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CONTROL OF LINEAR MOTORS WITH GAS LUBRICATION OF THE COORDINATE MEASUREMENT SYSTEM

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Introduction

The measurement of geometric dimensions of details and complex spatial surfaces with specified accuracy is extremely important in contemporary reality of development of industrial production in Ukraine. The quality and reliability of the functioning of components and details of complex mechanisms and machines depend on the accuracy of their manufacturing. The three-coordinate information and measurement systems for measuring of mechanical values, details, components, and spatial surfaces of various configurations are used increasingly due to the intensive development of instruments production.

However, the existing coordinate measuring machines are not so productive; they are characterized by low accuracy, reliability, and resistance to disturbance; they cannot be used as content elements of flexible production systems. These measuring tools cannot be used for control of objects with complex spatial surface and do not satisfy contemporary requirements for accuracy and velocity of measurements. The largest numbers of known coordinate measuring machines are used now in laboratory conditions only; they are not adapted for the work in conditions of manufacturing workshops.

One of the important stages of the analysis of coordinate measuring systems is examination of their functioning in conditions of manufacturing workshops, with deviations from the normal conditions of the devices exploitations.

In the branch of measurements of mechanical values and parameters of movements the methods and technical devices of coordinate measuring systems are the only possible means for non-contact control of movement, velocity and acceleration.

In process of many practical problems solution, that arise in instrument engineering, as well as during scientific research, the information deficiency exists due to the mode of measurements of the function of the state and other factors. This deficit is replenished by the solution of the problems of control of distributed systems, such as mathematical modeling, estimation of the state and parameters of stochastic processes, minimization of the number of the points for measurement and optimization of their location in spatial domain.

Among numerical requirements for measurements of mechanical characteristics, the need to ensure high static and dynamic characteristics regarding control under the influence of destabilizing factors with minimal deviations in transient modes of operations is important. One of the methods of disturbing factors compensation is the use of gas-lubricated linear motors in-built into the dampers of the coordinate measuring system.

Purpose

The purpose of present study is to investigate the effect of non-Gaussian disturbance on the control system of linear motors with gas lubrication of the coordinate-measuring system during the measurements of mechanical characteristics of objects with complex spatial surface.

Main part

The case of measurement (estimation) of a vector information parameter under the influence of additive interference with a non-Gaussian probability distribution density (PDD) is studied.

The sampling of random process $X_h = X(t_h)$, h=1,...,H, is a mixture of useful signals $S(\lambda,t_h)$, which contains information about the measured information parameters λ={λ$_1$,…,λ$_m$}, and, in addition, in general case, non-Gaussian disturbance n_h [1, 3, 4, 5]

$$X(i_h)=S(\lambda, t_h)+n_h.$$

Let's suppose that the results of mechanical characteristics measurements are the functions of sufficient statistics, and they have the asymptotic properties of sufficiency, shiftless and normality, as well as fulfilled regularity conditions.

We also suppose that additive disturbance and measurement parameters are independent.

The task of determining the optimal filtering of random process of mechanical characteristics measurements is in transforming of the signal ξ in order to most accurately reproduce S. The density of the distribution $p(x/\lambda)$ depends on the vector parameter λ={λ_1,…,λ_s}, $p \geq 2$. To obtain the expansion into a power polynomial relative to x:

$$ln\, p_s\,(x/\lambda) = \int_a^\lambda \sum_{i=1}^s k_i(t)\big[x^i - m_i(t)\big]dt + c_s(x), \quad (1)$$

where, $c_s(x)$ – is a function that does not depend on λ, and for the sequence to converge x for the sequence $\{ln\, p_s\,(x/\lambda)\}$ to the logarithm of the distribution density $ln\, p\,(x/\lambda)$ for $s \to \infty$ it is necessary to determine $k_i(\lambda)$ from the solution of the system of linear algebraic equations [1]:

$$\sum_{j=1}^s k_j(\lambda) F_{ij}(\lambda) = \frac{d}{d\lambda} m_i(\lambda),\ i = \overline{1, s},$$

