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ANALYSIS OF LOAD CAPACITY, WEAR AND LIFE WORM TRANSMISSION WITH INVOLUTE WORM CORRECTED WITH TWO- AND TREE PAIR ENGAGEMENT

The paper presents a method for determining the effect of teeth correction in an involute worm gear on the contact strength, wear and life of teeth of the worm wheel. The cases of two- and three-pairy of engagement are considered. The regularities regarding the effect of correction on contact and tribocontact parameters are established. When a positive correction coefficient is applied, the maximum contact pressures and wheel teeth wear decrease while the gear life increases; when the correction coefficient is negative, the trend is opposite.

Key words: *Worm gear, involute worm, parity of engagement, wheel teeth correction, contact strength, wear, life*

Introduction. Archimedes worm gears are widely used in various devices and equipment. On meshing a sliding friction is generated, which leads to the wear of teeth of the worm wheel [1] . For this reason, it is required that the life of such a gear and the wear of its wheel be determined already at the stage of design. The literature reports methods for determining abrasive wear of worm gear teeth [2 – 5]; however, the proposed methods cannot be used for uncorrected and corrected gears. The studies [2; 3] report the results regarding determination of teeth wear in an uncorrected gear according to a modified Archard law which takes into account the changes in contact pressures and oil film thickness in the contact zone based on elastohydrodynamic lubrication theory. A method [6; 7] was developed for determination of contact parameters in uncorrected worm gears; later on, it was generalized and applied to corrected worm gears.

The article deals with its generalization on the corrected transmission and its influence on:

– contact strength - maximum contact pressure,
– linear wear of the teeth of the wheel during one interaction;
– minimum durability of gears to permissible wear of teeth.

Method solution. According to the tribokinetic model of wear at friction slip [6] postulated frictional-fatigue fracture of the surface layer by the specific force of friction that occurs in tribocontact. It is accepted that the amount of wear uniquely depends on the level of the specific frictional force when slipping, which occurs in the zone of frictional contact. According to [6; 7] the function of liner wear of teeth of a worm gear per one revolution is calculated from the formula

$$h'_{2j} = \frac{\nu_j t'_j \left(f p_{j\max}^{(w)} \right)^{m_2}}{C_2 \left(\tau_{s2} \right)^{m_2}} \;, \tag{1}$$

where $t'_j = 2b_j / v_j$ is the time of contact between the meshing components at j-th point on the friction path with a length of $2b_j$; v_j is the velocity of slide at j-th point of meshing at the height of worm coils; f is the sliding friction coefficient; C_2, m_2 are the indicators of wear resistance of material of the worm wheel in a selected pair and the conditions of wear as determined in the experimental tests; $\tau_{s2} \approx 0,35 R_{m2}$ is the temporary shear strength of material the worm wheel; R_{m2} is the temporary tensile strength of material of the worm wheel; $2b_j^{(w)} = 2.256\sqrt{\Theta N' \rho_{zj} / bw}$ is the width of contact area; $p_{j\max}^{(w)} = 0,564\sqrt{N' / w\theta \rho_{zj} b}$ are the maximum contact pressured determined according to the Hertz formula depending on the number of meshing pairs w of teeth of the worm wheel; w is the number of engaged tooth pairs; $\theta = \left(1-\mu_1^2\right)/E_1 + \left(1-\mu_1^2\right)/E_2$ is the Kirchhoff modulus; μ, E are the Poisson ratio and Young modulus of materials of the worm wheel, respectively; b is the worm wheel width; ρ_{zj} is the radius of curvature of teeth of the worm wheel at j-th point of meshing

$$\rho_{zj} = \frac{\rho_{1j}\rho_{2j}}{\rho_{1j} + \rho_{2j}}. \tag{2}$$

Curvature radius both of involute worm thread profiles ρ_{1j} and of the worm wheel teeth ρ_{2j}

$$\rho_{1j} = -\frac{r_b tg\alpha_{cj}}{\cos^3 \alpha_{pxj} tg\gamma_b \cos^2\left(\alpha_{cj} + \varepsilon_j\right)},$$

$$\rho_{2j} = \frac{\rho_{1j} r_2 \sin\alpha_{pxj} + \rho_{1j} e_{pAj} - e_{pAj}^2}{r_2 \sin\alpha_{pxj} + \rho_{1j} - e_{pAj}}; \tag{3}$$

$$r_{f_1} = 0,5\left(d_1 - 2h_{f_1}\right), h_{f_1} = 1,2m \text{ (where } \gamma \le 15^\circ), \; h_{f_1} = 1,2m_n \text{ (where } \gamma \rangle 15^\circ);$$

$$tg\gamma = mz_1 / d_1, \quad d_1 = qm;$$

$$r_{a_1} = 0,5\left(d_1 + 2h_{a_1}\right), \; h_{a_1} = m \text{ (where } \gamma \le 15^\circ), \; h_{a_1} = m_n \text{ (where } \gamma \rangle 15^\circ);$$

