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OPTIMAL CHARACTER AND DIFFERENT NATURE OF FLOWS IN LAMINAR BOUNDARY LAYERS OF INCOMPRESSIBLE FLUID FLOW

The paper presents an original approach to the study of the problem of internal friction arising from the motion of a rigid body in an incompressible fluid. This approach takes into account the spatial variability of molecular viscosity in the boundary layer region, and the solution of the problem is based on the use of an extreme for the fluid flow rate functional. The spatial variability of molecular viscosity in the boundary layer, by a well-known analogy with the theory of heat conduction, is based on the absence of a spatial isotropy of the medium. It is shown that molecular viscosity depends on the nature of the flow - on how many forces act on the fluid. So, if the flow is unsteady and non-gradient or steady and gradient, then both of these flows are subject to the action of two forces. In such flows, the molecular viscosity due to the extreme of the fluid flow rate is a constant value. It has been fond that the distribution of velocity in a gradient stationary boundary layer has a parabolic distribution law, and all existing theories are described by this law quite accurately, with an error of maximum 5%. At the same time, in a laminar non-gradient boundary layer, only the force of internal friction acts on the fluid. This causes the spatial variability of molecular viscosity: shear stress can be constant not only due to the linearity of the velocity distribution, which is not observed in the boundary layer, but also due to the variability of molecular viscosity. The resulting exponential velocity distribution in a non-gradient boundary layer is in complete agreement with those in the problems solved by Stokes, and is also confirmed experimentally. The paper also points out that the exponential law is consistent with modern data obtained by direct numerical simulation (DNS) for flows with Low Reynolds numbers – both single-phase and two-phase, in the presence of particles inside the fluid.

Keywords: *internal friction, boundary layer, incompressible flow, variable molecular viscosity, Low Reynolds numbers, analytical solutions, calculus of variations.*

Introduction. When a body moves in a fluid, two phases interact, which causes the formation of a laminar or turbulent boundary layer, a domain of fluid adjacent to the moving body. In aviation and astronautics, the characteristics of the boundary layer play an important role, since they determine the drag force in flight and are partly responsible for the lift force. When studying the turbulent boundary layer, it turned out that not only the turbulence closure problem has not yet been solved, but there is also no clear theory of the laminar boundary layer, which would be directly based on the Navier-Stokes equations [1]. This work is devoted precisely to the development of a unified approach to the description of a laminar boundary layer of an incompressible flow. Before a brief review of the existing theories, we note that in modern numerical studies, flows are not divided into laminar and turbulent, and laminar flows are classified as Low- Reynolds number ones. If for a turbulent flow it is a priori assumed that the eddy viscosity is not constant, then we assume, by analogy with the DNS, that the molecular viscosity can also be variable. This is a physical assumption based on the absence of spatial isotropy in the boundary layer. For interest in this work, we cite a well-known fact: the laminar boundary layers on a rotating disk and when a rotating fluid flows around an immobile disk are completely different. Firstly, they are described by different self-similar solutions, and secondly, the thickness of the boundary layers differs by about a factor

of two [2] (immediately after formula 11.8). Now it is easy to understand that the problem of the motion of a body in a fluid at rest at infinity and the problem of fluid flow past an immobile body also have a different nature and are irreversible. In the first case, when the fluid flow is entirely due to and determined by the velocity shear (internal friction), a non-gradient boundary layer is formed. In the second case, the fluid flows past an immobile body and there are two forces: the force of the longitudinal pressure gradient and the force of viscous shear stresses.