where, $F_{ij}(\lambda) = m_{i+j}(\lambda) - m_i(\lambda) m_j(\lambda)$ $i, j = \overline{1, s}$, $m_i(\lambda)$, $m_j(\lambda)$ – are the moments. According to (1), for finding the estimation of the vector parameter, the approximation of compatible density of the distribution of independent sample values will be:

$$p_{sn}(x/\lambda) = exp\{\sum_{i=1}^s h_{mi}(\lambda) \cdot t_{in} + h_0(\lambda) + c_s(x, \lambda/\lambda_m)\}, \quad (2)$$

where the following designations were used:

$$h_{mi}(\lambda) = \int_a^\lambda k_{im} dt,$$
$$h_0(\lambda) = n \int_a^\lambda \sum_{i=\lambda}^s k_i(t) b_i(t) dt,$$
$$t_{in} = \sum_{r=1}^n c_s(x_r),$$

where, $c_s(x, \lambda/\lambda_m)$ – is a function independent on the component vector parameter.

According to the Kramer-Rao theorem, the dispersion of any shiftless estimation is determined by the inequality [1]:

$$G_\lambda^2 \geq [-m\{d^2\, ln\, W_n\,(\lambda)/d\lambda^2\}]^{-1}, \quad (3)$$

where $W_n(\lambda)$ – is a likelihood function.

Let's assume that the logarithm of the likelihood function (LLF) exists and it is:

$$B_n = lnW_n\{X_h - S(\lambda, t_h)\}, \quad (4)$$

where $W_n\{*\}$ – is the PDD of additive disturbance.

The estimation of the accuracy of the vector information parameter measurement let's examine on the base of example of random vibration processes that take place during the vibration of the basement in flexible manufacturing systems, and the measurement of the frequency – ω, the derivative of the frequency – ω' and the phase – φ of the useful signal of the sensors:

$$S(\lambda, t_h) = U_{mh}\, cos\big[(\omega + 0{,}2\omega' t_h) t_h + \phi\big]. \quad (5)$$

Let's represent the useful signal (2) $S(\lambda, t_h)$ as:

$$S(\lambda, t_h) = U_{mh}\, cos(\lambda_1 + \lambda_2 t_h + \lambda_3 t_h^2), \quad (6)$$

where $\lambda_1 = \varphi$; $\lambda_2 = \omega'$; $\lambda_3 = \omega$.

For the measured signal we define the derivatives:

$$S'_{\lambda i}(\lambda, t_h) = U_{mh} t_h^{i-1} \sin(\hat{\lambda}_1 + \hat{\lambda}_2 t_h + \hat{\lambda}_3 t_h^2);\ i=1, 2, 3. \quad (7)$$

Estimating the information parameters through the maximum of the posterior density of the probability distribution (PDPD), three equations have to be performed:

$$\frac{d}{d\lambda_1} \ln W_y(\lambda)\Big|_{\lambda_1=\hat{\lambda}} = 0;\ \frac{d}{d\lambda_2} \ln W_y(\lambda)\Big|_{\lambda_2=\hat{\lambda}} = 0;\ \frac{d}{d\lambda_3} \ln W_y(\lambda)\Big|_{\lambda_3=\hat{\lambda}} = 0. \quad (8)$$

The lower limit of the Kramer-Rao inequality for the dispersion of shiftless common parameters of the useful signal $\lambda = \{\varphi, \omega, \omega'\}$, let's write as [4]:

$$G_{\lambda ij}^2 \geq |I_{ij}|/|I|\ ;\ i, j=1,2,3, \quad (9)$$

where $|I_{ij}|$ – is the algebraic complement of element I_{ij} of Fisher's information matrix; $\|I\|$; $|I|$ – is the determinant of the matrix.