$$r_2 = 0,5 z_2 m, \; r_2 = 0,5 d_2, \; z_2 = u z_1;$$

$$r_b = 0,5 d_1 \cos\alpha_c, \; tg\alpha_c = tg\alpha_n / \sin\gamma, \quad q = 2\left(1 + \sqrt{z_2}\right); \; \alpha_n = \alpha = 20^\circ;$$

$$\alpha_{cj} = arctg\frac{\sqrt{x^2 - r_b^2}}{r_b}; \alpha_{pxj} = arctg\left(-tg\gamma_b \frac{\sqrt{x^2 - r_b^2}}{x}\right); tg\gamma_b = \frac{mz_1}{d_1 \cos\alpha_c};$$

$$\varepsilon_j = \frac{180}{\pi}\frac{\sqrt{x^2 - r_b^2}}{r_b}; e_{pAj} = \frac{r_1 - x}{\sin\alpha_{pxj}}, \; r_1 = 0,5 d_1, \; b = 2m\sqrt{q+1},$$

where r_{f1} is the radius of a circle of worm cavity; d_1 is the reference diameter of the worm; h_{f_1} is the height of worm thread base; m is the axial modulus of meshing; $m_n = m\cos\gamma$ is the normal modulus of meshing; γ is the angle of elevation of the screw line of worm coils; z_1 is the number of worm coils; q is the diametral quotient of the worm gear; r_{a1} is the radius of a circle of worm coil prongs; h_{a1} is the height of head of the worm coil; d_2 is the reference diameter of the worm wheel; z_2 is the number of teeth in the worm wheel; u is the gear ratio; r_b is the radius of a basic circle of worm coils; α_c is the face pressure angle; $\alpha_n = \alpha = 20^\circ$ is the angle of meshing; α_{cj} is the face pressure angle of j-th point; ε is an angular coordinate in every pitch (degrees); e_{pA} is the distance of j point from the contact point.

The section of meshing $[x_A, x_B]$ must be divided proportionately into smaller sections: $x_A = j_A = j_1,\ x_2 = j_2,\quad x_3 = j_3, ..., x_B = j_n = j_B,\ x_A \langle x \langle x_B$, $x_A = r_{f_1} + 0,2m,\ x_B = r_{a_1}$.

The slipping velocity v_j is determined according to the formula:

$$\nu_j = \frac{\omega_1 x}{\cos\gamma_A}, \tag{4}$$

where $tg\gamma_A = mz_1/2x$; $\omega_1 = \pi n_1/30$ is the angular velocity of the worm; n_1 is the number of revolutions of the worm.

The wear of the worm wheel teeth within one hour of gear operation is [6]:

$$\bar{h}_{2j} = 60 n_2 h'_{2j}, \quad n_2 = n_1/u, \tag{5}$$

where n_2 is the number of revolutions of the worm wheel per minute; h'_{2j}, $n_2 h'_{2j}$ are the linear wear of the worm wheel teeth within one revolution and one minute of operation, respectively.

The life of the worm gear for the acceptable wear h_{2*} of the worm wheel teeth is calculated according to the formula:

$$t_* = \left(h_{2*}/\bar{h}_{2j}\right). \tag{6}$$

The meshing force is calculated according to the formula

$$N' = \frac{2T}{d_1 \cos\alpha_{pxj} \sin\left(\gamma + \rho'\right)}, \tag{7}$$

where $T = 9550 \cdot 10^3 (N/n_1)$ is the torque transmitted by the worm; $\rho' = arctg\left(f/\cos\alpha\right)$ is the friction angle; N is the transmitted power.

As a result of the teeth correction, the profile of the worm wheel teeth undergoes displacement (positive or negative) with respect to the initial profile (uncorrected) and formation of a tooth profile by some other section of the involute. As a result, the contact pressures and teeth wear decrease while the gear life increases. In worm gears, we can only apply correction to the worm wheel teeth. Hence, the applied displacement of the gear-cutting hob is:

$$\xi = x_2 m, \tag{8}$$

where $x_2 \le \pm 1$ is the correction correlation.

The interaxial distance is:

$$a_{wk} = a_w + x_2 m, \tag{9}$$

where $a_w = r_1 + r_2$ is the interaxial distance in the uncorrected gear.