Problem state. Stokes [3] considered the problem of the forced motion of a fluid under the action of harmonic oscillations of a plane. An exponential dependence of the decrease in velocity with distance from the oscillating plane is obtained. Stokes, in order to convince everyone that he is right, refers to experiments, where this dependence is confirmed. This problem is presented as the first one in [3]. A quite reasonable question arises: why Stokes did not consider as the first and simplest problem the steady motion of a plane, which, according to Stokes, has a linear solution with respect to the fluid velocity. The answer is that, as is known [4], the laminar boundary layer has a nonlinear character. The exponential dependence as a good approximation for viscous and intermediate sub-layers of a turbulent boundary layer is mentioned in the work of Van Drist [5]. The mistake of Van Drist, however, like everyone else, was the belief in the reversibility of flows (problems), the impossibility of which has already been mentioned above. One conclusion can be drawn from the above: in a non-gradient boundary layer caused by pure shear, there is an exponential decrease in velocity with distance from the moving wall. Then what about the well-known theories of the boundary layer by Prandtl [6], Blasius [7], Karman [8], Pohlhausen [9], as well as the lesser known theories of Sohrab [10], Weyburn [11], [12] Abdul-Gafor [13]? As will be shown further, all theories are consistent. The answer lies in the different structure of laminar non-gradient and gradient boundary layers. All existing theories have been influenced by the erroneous opinion about the reversibility of flows - gradient and non-gradient. Further, using the calculus of variations, it is shown that under experimental conditions, when a gradient shear flow is created, the molecular viscosity has a constant value. And this is not a hypothesis, but a result obtained from an analytical solution for the velocity field. What, the constancy of viscosity, cannot be said about a purely shear flow in a non-gradient boundary layer. There, inside the boundary layer, the viscosity increases from a minimum value at the moving plane to a maximum value at the outer, even if blurred, boundary of the laminar boundary layer.

Formulation of the problem. The paper considers the optimal character and different nature of incompressible flows in laminar boundary layers:
– non-gradient boundary layer formed during the steady motion of an infinite plane;
– gradient boundary layer in the flow past an immobile plane.

The purpose of the work is to obtain analytical dependences for the distribution of the velocity field in gradient and non-gradient laminar boundary layers of an incompressible fluid flow, as well as to reveal their common nature and differences in properties.

Non-gradient laminar boundary layer caused by the steady motion of an infinite plane. In order to understand and compare the results obtained below, we present solutions to related problems considered by Stokes. As already mentioned in the introduction, Stokes [3] solved the problem of oscillations of an infinite plane with frequency n and amplitude c in a viscous fluid. If we denote, according to Stokes, for

x direction of the normal to the plane, then we obtain for the problem the corresponding Navier-Stokes equations (in Cartesian coordinates) and the boundary condition

$$\frac{\partial p}{\partial x} = 0 \,, \frac{\partial v}{\partial t} = \mu' \frac{\partial^2 V}{\partial^2 x} = \nu \frac{\partial^2 V}{\partial^2 x}, \tag{1}$$

$$V\big|_{x=0} = c \sin nt. \tag{2}$$

In Stokes' paper [3], these are equations (8) and (9), respectively. v is longitudinal velocity, and $\mu' = \nu$ are various designations for the coefficient of molecular viscosity. The solution obtained by Stokes is

$$V = c \exp\left(-\sqrt{n/2\mu'}\, x\right) \sin\left(nt - \sqrt{n/2\mu'}\, x\right). \tag{3}$$

The solution in the form (3) does not allow one to pass to the limit to a steady motion corresponding to $n \to 0$, since in this case it is equal to zero. Apparently, therefore, Schlichting (see 5.25 in [1]) gives another boundary condition in the form

$$V\big|_{x=0} = c \cdot \cos nt \tag{4}$$

and the corresponding to (4) solution ([2], 5.26)

$$V = c \exp\left(-\sqrt{n/2\mu'}\, x\right) \cos\left(nt - \sqrt{n/2\mu'}\, x\right). \tag{5}$$

Now, in the limit $n \to 0$ from (5), it turns out that the entire space must move at a constant velocity equal to the velocity of the plane. Stokes, apparently, understood the impossibility, from the point of view of conservation laws in physics, to force the entire half-space to acquire the velocity of a moving plane, and even more so to keep moving at this velocity. Trying to save the situation when the theory does not provide experimental confirmation of the solution of the simplest problem, Stokes obtained a self-similar solution to the problem of instantaneously setting the plane in motion. The boundary condition is now expressed as a Heaviside function

$$V\big|_{x=0} = c \cdot H(t) = \begin{cases} 0, t \le 0; \\ c, t > 0. \end{cases} \tag{6}$$

