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Klipa A.M., PhD in Engineering

ESTIMATION PROCEDURE OF SENSOR BIASES ON THE BASIS OF FLIGHT TEST DATA

National Aviation University

The causes of sensor biases during small aircraft operation are considered. The estimation problem of these biases on the basis of flight test data is solved. It is proposed to estimate sensor biases as a result of simultaneous application of extended Kalman filter and accelerated Kesten stochastic approximation during optimization of the likelihood function. The proposed procedure of sensor biases estimation was checked on the "benchmark" model of lateral motion of small piloted aircraft DHC-2 "Beaver".

Introduction

Estimation of aerodynamic characteristics of any aircraft on the basis of flight test data in the presence of measurement noise and systematic errors, which are caused by biases of measuring systems and devices, is a real problem, especially for small planes of general aviation and Unmanned Aerial Vehicles (UAV). For this class of aircrafts it is impossible to apply an efficient vibroprotection of sensors that causes a high level of measurement noise [1]. Moreover, strict technical, economical and constructional requirements exclude application of expensive sensors, which have high accuracy, while cheap and less accurate (micromechanical and fiber-optic) have significant biases (systematic errors) of output signals. So far as measurement noise and sensor biases causes to biased estimations of small aircraft dynamic model parameters, that is why minimization of these factors negative influence during identification procedure is a very actual problem.

Many works are devoted to the identification problem of aircraft dynamic characteristics, main of them are [2-4]. But in these works the task of sensor bias estimation was not stated. In this paper the causes of sensor biases occurrence during operation of small aircraft are considered, on the basis of works [1, 5]; the task of sensor bias estimation during the identification of small aircraft dynamic characteristics is solved and comparative analysis of sensor bias estimation results obtained without application of accelerated Kesten stochastic approximation and with it, which was not implemented in [6, 7], is performed.

Problem statement

It is necessary to consider causes of sensor bias appearance during operation of small piloted aircraft and UAVs; to estimate sensor biases without application of accelerated Kesten stochastic approximation and with it during the performance of parametrical identification procedure (determination of aircraft dynamic characteristics, which enter to the state space model [8] linearly) on the basis of flight test data in the presence of measurement noise and sensor biases. As it shown in [7], as an identification criterion it is better to use the negative logarithm of maximum likelihood function (LLF) $P(\mathbf{y}|\boldsymbol{\theta})$ [9]:

$$J(\boldsymbol{\theta}) = -\ln P(\mathbf{y}|\boldsymbol{\theta}) = = 0.5\left\{\sum_{i=1}^{N}(\mathbf{y}_i - \hat{\mathbf{y}}_i)^T \mathbf{R}_{in}^{-1}(\mathbf{y}_i - \hat{\mathbf{y}}_i) + + N\ln|\mathbf{R}_{in}| + lN\ln(2\pi)\right\} \quad (1)$$

where $P(\mathbf{y}|\boldsymbol{\theta})$ is the maximum likelihood function; $\mathbf{y}_i$ and $\hat{\mathbf{y}}_i$ are the i-th element of output variable vector of model and its estimation respectively; $(\mathbf{y}_i - \hat{\mathbf{y}}_i)$ is the i-th vector of innovations; $|\mathbf{R}_{in}|$ is the Frobenius norm of innovation matrix; N is the number of measurement points (it depends on the length of realization); l is the length of output vector $\hat{\mathbf{y}}$ (it depends on the number of measured values).

Problem solution

Since the output signal of sensors contains not only the useful component, but systematic and random errors also, practically all data processing operations have statistical nature. Mix of the useful signal and the errors on the output of sensors is complex in general case (the components of this mix can be correlated). However, for majority of practical tasks of measuring device analysis this mix is presented as additive [5], that is independence of useful signal and errors is supposed. Thereby, output signal of measurement devices in general case is the following:

$$\mathbf{y}(t) = \hat{\text{O}}(\mathbf{y}_0(t), \varsigma(t))$$

where $\mathbf{y}(t)$ is the output signal of measuring devices; $\mathbf{y}_0(t)$ is the useful component of output signal; $\varsigma(t)$ is the error; $\hat{O}(\mathbf{y}_0(t), \varsigma(t))$ is some non-linear function which is possible to take as $\mathbf{y}(t) = \mathbf{y}_0(t) + \varsigma(t)$.