The reference diameter of the worm in the uncorrected gear

$$d_{w1} = d_1 + x_2 m. \tag{10}$$

Therefore, the distance e_{pAj} between the j-th point of contact and the point of contact in [7] will be:

$$e_{pAj} = \frac{r_{w1} - x}{\sin\alpha_{pxj}}, r_{w1} = d_{w1}/2 .$$

For the uncorrected worm gear, the force in meshing

$$N' = \frac{2T}{d_{w1}\cos\alpha_{pxj}\sin(\gamma + \rho')} .. \tag{11}$$

Numerical solution. Other geometrical parameters are determined according to the formulae for the uncorrected worm gear. The computations were performed using the following set of data: N = 3.5 kW, n_1 = 1410 rpm, m = 6 mm, z_1 = 2, u = 25.5, f = 0.05, q = 8; worm- hardened steel grade 45 (HRC 50) described by $E_1 = 2.1\cdot10^5$ MPa, μ_1 = 0.3; worm wheel ring – bronze CuSn6Zn6Pb6 described by $E_2 = 1.1\cdot10^5$ MPa, μ_2 = 0.34; $C_2 = 7.6\cdot10^6$, m_2 = 0.88; τ_{s2} = 75 MPa; for j = 1; 2; 3; 4 and 5, respectively x = 18; 20; 22; 24 and 26 mm; h_{2*} = 0.5 mm; with double-pair and three-pair of engagement.

The results of the numerical solution are given in Figs. 1 – 4. Fig. 1 shows the relationships between the maximum contact pressures $p_{\max}$ and the displacement coefficient x_2 on entering the mesh (j = 1) and on leaving the mesh (j = 5). It was found that the positive correction of the worm wheel teeth leads to reduced pressures, while the negative correction results in their increase compared to the observations made regarding the uncorrected gear. This results in an increase in the teeth curvature summary radius $\rho_z = \rho$ at their acceptable wear h_{2*} = 0.5 mm when x_2 is positive and it decrease at negative values (Fig. 3).

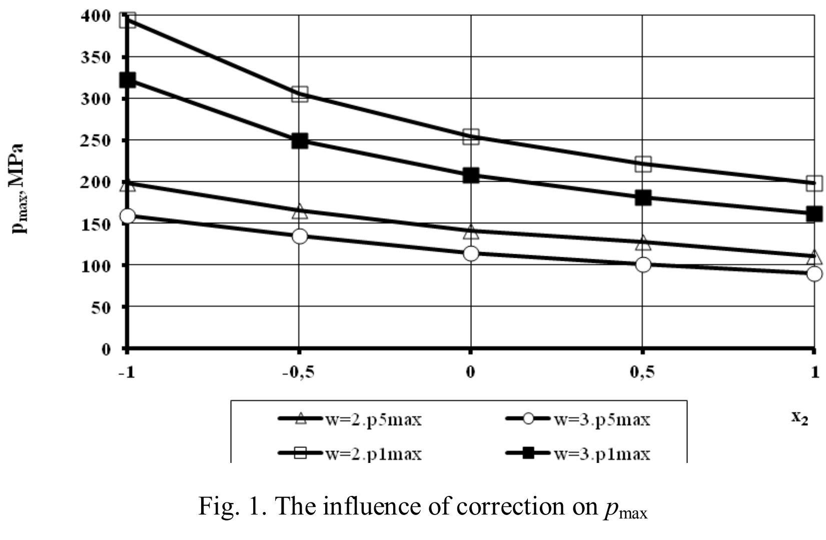

Fig. 1. The influence of correction on $p_{\max}$

Fig. 2 shows the minimal life $t_{\min}$ of the corrected gear at the entrance to the engagement (j = 1; x = 18 mm) in the implementation of two-pair and three-pair gearing. Positive correction of worm wheel teeth increases durability, while negative correction leads to decrease.

There is an increase in the reduced radius of curvature ρ with an increase in the magnitude of the displacement factor x_2 (Fig. 3).