By analogy with heat conduction, using the self-similar coordinate in the form

$$\eta = y / 2\sqrt{\nu t} \tag{7}$$

Stokes [3] obtained the corresponding self-similar solution

$$V_x = c\left(1 - \operatorname{erf}(\eta)\right). \tag{8}$$

For subsequent comparison with the solution obtained below, it is important to note that the introduction into (1) of the scales of quantities

$$[V] = c; \ \ [t] = 1/n; \ \ [x] = \sqrt{2\mu'/n}$$

leads to the following representation of dimensionless solutions (3) and (5)

$$V = \exp(-x)\sin(t-x), \quad V = \exp(-x)\cos(t-x). \tag{9}$$

In the course of work, we logically approached the emerging question about the nature of the solution in the case of the simplest problem

$$\frac{\partial p}{\partial y} = 0, \quad 0 = \nu \frac{\partial^2 V_x}{\partial y^2}, \ V_x\big|_{y=0} = c. \tag{10}$$

In relations (10), the coordinates x and y are swapped and index x is added to the velocity. Problem (10) corresponds, according to Stokes, to the steady motion of the plane along the axis x. What is wrong? Why is it impossible to obtain an experimentally confirmed exponential law from the formulation and solution of a simple problem (10)? To understand the reason for this discrepancy, it is necessary to find out what laws of physics the Navier-Stokes equations are based on. In deriving his equations, Stokes used an analogy with the laws of heat conduction. This analogy is based on the mathematical equivalence of the following equations

$$\frac{\partial V_x}{\partial t} = \frac{\partial}{\partial y}\left(\nu(y)\frac{\partial V_x}{\partial y}\right) \text{ as well as } \frac{\partial T}{\partial t} = \frac{\partial}{\partial y}\left(k(y)\frac{\partial T}{\partial y}\right). \tag{11}$$

In (11) $\nu(y)$ and $k(y)$ are molecular diffusion and thermal conductivity, respectively. As is known, the constancy of thermal conductivity $k(y)$ in (11) is based on the spatial isotropy of the medium, which just does not take place in the boundary layer, the domain of influence of one phase (body) on another (fluid).

Consider a plane moving at a constant velocity in an infinite domain of an incompressible fluid. Let us assume that the molecular viscosity of an incompressible fluid is a variable, at least within the boundary layer. We will consider it as a function of the distance to the solid plane

$$\mu = \mu(y). \tag{12}$$

The Navier-Stokes equation for this problem in accordance with (12) has the form

$$0 = \frac{d}{dy}\left(\mu(y)\frac{dV_x}{dy}\right). \tag{13}$$

Let be $[y]=\delta,\ \ [V_x]=U,\ \ \ [\mu]=\mu_0 = \max\mu$ the scales of the values of this problem. If they are taken out as factors on the right side of equation (13) and divided by a common constant factor (product), then the equation will not change. Therefore, hereafter we will consider dimensionless quantities, and we will leave their designations the same. Equation (13) now contains two unknown functions: $\mu(y)$ as well as $V_x(y)$. To close the problem, we assume that the fluid flow rate extreme condition is satisfied. In other words, among all possible incompressible fluid flows, caused by the friction of the plane with a dimensionless velocity

$$V_x|_{y=0} = 1 \tag{14}$$

we will consider realizing only that flow for which

$$J = \int_0^\infty V_x dy \Rightarrow ext. \tag{15}$$

Expression (15) agrees with the original interpretation of the Pierre Maupertuis principle [14]. According to Maupertuis, if denoted by m, V, S mass, speed and distance respectively, then the action in the form

$$I = mVs$$

reaches the minimum value for the true trajectory. This is the first formulated variation principle of least action. If for a steady motion and an infinite plane, as well as for a homogeneous fluid $m=const,\ s=const$, then formula (15) will mean the minimum flow rate in the boundary layer. A good overview of the applications of the calculus of variations for fluid flows is given in [15]. To the boundary condition (14) one should add the second, obvious condition

$$V_x(y) \to 0, \quad y \to +\infty. \tag{16}$$

To find the unknown function $\mu(y)$, when $V_x(y)$ is already defined, it is sufficient, according to (13), to formulate one boundary condition. As will be shown further, it is reasonable to put

$$\mu\big|_{y=\delta} = 1. \tag{17}$$

In equation (17) δ is an approximate value of the boundary layer thickness at which the velocity decreases by about 100 times. Thus, the problem is determined by relations (13)–(17). Since any viscosity of a fluid appears only in the presence of a velocity gradient, it is possible assuredly to write

