\pi}{2} - R_1\varphi_1)\left(-f_2'\right) - f_1'(f_1 + \xi)\cos\beta \right] n^{-1} = 0, \\ F_2^* &= \left[(y_0 - f_2 \cos\beta + \frac{\pi}{2} - R_2\varphi_2) \cdot \left(-f_2'\right) + f_1'(f_1 + \xi)\cos\beta \right] n^{-1} = 0. \end{aligned} \tag{12}$$

As it follows from (9), (10), (11), (12)

$$\begin{aligned} \bar{V}^{p1} &= (\Omega_1 \cos\beta)\bar{i} + (f_1 + \xi)\bar{j} + 0 \cdot \bar{k}, \\ \bar{V}^{p2} &= (-\Omega_1 \cos\beta)\bar{i} - (f_1 + \xi)j + 0 \cdot \bar{k}, \end{aligned} \tag{13}$$

where $\Omega_1 = \dfrac{(f_1 + \xi) f_1'}{f_2'}$.

Equations (11) and (12) are additional terms for parameter λ, μ, φ_i connection. Equations of teeth surfaces for gear and rack meshing in XYZ fixed coordinate system as well as meshing of pinion and gear teeth using (11), (12) and (4), (5) can be written down:

- for meshing of convex side of pinion and concave side of gear teeth

$$x = f_1 + \xi, \quad y = -\Omega_1 \cos\beta, \quad z = z_0 - f_2 \sin\beta; \tag{14}$$

- for meshing of concave side of pinion teeth and convex – gear teeth

$$x = f_1 + \xi, \quad y = \Omega_1 \cos\beta, \quad z = z_0 + f_2 \sin\beta. \tag{15}$$

The equations (14) and (15) within $\varphi_i = const$ determine instantaneous contact lineы on the surface of action. The first two equations (14) and (15) within $z = const$ determine line of action in transversal plane. Using (14) and (15) within $\mu = const$ the equations of the line of action in tooth normal cross-section can be obtained.

If teeth surfaces of racks for cutting pinion's teeth and gear's teeth are non-congruous, then we have a case of pinion and gear point meshing. If reference profiles are non-congruous, then contact points moves from one side of the tooth to another within $\lambda = const$. If rack surfaces are non-congruous in longitude direction, then contact point moves along teeth height within $\mu = const$. We have an analogue of Novikov's meshing in the first case and localization of pinion and gear teeth contact in the second one.

3. Surface geometry of pinion and gear teeth within reference profile shift. We will obtain equations of teeth surfaces, while writing down coordinates of meshing surfaces (14) and (15) in coordinate systems $X_1Y_1Z_1$ and $X_2Y_2Z_2$ (Fig. 4), connected with pinion and gear. While making such a transition, we have:

- the equations of surfaces of convex side of pinion teeth and concave side of gear teeth using eq. (14)

$$\begin{aligned} x_1 &= (f_1 + \xi + R_1)\cos\varphi_1 + \Omega_1 \cos\beta \sin\varphi_1, \\ y_1 &= (f_1 + \xi + R_1)\sin\varphi_1 - \Omega_1 \cos\beta \cos\varphi_1, \\ z_1 &= z_0 - f_2 \sin\beta; \end{aligned} \tag{16}$$

$$\begin{aligned} x_2 &= (f_1 + \xi - R_2)\cos\varphi_2 - \Omega_1 \cos\beta \sin\varphi_2, \\ y_2 &= -(f_1 + \xi - R_2)\sin\varphi_2 - \Omega_1 \cos\beta \sin\varphi_2, \\ z_2 &= z_0 - f_2 \sin\beta; \end{aligned} \tag{17}$$

- the equations of surfaces of concave side of pinion teeth and convex side of gear teeth using eq. (15)

$$\begin{aligned} x_1 &= (f_1 + \xi + R_1)\cos\varphi_1 - \Omega_1 \cos\beta \sin\varphi_1, \\ y_1 &= (f_1 + \xi + R_1)\sin\varphi_1 + \Omega_1 \cos\beta \cos\varphi_1, \\ z_1 &= z_0 + f_2 \sin\beta; \end{aligned} \tag{18}$$

$$\begin{aligned} x_2 &= (f_1 + \xi - R_2)\cos\varphi_2 + \Omega_1 \cos\beta \sin\varphi_2, \\ y_2 &= -(f_1 + \xi - R_2)\sin\varphi_2 + \Omega_1 \cos\beta \sin\varphi_2, \\ z_2 &= z_0 + f_2 \sin\beta. \end{aligned} \tag{19}$$

In equations (16) – (19) variables λ, μ, φ_1, φ_2 are connected by relations (11), (12). Profiles of pinion and gear teeth in normal cross-section are identified using equations (16) – (19) within $\mu = const$, in transverse cross-section within $z_1 = const$, $z_2 = const$. These equations determine coordinates of instantaneous contact lines of the pinion and gear teeth working surfaces within $\varphi_1 = const$, $\varphi_2 = \varphi_1 / u$ (u is gear ratio).