For the aircraft considered in this paper errors caused by delay have very small values; therefore they may be neglected [5].

Taking into account the mentioned above a sensor model can be presented as:

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{b} + \xi \quad (2)$$

where $\mathbf{b}$ is the vector of sensor biases; ξ is the vector of Gauss δ-correlated random errors of measurement:

$$E\xi(t) = 0; \quad E\left[\xi(t)\xi^T(t+\tau)\right] = \mathbf{R}\delta(\tau)$$

where E is the sign of mathematical expectation; $\mathbf{R}$ is the covariance matrix of measurement errors, $\delta(\tau)$ is Dirac function; τ is time shift.

Especially high values of biases take place in signals measured with the help of accelerometers and angular rate sensors [5]. The main cause of biases presence in signals from accelerometers is absence of gyro stabilization, since accelerometers are installed on the body of small piloted aircraft or UAV directly [1]. In angular rate sensors, except absence of gyro stabilization, the temperature compensation which is also causes significant values of biases is absent [5]. Absence of vibroprotection in these sensors results into raise of random errors level [1, 5].

Since the values of sensor biases are changing quiet slowly during the flight, their values may be assumed as constant [10] during flight test (20-60 seconds).

The state space model of aircraft dynamics with constant coefficients [8] and with taken into account the sensor model (2) is the following:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}; \\ \mathbf{y} &= \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} + \mathbf{b} + \xi \end{aligned} \quad (3)$$

where $\mathbf{A}$ is the $n \times n$ state matrix; $\mathbf{x}$ is the $n \times 1$ state vector; $\mathbf{B}$ is the $n \times m$ control matrix, $\mathbf{u}$ is the $n \times m$ control vector; $\mathbf{C}$ is the $l \times n$ measurement matrix; $\mathbf{y}$ is the $l \times n$ measurement vector; $\mathbf{D}$ is the $l \times m$ matrix of direct transfer of control from input to output; $\mathbf{b}$ is the vector with size $\mu \times 1$; ξ is the vector with size $l \times 1$.

Since it is required to estimate sensor biases under unknown parameters of aircraft model (3), in [6] it is proposed to extend the state space of this model by means of including in it so-called "dummy" variables (sensor biases):

$$\mathbf{b} = [b_1,\ b_2,\ \ldots,\ b_\mu]^T.$$

After extension of state space the input vector $\mathbf{u}_{ext}$, the state vector $\mathbf{x}_{ext}$, the output (measurement) vector $\mathbf{y}_{ext}$ are the following:

$$\begin{aligned} \mathbf{u}_{ext} &= \mathbf{u} = [u_1,\ \ldots,\ u_m]^T, \\ \mathbf{x}_{ext} &= [\mathbf{x},\ \mathbf{b}]^T = \\ &= [x_1,\ \ldots,\ x_n,\ b_1,\ \ldots,\ b_\mu]^T, \\ \mathbf{y}_{ext} &= \mathbf{y} = [y_1,\ \ldots,\ y_l]^T \end{aligned} \quad (4)$$

and four matrices which correspond to the state space model (3) look like:

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_0 & \mathbf{O}_{n\times\mu} \\ \mathbf{O}_{\mu\times n} & \mathbf{O}_{\mu\times\mu} \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} \mathbf{B}_0 \\ \mathbf{O}_{\mu\times m} \end{bmatrix};$$
$$\mathbf{C} = \begin{bmatrix} \mathbf{E}_{n\times n} & \begin{matrix} \mathbf{E}_{\mu\times\mu} \\ \mathbf{O}_{(n-\mu)\times\mu} \end{matrix} \end{bmatrix}; \quad \mathbf{D} = [\mathbf{O}_{n\times m}] \quad (5)$$

where

$$\mathbf{A}_0 = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \vdots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix};$$

$$\mathbf{B}_0 = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \vdots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{bmatrix};$$

$\mathbf{O}_{i\times j}$ is the $i \times j$ zero matrix; $\mathbf{E}_{i\times j}$ is the $i \times j$ unit diagonal matrix.