$$\mu(y) = f\left(dV_x / dy\right) \tag{18}$$

where f means certain function . On the other hand, we use an equation similar to the turbulent flow equation, where viscosity is a function of coordinate. By analogy, we assume that viscosity is a function of velocity

$$\mu(y) = \mu\big(V_x(y)\big). \tag{19}$$

Comparing relations (18) and (19), it is easy to conclude that

$$V_x(y) = \mu^{-1}\big(\mu(V_x)\big) = \mu^{-1}\big(f\left(dV_x / dy\right)\big) = g\left(dV_x / dy\right). \tag{20}$$

The Euler equation for the extreme of the functional according to (15) and (20) has the form

$$-\frac{d}{dy}\left(\partial g \,/\, \partial\left(\frac{dV_x}{dy}\right)\right) = 0. \tag{21}$$

From (21) it immediately follows that

$$\frac{\partial g}{\partial\left(\dfrac{dV_x}{dy}\right)} = C_1. \tag{22}$$

Using the invariance property of the first differential and replacing the function g by the V_x we obtain from (22)

$$\frac{dV_x}{dy} = C_1 \frac{d^2 V_x}{dy^2}. \tag{23}$$

The general solution of equation (23) is

$$V_x(y) = A + B \exp\left(y / C_1\right). \tag{24}$$

For the case of the considered motion of the plane, according to the boundary conditions (14) and (16), the velocity distribution has the form

$$V_x(y) = \exp\left(y / C_1\right). \tag{25}$$

The value of the constant C_1 is determined from the condition of meeting the Stokes solution (9). This meeting gives $C_1 = -1$ and therefore

$$V_x = \exp(-y). \tag{26}$$

Solution (26) testifies to the correctness of using the calculus of variations approach for the problem under consideration. Moreover, a comparison with the experimental and theoretical distributions (see Fig. 1, *a*) shows that, for the dimensionless value $y = 5$ a

decrease in the velocity value by about 100 is obtained (see [2,4]). To obtain the velocity profile given in all sources for the inverse problem for the considered problem of flow past an immobile plane, it is sufficient to subtract the right side of equality (26) from unity:

$$u = 1 - \exp(-y). \qquad (27)$$

For convenience of comparison with other results, expression (27) is shown in Fig. 1, *b-c*). The value $1/C_1 = -0.0175$ used in fig. 1, *b-c*) to match with the internal scales of the boundary layer. We point to one non-trivial detail. According to (26), the viscosity of a fluid can increase to infinity, since it must be determined by an exponential function with a positive argument tending to infinity. However, in such cases, one resorts to describing the solution in other, initial quantities. The Navier-Stokes equations are derived in stresses that maintain across boundary layer. And this condition is just fulfilled. A similar example in fluid mechanics would be the point vortex model, where the velocity on the axis goes to infinity, but in this case circulation is used, which is a constant.
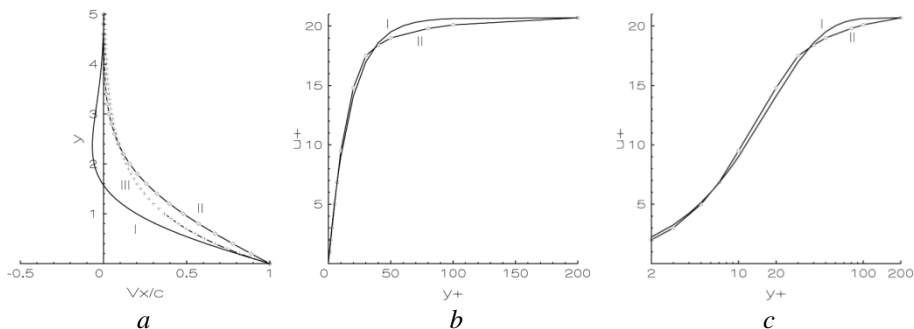

Fig.1. Distribution of the longitudinal component of the velocity: comparison of the solution. *a* – (26) (III) with known close solutions: amplitude of solution (9) (I), self-similar solution (8) (II) [3]; *b – c* – (27) with solutions at low Reynolds numbers obtained using DNS [16]

It should be noted that the exponential solution in the form (27) can also approximate two-phase low-Reynolds turbulent flows [17]. In this case, in the formula (27) $1/C_1 = -0.05$.