The elements of the information matrix at $i \neq j$ take into account the mutual dependence of the evaluated parameters. If the measured parameters do not depend on each other, the information matrix is simplified, and we have:

$$[I] = \begin{bmatrix} I_{11} & 0 & 0 \\ 0 & I_{22} & 0 \\ 0 & 0 & I_{33} \end{bmatrix}. \quad (10)$$

Dispersion of the parameters for this case will be

$$G_{ai}^2 = G_{aii}^2 = |I_{ii}|/|I|;\ i=1, 2, 3.$$

The elements of the matrix (10) can be expressed by the ratio:

$$I_{ij} = \sum_{h=1}^{H} h'_{\lambda i}(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, t_l, t_h) S'_{\lambda i}(\hat{\lambda}_1, \hat{\lambda}_2, \hat{\lambda}_3, t_h)\Delta + I_{\lambda\phi ij}. \quad (11)$$

where $H = T\Delta$ – is the whole part of the ratio;; $\Delta = h - (h-1)$ – interval of time counts; t_l – beginning of the time of measurement; $S'_{\lambda j}$ – derivative of the processed useful signal according to the estimated parameter λ_j ($j = 1,2,3$), due to $\lambda = \hat{\lambda}$; $I_{\lambda\phi ij}$ – component of Fisher's information matrix; $h'_{\lambda i}$ – derivative for parameter λ_i($i = 1,2,3$), conditioned for $\lambda = \hat{\lambda}$ from the weight function $h(\lambda, t_l, t_h)$, which is a solution of the equation:

$$\sum_{h=1}^{H} R_n(t_h - t_{h-1}) h(\lambda, t_l, t_h)\Delta = S(\lambda, t_h), \quad (12)$$

where $R_n(t_h - t_{h-1})$ – is a correlation function for disturbance, $h(\lambda, t_l, t_h) = \mu_n^2 N_n^{-2} S(\lambda, t_h)$, where $\mu_n^2 \geq 1$ – is the coefficient that takes into account the difference between PDD of additive disturbance and Gaussian one [3]; N_n^2 – spectral density of disturbance.

Then the relation (11) will be:

$$J_{ij} = \mu_n^2 G_{n\Delta}^{-2} \sum_{h=1}^{H} S'_{\lambda i}(\hat{\lambda}, t_h) S'_{\lambda j}(\hat{\lambda}_l, t_h) + I_{\lambda\phi ij}, \quad (13)$$

where $G_{n\Delta}^2 = N_n^2/\Delta$ – is dispersion of disturbance.

Let's put into (13) the value of derivative $S_{\lambda i}(\hat{\lambda}, t_h)$, for $0 < t \leq T$, and we will have:

$$I_{ij} = 0{,}2\mu_n^2 G_{n\Delta}^{-2} \sum_{h=1}^{H} U_{mh}^2 t_h^{i+j-2}\left[1 + \sin 2(\hat{\lambda}_1 + \hat{\lambda}_2 t_h + \hat{\lambda}_3 t_h^2)\right] + I_{\lambda\phi ij}. \quad (14)$$

For a large number of measurements will be:

$$I_{ij} = \mu_n^2 \frac{U_{mh}^2}{2G_n^2}(-1)^{i+j-2} T^{i+j-2}(i+j-1)^{-1} + I_{\lambda\phi ij}, \quad (15)$$

where, $U_m = H^{-1}\sum_{h=1}^{H} U_{mh}$; $G_n^2 = N_n^2(\Delta H)^{-1} = N_{nn}^2/T$ – is the dispersion of additive disturbance.

We signify $A = U_m^2/2G_n^2$, that play the role of generalized signal/disturbance ratio, and the matrix $[I]$ will be written as:

$$[I] = \mu_n^2\rho\begin{bmatrix} 1 & -0{,}2T & 1/5T^2 \\ -0{,}2T & 1/3T^2 & -0{,}25T^3 \\ 1/5T^2 & -0{,}25T^3 & 0{,}5T^4 \end{bmatrix} + \begin{bmatrix} I_{\lambda\phi11} & I_{\lambda\phi12} & I_{\lambda\phi13} \\ I_{\lambda\phi21} & I_{\lambda\phi22} & I_{\lambda\phi23} \\ I_{\lambda\phi31} & I_{\lambda\phi32} & I_{\lambda\phi33} \end{bmatrix}. \quad (16)$$