By the extension (4)-(5) Hamiltonian matrix associated with the Riccati equation for synthesis of optimum observer [8] for the system (3) will have 2μ zero eigenvalues. Solution of the optimum observer synthesis problem in the presence of singular Hamiltonian matrix having high order multiple zero eigenvalues is practically impossible. Because of it in [6] it is suggested to apply the randomization method to the "dummy" state variables. As a result of randomization Hamiltonian is a non-singular matrix and task of optimal observer synthesis is solved with the help of standard algorithm based on the stationary Kalman filtration [6, 8].

The first step of this procedure contains the determination of the covariance matrices of the state variables $\mathbf{Q}$ and the measurements $\mathbf{R}$. The matrix $\mathbf{R}$ could be easily determined on the basis of known r.m.s. of the sensors. The matrix $\mathbf{Q}$ requires some procedures for its estimation. The simplest one is the running Kalman filtering procedure several times to receive the best value of LLF after each execution. Then it is necessary to determine the covariance matrix of innovations $\mathbf{R}_{in}$ for Kalman filtering:

$$\mathbf{R}_{in} = \mathbf{CPC}^T + \mathbf{R} \qquad (6)$$

where $\mathbf{P}$ is the solution of the Riccati equation.

In [2, 3] several methods for iterative determination of this matrix are discussed. The problem is the fact that $\mathbf{R}_{in}$ in (6) depends on the covariance matrix $\mathbf{P}$ of state variables, which could be defined later as the Riccati equation solution that depends on the matrix $\mathbf{R}_{in}$. In [6] it is proposed the simplest way for solution of this problem. As the first step before the solution of Riccati equation, the matrix $\mathbf{R}_{in}$ could be approximated as [6]:

$$\mathbf{R}_{in} = \mathbf{CQC}^T + \mathbf{R}.$$

In this case it is possible to solve the Riccati equation. This approximation could be successfully used for Kalman filtering itself as well as at each step of the optimization procedure.

As it is offered in [2, 3], the better results (in comparison with purely discrete case) could be received using the first order approximation of the discrete Riccati equation by the continuous Riccati equation:

$$\mathbf{AP} + \mathbf{PA}^T - (1/d)\mathbf{PC}^T(\mathbf{R}_{in})^{-1}\mathbf{CP} + \mathbf{Q} = 0$$

where d is the sampling interval.

Solution of this equation gives the state variables covariance matrix $\mathbf{P}$, which then is used to determine Kalman gain matrix $\mathbf{K}$:

$$\mathbf{K} = \mathbf{PC}^\mathrm{T}(\mathbf{R}_{in})^{-1}.$$

For calculation of updated state space vector it is used well-known procedure of standard Kalman filtration [8].

Since application of stationary Kalman filtration to "dummy" variables gives their rough estimation, in [6] it is suggested to use a combination: extended Kalman filter (for estimation of real state variables and rough estimation of "dummy" state variables, which are sensor biases) and accelerated Kesten stochastic approximation (for refining of the data obtained as a results of extended Kalman filtration).

The advantages of such combination are noted in particular in [10]. In this connection in [6] it is proposed to use additional correction for state variables that concern to bias b_j. This correction is determined by algorithm of accelerated Kesten stochastic approximation [11]:

$$\hat{\hat{x}}_{bj}(i+1) = \hat{x}_{bj}(i+1) + \gamma(i)\cdot\left(y_{bj} - \hat{y}_{bj}\right)$$

where $\hat{x}_{bj}(i+1)$ is the state variable, which concerns to j-th bias on (i+1)-th step; $\gamma(i)$ is the gain of the stochastic approximation on i-th step that could be defined as [11]:

if $i \le 2$, then $\gamma(i) = 1/i$;

if $i > 2$, then $\gamma(i) = 1/\left(2 + \lambda(i)\right)$;

where $\lambda(i) = \lambda(i-1) - 0.5\left(\varphi(i) - 1\right)$.

