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COMPARATIVE ANALYSIS OF MEASUREMENTS VERACITY IN HETEROGENEOUS AND HOMOGENEOUS AIRSPACE COMPLEXES

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The method and engineering technique for the comparative analysis of measurements results veracity in heterogeneous and homogeneous airspace complexes are offered. Index parameters of veracity increase at integrating of the information measurement systems are entered. Numerical examples and results of experimental researches of measurements results veracity in heterogeneous and homogeneous complexes are given

Introduction

It is generally known [1–3], measurement is operation of physical value finding by an experimental way with help special hardware. Measurements provide direct communication between a theory and experiment, high veracity of scientific researches, optimum quality and efficiency of the use of goods and services management of production.

Actual directions of development of metrology and allied subjects are:

- development of new methods, information technologies and techniques of measurements,
- creation of the modern measurement systems and complexes for the effective technological process control, upgrading goods and services;
- further increase of accuracy, veracity, fast-acting, sensitiveness, degree of automation and comfort of the use of measurement facilities;
- expansion of range and possibilities of measurement facilities and others.

In work [4] method and engineering technique of veracity definition of measurement results in heterogeneous and homogeneous aerospace complexes are offered. However the comparative analysis of measurements results veracity in heterogeneous and homogeneous airspace complexes was not executed.

The purpose work consists in developing these method and a technique for the comparative analysis of measurements results veracity in heterogeneous and homogeneous airspace complexes in the given typical measurement conditions.

For achievement of purpose is posed and is decided the problems problem substantiation, determination of necessary basic data, choice of task decision method, expected results prognostication, imitation experiment planning, experiment results processing and their comparison with theoretical results, engineering technique development of veracity comparative analysis of measurement results in heterogeneous and homogeneous complexes.

Problem statement

The typical measurements conditions us as the known basic decided problem data, a priori probabilistic characteristics of the measured physical values and ratios signal-noise in measurements for optimum heterogeneous and homogeneous complexes. For the comparative analysis of veracity measurements results in heterogeneous and homogeneous complexes in the modes of search and monitoring we use method which was offered by us before in work [4]. Errors probabilities of the first and second kind, complete probabilities of error measurement and correct measurement, index characteristics of measurements veracity increase, serve as the results of comparative analysis. For conducting of imitation experiment and data processing about veracity of measurements with equal accuracy the system of automation of the scientific researches *MathCAD* 14 is used.

Statement of basic material

The problem decides with help of before offered methods of the indirect measurements with equal and different accuracy on parallel and recurrence ways [3, 4].

In a parallel way the measurement result is optimum, on the criterion of maximum of accuracy, estimation of the measured value of an form:

$$X = \sum_{k=1}^{m} g_i \times Y_i, \qquad (1)$$

where the optimum value of weighting factor g_{iopt} for the i – th result of the Y_i indirect measurements is determined on a formula

$$g_{i,\,opt\,i} = 1/D_i \Big/ \sum_{k=1}^{m} 1/D_k, \qquad (2)$$

where D_i is dispersion of additive error of the i – th system of complex, m is common number of the systems in a complex.

The minimum dispersion value of optimum estimation in a parallel way is determined on a formula

$$D_{\min}(X) = 1 \Big/ \sum_{k=1}^{m} 1/D_k\ , \qquad (3)$$

In a recurrence way the measurement result is optimum, on the criterion of maximum accuracy, estimation of the measured value of an form:

$$X_{opt}(k+1) = X_{opt}(k) - \frac{g_{k,opt} D_k}{g_{k,opt} D_k + D_{k+1}} [X_{opt}(k) - Y_{k+1}], \qquad (4)$$
$$k = 1, m-1,$$

where optimum value of weighting factor of g_i for $(k+1)$ – th result of the Y_k indirect measurements determine on a formula

$$g_{opt}(k+1) = \frac{g_{opt}(k) D_k}{g_{opt}(k) D_k + D_{k+1}}\ . \qquad (5)$$

The minimum dispersion value of optimum estimation in a recurrence way is determined on a formula