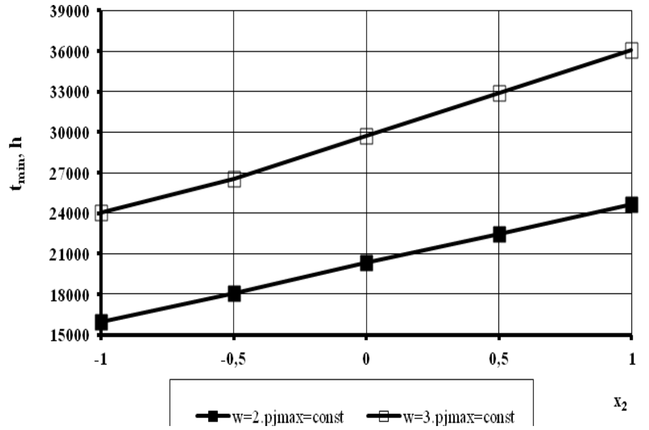

Fig. 2. The influence of correction on gear life $t_{\min}$

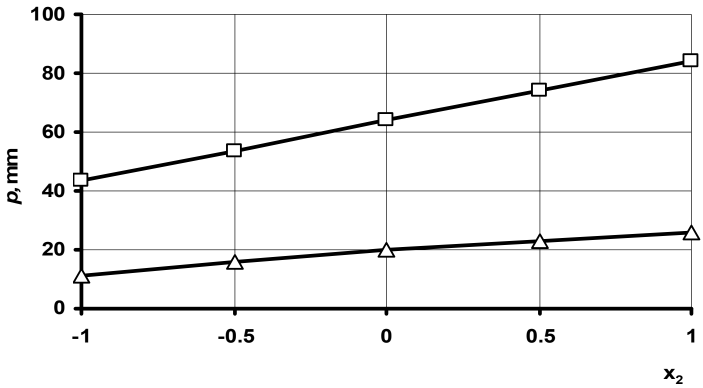

Fig. 3. The influence of correction on reduced curvature radii change :
□ – ρ_{zmax} by $j = 1$;△ – ρ_{zmin} by $j = 5$

The effect of teeth correction of the linear wear $\overline{h}_2$ of the worm wheel teeth for $w = 2$ during one hour of gear operation is illustrated in Fig. 4.

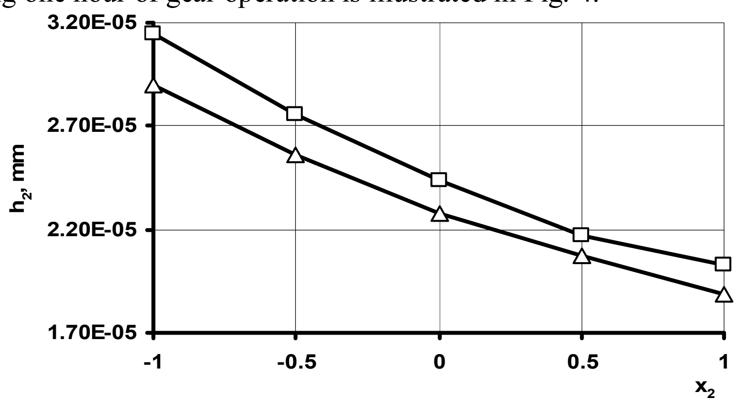

Fig. 4. Gear teeth wear versus x_2: □ – $\overline{h}_{2\max}$ at $j = 1$; △ – $\overline{h}_{2\min}$ at $j = 5$

As a result of the applied teeth correction, when $x_2 > 0$, ρ_z increases and so the wear $\overline{h}_2$ (Fig. 4) decreases and the gear life $t_{\min}$ increases (Fig. 2); when $x_2 < 0$, ρ_z decreases, which causes consequences which are undesired from a practical point of view. It must be stressed that the linear wear of teeth on entering the mesh (j = 1) and on leaving the mesh (j = 5) is similar, although the pressures $p_{\max}$ (see Fig. 1) differ to a significant degree.

The applied teeth correction does not affect the sliding velocity v_j in meshing and its value only depends on the position of contact point j at the height of the tooth.

The discovered qualitative and quantitative relationships with respect to the effect of teeth correction on the load capacity, wear and life velocity of the gear are reported in the Conclusions section below.

Conclusions

1. It has been found that, in contrast to the uncorrected gear, when the correction coefficient x_2 is positive, the maximum contact pressures decrease; when the correction coefficient has a negative value – the pressures increase.

2. The life of the gear increases at $x_2 > 0$ while at $x_2 < 0$ it decreases.

3.The radii of teeth curvature increase with an increase in x_2 and they decrease with a decrease in x_2.

4. The wear of the worm wheel teeth decreases at $x_2 > 0$ and increases at $x_2 < 0$ compared to the case when $x_2 = 0$.

5. With three-pair gearing, $p_{\max}$ will be lower up to 1.22 times than with two-pair, and $t_{\min}$ increases to 1.5 times.

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АНАЛІЗ НАВАНТАЖУВАЛЬНОЇ ЗДАТНОСТІ, ЗНОШУВАННЯ І РЕСУРСУ ЧЕРВЯЧНОЇ ПЕРЕДАЧІ З ЕВОЛЬВЕНТНИМ ЧЕРВ'ЯКОМ З КОРИГОВАНИМ ДВО- І ТРИПАРНИМ ЗАЧЕПЛЕННЯМ

У статті наведено метод визначення впливу корекції зубів черв'ячного колеса у евольвентній черв'ячній передачі на контактну міцність, зношування зубів черв'ячного колеса та довговічність передачі. Розглядаються випадки дво- та трипарного зачеплення. Встановлено закономірності впливу коригування на контактні та трибоконтактні параметри. При додатних коефіцієнтах корекції максимальні контактні тиски та зношування зубів колеса зменшуються при одночасному збільшенні довговічності передачі; при від'ємному коефіцієнті корекції тенденція є оберненою.

Ключові слова: черв'ячна передача, евольвентний черв'як, парність зачеплення, коригування зубів колеса, контактні тиски, зношування, довговічність