Function $\varphi(i)$ is defined by the following expressions:

$\varphi(i) = 0$, if

$\left((y-\hat{y})_i - (y-\hat{y})_{i-1}\right)\left((y-\hat{y})_{i-1} - (y-\hat{y})_{i-2}\right) \le 0$;

$\varphi(i) = 1$, if

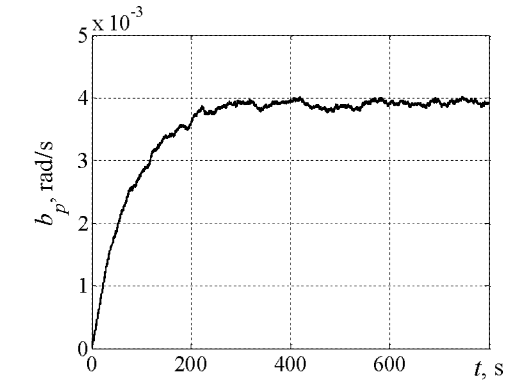
$\left((y-\hat{y})_i - (y-\hat{y})_{i-1}\right)\left((y-\hat{y})_{i-1} - (y-\hat{y})_{i-2}\right) > 0$.

It is necessary to notice, that, if the estimation process is optimal, then the innovation vector $\left(\mathbf{y}_{i+1} - \hat{\mathbf{y}}_{i+1}\right)$ in the steady-state mode should have properties of the white noise. Efficiency confirmation of the sensor bias estimation procedure is that the correlation functions of all estimated biases tend to delta-functions.

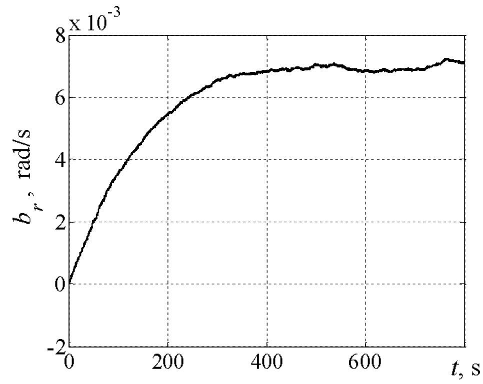
Presented procedure of sensor biases estimation during the parametrical identification of small aircraft model was checked on the "benchmark" dynamic model of lateral motion of small piloted aircraft DHC-2 "Beaver" [12]. Mathematical model of this motion in state space [13] is described by the following vectors: $\mathbf{x} = [p, r, v]^T$ where p, r are roll and yaw rates respectively; v is lateral velocity; $\mathbf{u} = [\delta\alpha, \delta r]^T$ where $\delta\alpha$, δr are aileron and rudder deflections respectively; $\mathbf{y} = [\dot{p}, \dot{r}, a_y, p, r]^T$ where $\dot{p}, \dot{r}$ are roll and yaw accelerations respectively; a_y is lateral acceleration; $\mathbf{b} = [b_p, b_r, b_{ay}]^T$ where b_p, b_r, b_{ay} are the biases of corresponding measured signals.

Values of sensor biases, estimated without accelerated Kesten stochastic approximation (fig. 1) and with its application (fig. 2) during the parametrical identification of lateral motion model of the aircraft "Beaver", converge, but the length of realization of output signals (time of flight test) which are necessary for the estimation procedure is differs more than 40 times.