The equations (16) – (19) determine within $\lambda = const$ the trace of contact point on teeth surface while it moves from one side to another. In common case equations (16) – (19) within $\lambda = const$ are essentially the equations of helical lines of variable pitch. Cylinder radiuses, on which these helical lines are placed can be determined from the first two equations (16) – (19):

$$R_{1b} = \sqrt{(f_1 + \xi + R_1)^2 + (\Omega_1 \cos\beta)^2}\,,$$
$$R_{2b} = \sqrt{(f_1 + \xi - R_2)^2 + (\Omega_1 \cos\beta)^2}\,. \qquad (20)$$

These equations can be used for the determination of meshing field borders, appropriate to the tops of pinion and gear teeth. It is necessary to take the values $R_{1b} = R_{a1}$, $R_{2b} = R_{a2}$ (R_{a1}, R_{a2} – radiuses of pinion teeth tops and gear) in (20), and the values of z_1 and z_2 from equations (16) – (19) within the determination of μ. Values of z_1 and z_2 are necessary to be taken less than gears width.

Two types of arched gears are possible. The first one is symmetric gears with arched teeth which are symmetric relatively to the plane XOZ (Fig. 4). The second one is asymmetric gears with arched teeth which working surface is located from one side of the XOZ plane (Fig. 4).

In the first case the values of z in equations (14) and (15) meet the condition $-B \le z \le B$ (B is the gear width). In the second one the condition is $B_1 \le z \le B$ (B_1 is the distance from XOZ plane to the nearest tooth face (Fig. 4).

Conclusions. Received results can be used for the determination of indices of loading ability and other meshing characteristics of cylindrical arched gears with generalized teeth geometry within reference profile shift.

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ГЕОМЕТРІЯ ЗУБЦІВ АРОЧНИХ ПЕРЕДАЧ ПРИ ЗМІЩЕНІ ВИХІДНОГО КОНТУРА

Досліджена геометрія зубців циліндричних зубчастих коліс, нарізаних рейковим інструментом, зубці яких профільовані довільними кривими в нормальному поперечному перерізі та в прямолінійному напрямку з урахуванням зміщення вихідного контуру. Зачеплення інструментальної рейки з арковими зубцями із зубчастими колесами з урахуванням зміщення вихідного контуру є аналогом процесу нарізання зубців за допомогою методу обкочування. Суть методу полягає в тому, що поверхні зубців рейкового інструменту обкочують поверхні зубців зубчастих коліс. Якщо поверхні зубців інструментальної рейки для нарізання зубчастих коліс та шестерен невідповідні, то у наявний випадок точкового зачеплення шестерні та колеса. Якщо початкові профілі несуперечливі, то контактна точка переміщується з однієї сторони зуба на іншу. Були отримані рівняння поверхонь зубців в системах координат $X_1Y_1Z_1$ і $X_2Y_2Z_2$, що пов'язані із шестірнею та колесом відповідно. Отримано рівняння поверхонь опуклої сторони зубців шестерні та увігнутої сторони зубців колеса. У загальному випадку ці рівняння є рівняннями гвинтових поверхонь змінної висоти. З використанням цих рівнянь можна визначати координати поверхонь опуклої сторони зубців шестерні та увігнутої сторони зубців колеса. Ці рівняння можуть також бути використані при визначенні границь поля зачеплення, що відповідають вершинам зубців шестерні та колеса. Рівняння можуть бути застосовані для симетричних та асиметричних відносно площини XOZ аркових зубців. Отримані результати можуть бути використані при визначенні показників навантажувальної здатності та інших характеристик циліндричних арочних передач із узагальненою геометрією зубців зі зміщенням вихідного контуру.

Ключові слова: вихідний контур; твірна поверхня; арочні передачі; зміщення вихідного контуру; зубчаста рейка.

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