$$D_{\min}(k+1) = g_{opt}(k+1) D_{k+1}, \qquad (6)$$

A homogeneous complex is the singular type of heterogeneous complex, in which all optimum weighting factors choose equal. Such choice is done because errors dispersions of all complex systems suppose identical and the arithmetical mean of sample serves as estimation. Using of unoptimum, equal on a value, weighting factors results in the value of dispersion

$$D_0 = \sum_{k=1}^{m} D_k \Big/ m^2\ . \qquad (7)$$

In the offered method of comparative analysis of measurements results veracity in heterogeneous and homogeneous complexes dispersions D_k of the measurement errors of the systems which form complexes serve as basic data. By them the optimum weighting factors $g_{k,opt}$ (2), (5) and minimum values of dispersions (3), are calculated, (6). The optimum values of weighting factors $g_{k,opt}$ are used in formulas (1), (4) for optimum processing of results of the indirect measurements Y_k.

Veracity of measurement is considered, as property of measurement means to state a correct measuring estimation of values in the given measurement conditions. As well as for systems of diagnosing in a role of the basic measurement parameter veracity it is convenient to choose full probability of correct measurement, that is full probability of that measuring value will be in the given interval of values. As well as at diagnosing [4], in measurements it is convenient to use conditional and full probabilities of mistakes of the first and second kinds, and at additional distinction of correct and wrong measurements, also conditional and full probabilities of mistakes of the third and fourth kinds [4].

For the calculation of measurement veracity parameters we use the traditional parities for probabilistic characteristics [4]. Complete error probability of measurement

$$Q(a,b,x)=\int_a^b f_1(x)\left[\int_{-\infty}^{\chi a-x} f_2(\xi)d\xi+\right.$$
$$\left.+\int_{\chi b-x}^{\infty} f_2(\xi)d\xi\right]dx+ \qquad (8)$$
$$+\int_{-\infty}^{a} f_1(x)\int_{\chi a-x}^{\chi b-x} f_2(\xi)d\xi dx+$$
$$+\int_b^{\infty} f_1(x)\int_{\chi a-x}^{\chi b-x} f_2(\xi)d\xi dx$$

where *a, b* are the given limits of the tolerance band for measuring values X,
$f_1(X)$, $f_2(\xi)$ are densities of distribution measuring values and errors of measurement of its values,
χ – the factor of optimum tolerance band expansion, by the choice of which in offered by us method [4] we minimize dispersion of measured value estimation and, accordingly, complete error probability

$$\chi(h)=h+1/h, \qquad (9)$$

where the signal/noise ratio in measurements of power

$$h=\sigma_x^2/\sigma_\xi^2 \, . \qquad (10)$$

The meaning of the measured value Yi in the moment of measurement t_i is got of a form

$$Y(t_i)=\mathrm{k}_{xy}X(t_i)+\xi(t_i)=Y_i, \qquad (11)$$

where k_{xy} is transmission factor of measurement transformer. Further we determine, whether measurement result $Y(t_i)$ got in the extended tolerance band$[\chi a,\chi b]$. If meaning $Y_i\in\lfloor\chi a,\chi b\rfloor$, we take decision optimum on the minimum error complete probability criterion on that the measured value X is in the given interval [*a, b*].

Probability P of the measured signal situation in the tolerance band in this method is assign to equal probability of implementation in the measurement moment t_i of the inequality $P(t_i)=P_i(a\le X\le b,t_i)$

$$P=\int_a^b f_1(x)dx=\mathrm{P}_i \, . \qquad (12)$$

The veracity parameters of the offered method of optimum measurements in heterogeneous and homogeneous complexes are determined on known parities taking into account two of principles features:

- optimum sample tolerance band in the indirect measurements,
- optimum meaning measured value estimation on the results got in the moment of measurement.

Event which consists that the measured value situated in the chosen tolerance band the, and on measurements results is made decision, that measured value is out of the tolerance band we will name the first kind measurement error.

Complete probability of first kind measurement error

$$\alpha=\int_a^b f_1(x)\left[\int_{-\infty}^{\chi a-x} f_2(\xi)d\xi\right]dx$$
$$+\int_a^b f_1(x)\left[\int_{\chi b-x}^{\infty} f_2(\xi)d\xi\right]dx. \qquad (13)$$

Event which consists that the measured value doesn't situated in the chosen tolerance band and on measurements results is made decision, that it situated in the tolerance band the we will name second kind measurement error.