Gradient laminar boundary layer: fluid flow past an immobile plane caused by a pressure gradient. The above studies cause natural bewilderment, since measurements in wind tunnels, as well as the theory of a laminar boundary layer, confirm the constancy of molecular viscosity in the boundary layer, and in the flow domain as a whole. To study this situation, it is necessary to turn to the physics of flows. Let's answer the main question: what forces cause the fluid to flow in each case? In the case of non-gradient flow, this is internal friction, which is entirely determined by the velocity gradient. Therefore, for the flow, functional integrand depends only on the derivative of the velocity. Otherwise, the situation is in the case of gradient flow. Obviously, now the flow is created by two forces (as in the non-steady problems solved by Stokes [3]): the pressure gradient and internal friction. The pressure gradient creates a velocity field which is perturbed by the experimental object (body) that forms the boundary layer. Blasius' mistake is that in the problem of fluid flow past an immobile plate, he excluded the force of the longitudinal pressure gradient ([2], see formulas 7.5). It turns out that the flow of the fluid is not caused by anything: there is no force that would make the fluid flow. Since everyone was interested in the inverse problem of the

motion of a plate (wing), where there really is no longitudinal pressure gradient, then, believing in the reversibility of phenomena, everyone agreed with Blasius. If we now assume that in the fluid flow rate functional the integrand formally depends not only on the derivative of the velocity, but also on the velocity itself (determined by the longitudinal pressure gradient), then, as will be shown now, the value of the molecular viscosity turns out to be constant. So, consider the gradient flow described by the equation

$$\frac{\partial p}{\partial x} = \frac{d}{dy}\left(\mu(y)\frac{dV_x}{dy} \right) \tag{28}$$

with conditions (14), (15), (17). Instead of condition (14), we use the condition

$$V_x(y=1) = 1. \tag{29}$$

Condition (29) means reaching the outer boundary (here, the thickness of the boundary layer is taken as the scale) of the gradient boundary layer. Of course, this is approximate, since there is no strict outer boundary for the boundary layer. Now suppose, due to the changed physics of the phenomenon (two forces instead of one), that

$$V_x(y) = g\left(V_x, dV_x \,/\, dy\right). \tag{30}$$

Relation (30) allows us to obtain, instead of (23), the corresponding Euler equation

$$1 - \frac{d}{dy}\left(\partial g \,/\, \partial\left(\frac{dV_x}{dy} \right) \right) = 0. \tag{31}$$

The solution of equation (31), taking into account the no-slip conditions and condition (29), has the form

$$V_x(y) = y(2-y). \tag{32}$$

Substitution (32) into the dimensionless Navier-Stokes equation corresponding to the gradient flow

$$-1 = \frac{d}{dy}\left(\mu(y)\frac{dV_x}{dy} \right),$$

leads to

$$y + C_1^* = \mu(y)\left(2 - 2y\right). \tag{33}$$

From expression (33) it follows that at $C_1^* = -1$ viscosity is constant

$$\mu = 1/2 = \text{Const}. \tag{34}$$

Hence, the main part of the gradient boundary layer is described by a parabolic profile, and in this case the molecular viscosity is constant there. Fig. 2 shows the results of known theories. All solutions, as well as experimental data [4], are quite close to the parabolic law (32). This explains the desire of all cited authors to bring their solutions closer to the parabolic law.

Conclusions. An original approach to the analytical description of a steady laminar boundary layer of an incompressible fluid is proposed, due to the lack of a physically confirmed solution to the simplest problem of the steady motion of an infinite plane in a fluid at rest at infinity. As shown above, Stokes, trying to close the system of equations, assumed a constant value of molecular viscosity in his model. Careful investigation revealed the cause of the discrepancy. Stokes used the analogy with Fourier's second law, where the thermal conductivity is a constant. But the constancy of thermal conductivity does not take place in the boundary layer, since the spatial isotropy of the medium is violated there. With the constancy of viscosity, the situation is much more complicated. As it turned out in the