If the PDD of measured parameters [4]:

$$W_\lambda(\lambda) = (2\pi G^2)^{-1{,}5}D^{-0{,}5}\,exp\{-(2G^2D)^{-0{,}5}\textstyle\sum_{i=1}^{3}\sum_{k=1}^{3}D_{ik}(\lambda_{i-a})(\lambda_{k-a})\}, \quad (17)$$

where G^2 and a – are the dispersion and average value; D – determinant of the matrix R_λ, the elements of which are the value of the normalized correlation function: D_{ik} – the algebraic complement of the element R_{ik} in the matrix, then the information matrix $[I_{\lambda\phi ij}]$ will be:

$$[I_{\lambda\phi ij}] = (2G^2D)^{-1}\begin{bmatrix} 2D_{11} & D_{12}+D_{21} & D_{13}+D_{31} \\ D_{21}+D_{12} & D_{22} & D_{23}+D_{32} \\ D_{31}+D_{13} & D_{32}+D_{23} & D_{33} \end{bmatrix}.$$

Let's determine the correlation matrix of errors (10) by the method of maximum likelihood [1]: so, we have:

$$[I] = \mu_n^2\rho\begin{bmatrix} 1 & -0{,}2T & 1/5T^2 \\ -0{,}2T & 1/3T^2 & -0{,}25T^3 \\ 1/5T^2 & -0{,}25T^3 & 0{,}5T^4 \end{bmatrix}.$$

The determinant of this matrix will be:

$$[I] = (\mu_n^2\rho)^3T^\sigma/720 \quad (18)$$

Substituting values (15), (18) into (9), under the influence of non-Gaussian additive disturbances we have for: frequency, frequency derivative, and phase, respectively,

$$G_\omega^2 \geq \{\mu_n^2\rho T^2/61\}^{-1}, \\ G_{\omega'}^2 \geq \{\mu_n^2\rho T^4/240\}^{-1}, \\ G_\phi^2 \geq \{\mu_n^2\rho/3\}^{-1} \quad (19)$$

If the object of measurement is installed on the rotary table, and it moves with a constant speed, i.e. $\omega' = 0$, we have:

$$G_\omega^2 \geq \{\mu_n^2\rho T^2/12\}^{-1};\; G_\phi^2 \geq \{\mu_n^2\rho/3\}^{-1}.$$

Inequality (19) demonstrates that the lower limits of the dispersion of estimates ω and φ are significantly lover if they are measured in the middle of the observation interval. The increase in the accuracy of such measurements is due to the fact that the off-diagonal elements I_{12}, I_{21}, I_{23}, I_{32} are equal to zero and only the elements I_{13}, I_{31} differ from zero. If non-diagonal elements in matrix $[I]$ (16) differ from zero, it is necessary to take into account the presence of correlation links between the estimation errors of individual measurement parameters.

Thus, with an increase of ρ, T, i μ_n^2, the reduced error of measurement decreases, and the measurement accuracy increases.

Conclusions

As we can see from (19) and taking into account the non-Gaussian nature of the additive disturbance; this allows us to increase significantly the accuracy of measured parameter. Moreover, the more the PDD of the additive disturbance differs from the Gaussian one, the more accurately we get the measurement result.

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CONTROL OF LINEAR MOTORS WITH GAS LUBRICATION OF THE COORDINATE MEASUREMENT SYSTEM

The problem of the influence of non-Gaussian disturbance on the control system during the measurement of coordinate measuring system of mechanical quantities of objects with a complex spatial surface is described in the article. It is shown that the accuracy of the filtering of the random process of measuring mechanical quantities is significantly affected if to take into account the disturbing factors with non-Gaussian probability distribution density.

***Keywords**: non-Gaussian interference; coordinate measuring system of mechanical values; disturbing factors; probability distribution density.*

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КЕРУВАННЯ ЛІНІЙНИМИ ДВИГУНАМИ З ГАЗОВИМ ЗМАЩЕННЯМ КООРДИНАТНО-ВИМІРЮВАЛЬНОЇ СИСТЕМИ

У статті розглянуто задачу впливу негаусової завади на систему керування при вимірюванні координатно-вимірювальної системи механічних величин об'єктів із складною просторовою поверхнею. Показано, що на точність фільтрації випадкового процесу вимірювання механічних величин значною мірою впливає врахування збурюючих факторів із негаусовою щільністю розподілення ймовірностей.

***Ключові слова**: негаусова завада; координатно-вимірювальна система механічних величин; збурюючі фактори; щільність розподілення ймовірностей.*