Complete probability of second kind error

$$\beta=\int_{-\infty}^{a} f_1(x)\left[\int_{\chi a-x}^{\chi b-x} f_2(\xi)d\xi\right]dx$$
$$+\int_b^{\infty} f_1(x)\left[\int_{\chi a-x}^{\chi b-x} f_2(\xi)d\xi\right]dx. \qquad (14)$$

Minimum meaning of measurement error complete probability

$$Q_{\min}=\alpha+\beta. \qquad (15)$$

Maximal meaning of correct measurement complete probability

$$D_{\max} = 1\text{-}Q_{\min} = 1\text{-}(\alpha + \beta). \quad (16)$$

Conditional probability of first kind error in measurements

$$P_{\alpha} = \alpha / P. \quad (17)$$

Conditional probability of second kind error in measurements

$$P_{\beta} = \beta/(1 - P). \quad (18)$$

It was before shown [7], that in diagnosing by single strobbing the rationed tolerance band is $\eta=(b-a)/\sigma_x$, absolute meaning of the tolerance band of $\delta=b-a$, mean square value of noise σ_{ξ}, relation a signal/noise $h = P_x / P_{\xi}$ and dynamic range ΔD of the measurements is connected by ratios:

$$\eta = \frac{\delta}{\sigma_{\xi}\sqrt{h}} = \frac{\delta}{\sigma_{\xi}\sqrt{P_x/P_{\xi}}} = \frac{\delta}{\sigma_{\xi}\sqrt{e^{\Delta D}}} = \frac{\delta/\sigma_{\xi}}{\sigma_x/\sigma_{\xi}}. \quad (19)$$

It ensues from ratios (19), that managing veracity of single measurement at fixed σ_{ξ} is possible by three ways: changing δ, ΔD or δ and ΔD simultaneous. At the fixed mean square value of σ_{ξ} additive error ξ efficiency measurements determine ratios:

$$\eta\sqrt{e^{\Delta D}} = \frac{\delta}{\sigma_{\xi}}, \; \sigma_{\xi}\eta = \delta/\sqrt{e^{\Delta D}}. \quad (20)$$

Ratios (20) are evidently shown, as the lack of dynamic measurement range can be "trucked" for the tolerance band width. At the large dynamic signal range is possible without fear to narrow assign tolerance band. Condition (20), in fact, is the condition of equivalence on error complete probability in measurements of three ways of management by the measurement information volume at the single measurement.

The measured value density of distribution and measurement error are assumed normal and they had a standard form:

$$f_1(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(m_x - x)^2}{2\sigma_x^2}},$$

$$f_2(\xi) = \frac{1}{\sqrt{2\pi}\sigma_{\xi}} e^{-\frac{(m_{\xi} - \xi)^2}{2\sigma_{\xi}^2}}, \quad (21)$$

where m_x and m_{ξ} – accordingly, expected values of the measured value and systematic error of measurement equipment.

Principle feature of measurements veracity estimation in heterogeneous and homogeneous aerospace complexes is using of optimum estimations (1), (4), which provide the minimum dispersion values of measurements error (3), (6) and, therefore, it have maximal ratios signal/noise (10) in measurements.

For the comparative analysis of measurement veracity parameters of in heterogeneous and homogeneous complexes we developed the comparative analysis engineering technique, which includes the following basic stages:

1. Selection of basic a priori data: supposed meaning of the measured random value m, its mean square values σ, meanings of the expectation values of m_i and dispersions D_i measurements errors of systems which form a heterogeneous or homogeneous complex.

2. Calculation of minimum dispersions value D_{min} of complexes on formulas (3), (6) and maximal ratios signal/noise on a formula (10).

3. Calculation of optimum expansion factor of tolerance band on a formula (9).

Note 1. Assign accuracy of calculations on this and other formulas of the indirect measurements is provided by the use of command *float* system *MathCAD* 14.

4. Choice of the measurements result situation in tolerance band required probability (12) and on it tolerance band length.

Note 2. Tolerance band length is recommended to choose multiple to the value σ, symmetric with regard to value m, using a priori values of m and σ for the measured value X.