course of the above studies, for a non-gradient boundary layer, the viscosity really, by analogy with thermal conductivity, must be variable in order to obtain an experimentally confirmed dependence of the exponential decrease. And for a gradient boundary layer, due to a different physics of the phenomenon (the presence of a longitudinal pressure gradient), it was possible, on the basis of the calculus of variations, to show that the viscosity inside the boundary layer is constant, and the velocity distribution is described by a well-known parabolic law, which with high accuracy (5%) correspond to all known boundary layer theories and experimental data. The solutions of non-stationary problems of plane oscillations and instantaneous setting of the plane in motion, obtained by Stokes, based on the constancy of viscosity and have the same property as the gradient steady boundary layer, namely: not one, but two forces act on the fluid - internal friction and inertia. Thus, to describe the laminar boundary layer of an incompressible fluid, in general, one should use the variable viscosity in the Navier-Stokes equations, as is done in direct numerical simulation (DNS). For the mathematical closure of the system of equations, the extreme condition of the fluid flow rate functional can be used.
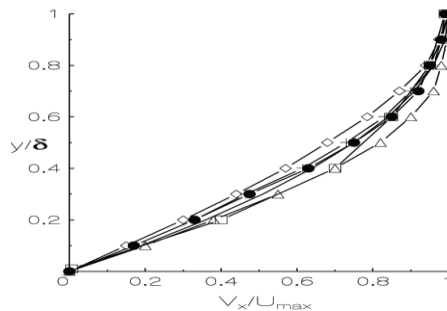

Fig. 2. Longitudinal velocity in a stationary laminar-gradient boundary layer of an incompressible fluid. • [7], diamond [8], □ is [4], Δ is [9], ∗ is [10], + is [13], -- is (32).

As a further study, it is reasonable to apply the used approach to the analytical description of the turbulent boundary layer.

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ОПТИМАЛЬНА ПРИРОДА ТА РІЗНІ ВЛАСТИВОСТІ ПРИМЕЖОВИХ ЛАМІНАРНИХ ШАРІВ НЕСТИСЛИВОЇ ТЕЧІЇ РІДИНИ

Наведено новий підхід щодо аналітичного опису стаціонарного ламінарного примежового шару нестисливої рідини на нескінченій площині. Цей підхід базується на відмові, в загальному випадку, від припущення про сталість молекулярної в'язкості усюди в області течії. Новизна полягає у використанні варіаційного числення для замикання рівняння Нав'є-Стокса, в якому вже присутня нова невідома функція — молекулярна в'язкість. Для замикання використовується умова екстремуму втрати рідини крізь переріз примежового шару. Ця умова корелює із першим в історії варіаційним принципом (найменшої дії) П'єра Моперт'юі. В залежності від типу течії, - градієнтної чи без-градієнтної, -- вдалося показати, що при градієнтній течії рідини в'язкість є сталою усюди, а в без-градієнтній течії змінюється по всій товщі примежового шару. Ця відмінність пояснюється різною кількістю сил, що створюють течію рідини в без-градієнтному та градієнтному примежових шарах. Це відповідно одна та дві сили. Наявність другої сили, а саме повздовжнього градієнту тиску, відповідає за сталість молекулярної в'язкості. Відсутність інших, крім сили внутрішнього тертя, сил дозволяє молекулярній в'язкості, за умови сталості дотичних напружень, бути змінною величиною, а профілю повздовжньої швидкості відрізнятись від лінійної функції, яка не спостерігається у примежовому шарі. Наведено порівняння із існуючими класичними та сучасними теоріями ламінарного примежового шару. Згідно із цими теоріями, профіль швидкості має разючу схожість (відхилення не перевищує 5%) із параболічним законом, отриманим у даній роботі, а також забезпечує сталість молекулярної в'язкості в середині примежового градієнтного ламінарного шару нестисливої рідини. В цілому в роботі робиться висновок про відсутність аналогії між градієнтним та без-градієнтним стаціонарними примежовими шарами нестисливої рідини. Течії у цих шарах описується зовсім різними функціональними розподілами – параболічним і експоненціальним.

Ключові слова: внутрішнє тертя, примежовий шар, нестислива течія, змінна молекулярна в'язкість, малі числа Рейнольдса, аналітичні розв'язки, варіаційне числення