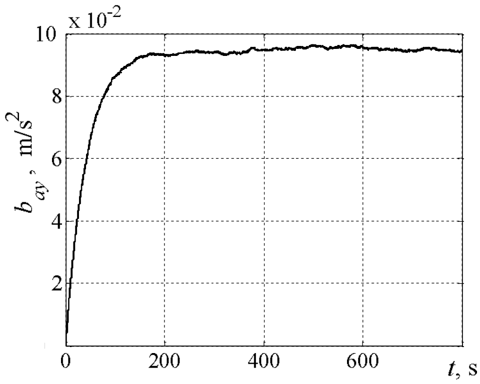
Time which is necessary to estimate b_r without stochastic approximation is approximately 600 seconds, and with accelerated Kesten stochastic approximation is 13 seconds).



a)

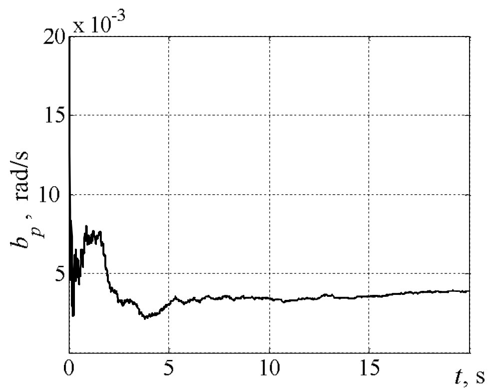


b)



c)

Fig. 1. Processes of sensor bias estimation without applying of accelerated Kesten stochastic approximation



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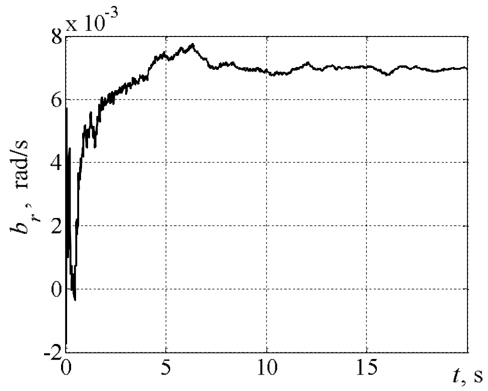

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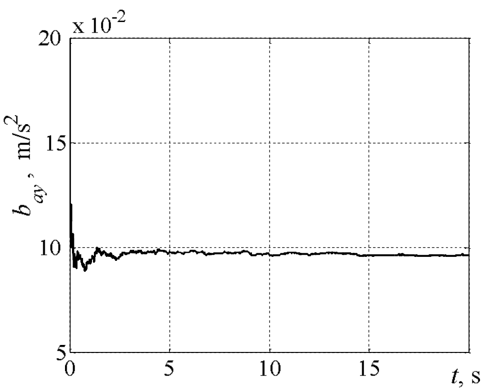

c)

Fig. 2. Processes of sensor bias estimation with applying of accelerated Kesten stochastic approximation

Results of the proposed sensor bias estimation procedure (with and without applying of accelerated Kesten stochastic approximation (AKSA)) on the basis of flight test data during the identification of dynamic characteristics of small piloted aircraft "Beaver" is presented in Table 1.

Table 1.

Results of Sensor Bias Estimation

Bias	Nominal value	Result of estimation without AKSA (necessary time for flight test is 800 s)		Result of estimation with AKSA (necessary time for flight test is 20 s)	
		Estimated value	Relative error, %	Estimated value	Relative error, %
b_p, rad/s	0.0040	0.003903	2.43	0.003765	5.88
b_r, rad/s	0.0070	0.006945	0.79	0.006966	0.49
b_{ay}, m/s^2	0.0950	0.094783	0.23	0.096798	1.89

Obtained results prove the efficiency of simultaneous application of extended Kalman filter and accelerated Kesten stochastic approximation during optimization of identification criteria (1).

Conclusions

The causes of sensor bias during small aircraft operation are considered. The estimation problem of these biases during the identification of dynamic characteristics of small aircraft is solved. The comparative analysis of results of sensor bias estimation obtained without application of accelerated Kesten stochastic approximation and with it is made. The proposed procedure was checked on the "benchmark" model of lateral motion of small piloted aircraft "Beaver". Relative error of sensor bias estimation is less than 2% for 2 biases b_r and b_{ay} and is less than 6% for b_p, moreover, relative error of lateral motion model parameter estimation is less than 5% for 12 parameters, but for other 3 ones it is less than 25%, that converge with the results of identifiability analysis [4] of lateral motion model for this plane [14].

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