Note 3. After measurements length and location of the tolerance band can be specified using a posteriori optimum estimations (1), (4).

5. Integration limits determination for measurements error. The minimum mean square values of measurement error of heterogeneous complex are used, which are got by operation of root extraction from dispersions (3), (6), (7).

Note 4. At the choice of integration limits it is necessary to watch after implementation with the required accuracy of normalization condition for the error distributing density.

6. Integrations are assign for implementation within the limits of the chosen tolerance bands for measured value change sequence and for measurement error with the step required for providing of calculations accuracy.

7. The calculations of veracity measurement parameters on formulas (8) are executed - (10), (12) - (18).

8. In cases of deficient measurements veracity measures are taken on providing of the assign veracity with the using condition (20), and also known ways of redundancy introduction [3].

9. The results of veracity comparative analysis are drew up as tables containing the results of calculation veracity parameters depending on the ratios signal/noise and numbers of the systems in a complex.

10. The index parameters of veracity increase are determined on the following formulas:

$$I_\alpha = \alpha_i / a_k, I_\beta = \beta_i / \beta_k$$
$$I_Q = Q_i / Q_k, I_D = D_k / D_i. \qquad (22)$$

Note 5. The veracity parameters in a numerator and denominator are assigned so that the value of index parameter always was more than 1 and showed the achieved effect.

11. Conclusions and practical recommendations are formulated on results the comparative analysis.

12. In the case of necessity specify the choice of basic a priori data and repeat calculations on points 2-10.

For illustration of this technique we will show, as executed the comparative analysis of measurements veracity in the modes of search and monitoring in automatic navigation aerospace complexes.

In the mode of search the extended range of measured parameter change is used for the exposure of existence of measurements object, for example findings of airplane in the area of air traffic control. In the mode of search measurement error dispersion, as a rule, is one order with dispersion of the measured values. Therefore search complexes of measurement object it is possible to name «rough measurement complexes».

After the measurements object discovery a complex passes to the mode of monitoring, in which mean square error value must be one order with the linear values of object of measurement. Therefore complexes of monitoring may by named «high-accuracy measurement complexes». In the monitoring mode of error measurement dispersion is in hundred and more than times less dispersion of the measured value.

Otherwise speaking, in the search mode measurements carry preliminary character, execute them at the relatively small ratios signal/noise, after the « measurement object capture» in the monitoring mode after an object measurements execute a signal/noise at the relatively large ratios. Therefore we will consider two typical examples of measurements veracity comparative analysis: for the search mode and for the monitoring mode.

Example 1. The experiment results which were executed by the method of measurements statistical imitation design in the mode of search are resulted in this instance. Comparison was executed for heterogeneous recurrent and parallel complexes, and also for a homogeneous complex proper to them. (In homogeneous complexes a parallel way and recurrent way give results identical on accuracy of measurements).

The experiment was executed at the following terms and suppositions:

1. In a role of standard of measurements the normally distributed random value was chosen X with the rationed expected value $M[X]=1$ and the dispersion $D_0=(\sigma_0)^2=(0.2)^2=0.04$. Consequently, the variation factor of the measured value was chosen equal 20%.

2. The optimum management by integration was executed for the $m=10$ measurement which have unequal accuracy systems in a complex which has the characteristic number $m0=11$.

3. The ranged row of errors dispersions of measurement systems was represented by arithmetic decreasing progression have form

$$Dk=D1-(k-1)d,\ to=1,m. \qquad (23)$$

4. Simplifying supposition is accepted that all measurement systems do not have a systematic error, expected value

$$M(\xi k)=0,\ to=1,\ m. \qquad (24)$$

5. For the the least exact system chosen 20 % mean square error equal on a value to the variation factor the measured random value, consequently, the ratio is the signal/noise of measurement for the first system

$$h_1 = \frac{D_0}{D_1} = 1, \qquad (25)$$

and $D1=0.04$.

6. The progression index d (23) is chosen so that to overcome a row from 10 systems of a different class of accuracy, including standard system which has the zeroing mean square value of error. Therefore d=0.004, and in a numeral kind progression was represented by a formula

$$D_k=0.04-(k-1)0.005,\ k=1,m, \qquad (26)$$

7. At the design of measurements simplifying supposition is accepted that all measurement transformers of values of physical value in electric signals are linear and have the identical factors of transmission

$$k_{xy,i}=1,\ i=1,m. \qquad (27)$$

8. The results of measurements are designed by realization of random values by the function (28) of the *MathCAD* system

$$Y_k=rnorm\{N,\ M[X],\ \sigma k\},\ k=1,m, \qquad (28)$$

in which the number of realization is chosen $N=1$, expected value $M[X]=1$, mean square error of $\sigma_k = \sqrt{D_k}$.

9. For the design of work of measurement complexes the program GK - m/m_0 was developed. It allows to simulate measurements of heterogeneous and homogeneous complexes at different sets of entrance these experiments and execute the imitation modeling of measurements at different volumes of the ***N*** samples and at a different number of the systems $m<m_0$ in a complex. Number of the systems in a complex are k=2.., $m;\ m=10$.

10. The Y_k Realizations, k=2,. m; it was formed by the function of random numbers generation (28) with the gaussian distributing with assign $N=1$, $M[X]$, σ_k.

11. In the experiment accuracy of modeling was assign «six symbols after a point».

12. The results of experiments were processed by the automation system of the scientific researches *MathCAD*.

13. For example, as a result of application of operation (28) in one model experiment with $k=8$ such realization of the indirect measurements was got:
$Y_1=1.037235$, $Y_2=0.838513$, $Y_3=1.366219$, $Y_4=0.804573$, $Y_5=1.076496$, $Y_6=0.946446$, $Y_7=1.042846$, $Y_8=1.063372$.

Relative efficiency of heterogeneous recurrent complexes in the modes of search and monitoring as compared to parallel recurrent complexes (continuous lines) and homogeneous complexes (stroking lines) is evidently illustrated by the graphs of fig.1-4 for the index efficiency parameters and results of calculations, which resulted in table 1,2.

For calculations and implementation of digital imitation statistical design the program RGK - m/m_0 was developed. On results researches and conducted calculations it is possible to do next conclusions.

Tabl. 1. Minimum relative errors and maximal ratios signal/noise in heterogeneous and homogeneous complexes in the mode of search with

P=0.9973, Δx=6 σ_1; m_0=11, k=2,10; $M[X]$=1; V_0=20%; D_0=0.04; D_1=D_0=0.04; d=0.004

K	Heterogeneous recurrent complex		Heterogeneous parallel complex		Homogeneous complex	
CF	σ_R	h_R	σ_P	h_P	σ_A	h_A
1	2	3	4	5	6	7
1	0.200000	1.000000	0.200000	1.000000	0.200000	1.000000
2	0.111417	3.222222	0.137699	2.111111	0.137840	2.105263
4	0.082333	5.900794	0.091385	4.789683	0.092195	4.705882
6	0.064659	9.567460	0.068778	8.456349	0.070711	8.000000
8	0.050963	15.400794	0.052908	14.289683	0.057009	12.307692
10	0.036273	30.400794	0.036955	29.289683	0.046900	18.181818

Conclusions

1. Comparative analysis of indexes of measurements veracity heterogeneous and homogeneous complexes allows to get comparative quantitative estimations of minimum values of optimum estimations mean square errors of measurements results, maximal ratios signal/noise in measurements (table 1, 2), errors probabilities of the first and second kind, complete probabilities of errors and correct decisions, generalized index efficiency parameters of recurrent complexes depending on the number of the systems in a complex (fig. 1-4).

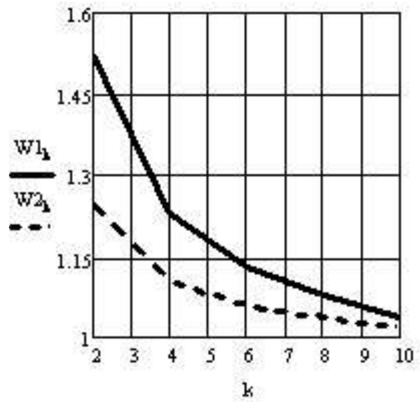

Fig. 1. Diagram of the ***h***-effectivness

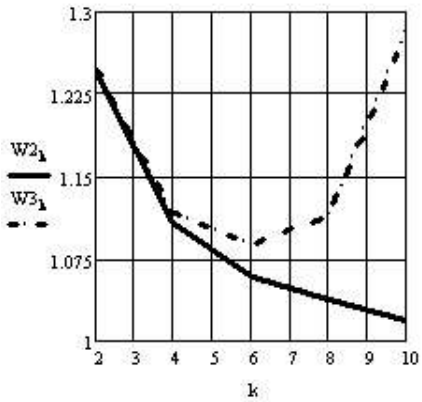

Fig 2. Diagram of the ***α***-effectivness

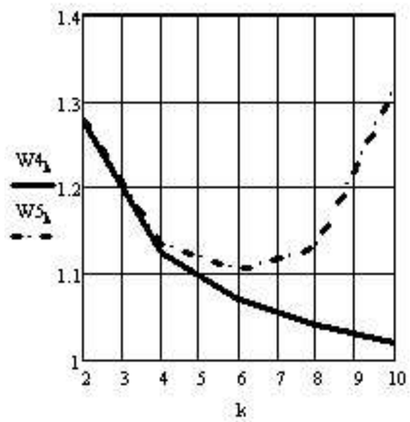

Fig. 3. Diagram of the ***β***-effectivness

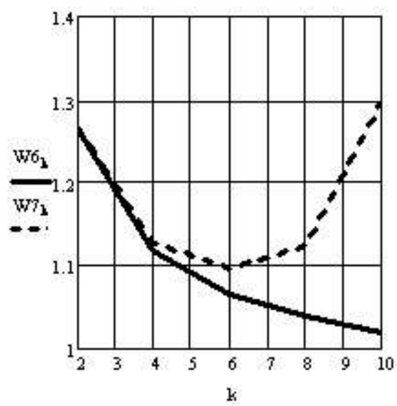

Fig. 4. Diagram of the ***Θ***-effectivness

Tabl. 2. Minimum relative errors and maximal ratios signal/noise in heterogeneous and homogeneous complexes in the monitoring mode with

P=0.9973, $\Delta x=6\sigma_1$; m_0=11, k=2,10; $M[X]$=1; V_0=20 %; D_0=0.04; $D_1=6.25 \times 10^{-4}$; $d=6.25 \times 10^{-5}$

K	Heterogeneous recurrent complex		Heterogeneous parallel complex		Homogeneous complex	
CF	$\sigma_R \times 10^{-2}$	h_R	$\sigma_P \times 10^{-2}$	h_P	$\sigma_A \times 10^{-2}$	h_A
1	2	3	4	5	6	7
1	2.500000	64.000	2.500000	64.000	2.500000	64.000
2	1.392715	206.222	1.720618	135.111	1.723006	134.737
4	1.029164	337.651	1.142317	306.540	1.152443	301.177
6	0.808242	612.317	0.859703	541.206	0.883883	512.000
8	0.637042	985.651	0.661346	914.540	0.712610	787.692
10	0.453417	1946.000	0.461937	1875.00	0.586302	1164.00

2. The results of comparative analysis show asymptotical efficiency of measurement complexes increasing with growth of systems number as compared to the single equally accuracy and not equally accuracy measurement systems.

3. Recurrent heterogeneous complexes are most effective as compared to parallel heterogeneous complexes and homogeneous complexes at any number of the systems in a complex. As the graphs of fig.1-4 show, recurrent heterogeneous complexes provide most profit in relation to parallel complexes at the small systems number in a complex. This property is substantial for construction of measurement complexes in practice. In the mode of monitoring heterogeneous complexes are less effective, because measurements are executed at the relatively high ratios signal/noise.

4. Parallel heterogeneous complexes asymptotical are approached on efficiency to recurrent as the number of the systems in a complex approaches the characteristic number of complex.

5. Homogeneous complexes are especially lost in efficiency to the heterogeneous complexes at the relatively small and relatively large number of the systems in a complex, i.e., when the differences of optimum values of weighting factors of heterogeneous complexes from the value 1/m most show up. Consequently, advantages of recurrent method as compared to a parallel method especially shows up in small size complexes which find wide practical application.

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