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FORMALIZED METHOD OF THE SOLUTION OF MULTI-CRITERIA PROBLEMS

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Introduction

The solution of simple optimization problems is quite formalized. When the quality of a solution is judged by one criterion, the problem has a single definite solution. If the task is multi-criteria (vector), then as a result of optimization, a set (Pareto area) of acceptable solutions is obtained. But of these, you usually need to choose only one. Since the points of the Pareto set are formally incomparable, then in order to solve the problem, it is fundamentally necessary to involve information about the preferences of the decision maker (DM). When solving a specific problem of vector optimization, the decision maker creates his own model of the objective function (utility function) in accordance with his preferences. Thus, the solution of multi-criteria problems is subjective in nature.

Nevertheless, it is possible, if not to eliminate, then at least significantly reduce the influence of subjective factors on the result of solving a multi-criteria problem [1-3]. It is assumed that there are some invariants, rules that are usually common to all decision makers, regardless of their individual inclinations, and which they equally adhere to in a given situation. The inevitable subjectivity of the decision maker has its limits [4-6]. In business decisions, a person must be rational in order to be able to convince others, explain the motives for his choice, the logic of his subjective model. Therefore, any preferences of the decision maker must be within a certain rational system. This is what makes formalization possible.

The works of [6-13] considered the existing consensus models of minimum costs to increase the efficiency of decision-making by decision-makers. Different consensus models are considered, for example, with distributional robust random constraint (DRO-MCCM). The presented studies recommend using these models in a supply chain management scenario involving pricing of new products. A comparison and analysis of the sensitivity of the proposed models and their effectiveness was carried out. However, formalized methods of solving multi-criteria problems remain unconsidered.

Main part

The optimization problem is to choose the conditions that allow the object of study in a given situation to show its best properties. The conditions on which the properties of an object depend are quantitatively expressed by some variables $x = \{x_i\}_{i=1}^{n}$, specified in the domain of X and called optimization arguments. The decision-making situation depends on external influences r . These influences do not depend on us, they are set from outside, but it is known that they can take their values from a compact set R. It is usually believed that calculations are carried out with a given and known vector of external influences $r^0 \in R$, on which, ultimately, depends specific decision-making situation.

In turn, each of the properties of the object from the area M is quantitatively described using the variable y_k, $k \in [1, s]$, the value of which characterizes the quality of the object in relation to this property. The indicators, called quality criteria, form a vector $y = \{y_k\}_{k=1}^{s} \in M$. Its components quantitatively express the assessment of the object properties for a given set of optimization arguments $x = \{x_i\}_{i=1}^{n} \in X$.

The objective function $y = f(x)$ connects the quality criteria vector with the optimization arguments. This function is a model of the decision maker's utility function in a given situation. With some reservations, the optimization problem is formulated as finding such a combination of arguments from the domain of their definition, in which the objective function acquires an extreme value:

$$x *= arg \underset{\substack{x \in X \\ y \in M}}{extr} f(x) \Big|_{r \in R}.$$

If, without loss of generality, we assume that "better" means "less", then in practice, for a fixed $r = r^o \in R$ and guaranteed $y \in M$, the expression $x *= arg \min_{x \in X} f(x)$ is applied.

For system linking in multi-criteria tasks, instead of $y = f(x)$, the scalar convolution of the vector of partial criteria $Y = f[y(x)]$ is usually used as an objective function, where y is the s -dimensional vector of criteria $y = \{y_k\}_{k=1}^{s}$. Scalar convolution acts as a tool for the act of criteria composition.

When solving a specific vector optimization problem, the decision maker chooses a model of the objective function in the form of a scalar convolution that is adequate to the given situation and assigns its parameters in accordance with his preferences. The most commonly used additive (linear) scalar convolution:

$$Y[y(x)] = \sum_{k=1}^{s} a_k y_k(x),$$

where a_k are the weight coefficients determined by the decision maker based on his utility function in a given situation. The Laplace principle in decision theory consists in the extremization of a linear scalar convolution. The principle of optimality is a rule that allows one to calculate a certain unified numerical measure of the effectiveness of a solution (the act of composition of criteria) based on the values of the criteria. The disadvantage (specificity) of using linear scalar convolution is the possibility of "compensating" one criterion at the expense of others.

In some cases, the decision maker considers the multiplicative scalar convolution adequate to the given situation:

$$Y[y(x)] = \prod_{k=1}^{s} y_k(x),$$

the extremization of which expresses Pascal's principle. This principle is adequate in tasks with a cumulative effect, when the action of some efficiency factors, as it were, enhances or reduces the influence of other factors. When maximizing partial criteria, the zero value of any of them completely suppresses the contribution of all others to the overall efficiency of the solution. In the aerospace industry, such an approach can be partly justified when each criterion (for example, reliability and safety) is critical and no improvement in other criteria can compensate for its low value. If at least one of the partial criteria is equal to zero, then the global criterion is also equal to zero.

The disadvantage of using multiplicative scalar convolution is that a very expensive and very efficient system can have the same score as a cheap and low efficient one. Let's compare such "weapon systems" as an atomic bomb and a slingshot, which, at a low cost, has some damaging factor. Guided by the multiplicative convolution, you can choose a slingshot to equip the army. Similarly to the Laplace principle, one can generalize the Pascal principle by introducing weight coefficients:

$$Y[y(x)] = \prod_{i=1}^{s} [y_i(x)]^{a_i}.$$

The Charnes-Cooper concept is based on the principle of "closer to the ideal (utopian) point". In the space of criteria, under given conditions and constraints, an a priori unknown ideal vector y^{id} is determined, for which the optimization problem is solved s times (according to the number of partial criteria), each time with one (next) criterion, as if there were no others at all. The sequence of "single-criterion" solutions of the original multi-criteria problem gives the coordinates of the unattainable ideal vector $y^{id} = \{y_k^{id}\}_{k=1}^{s}$.

After that, the objective function $Y(y)$ is introduced as a measure of approximation to the ideal vector in the space of optimized criteria in the form of some non-negative function of the vector $y^{id} - y$, for example, in the form of the square of the Euclidean norm of this vector:

$$Y(y) = \left\| \frac{y^{id} - y}{y^{id}} \right\| = \sum_{k=1}^{s} \left[\frac{y_k^{id} - y_k}{y_k^{id}} \right]^2 .$$

The disadvantage of this method is the cumbersome procedure for determining the coordinates of an ideal vector. It is important that the possibility of violation of restrictions is not excluded. In addition to those listed, other types of scalar convolutions are also used in practical research [1].

One of the most important provisions of the theory of decision making under many criteria is that there is no best solution in some absolute sense. The decision taken can be considered the best only for the given decision maker (DM) in accordance with the goal set by him and taking into account the specific situation. Normative models for solving multi-criteria problems are based on the hypothesis of the existence in the minds of decision makers of a certain utility function [4], measured both in nominal and ordinal scales. The reflection of this utility function is the trade-off scheme and its model in a given situation – a scalar convolution of partial criteria $Y[y(x)]$, which allows constructively solving the problem of multi-criteria optimization.

In the concept of optimality, in addition to criteria, restrictions play an equally important role, both in terms of optimization arguments and in terms of decision efficiency criteria. Even small changes can significantly affect the solution [14]. In addition, the very concept of a decision-making situation is evaluated by a measure of the dangerous approximation of individual partial criteria to their maximum permissible values. This is the basis for a possible approach to the formalization of the solution of multi-criteria problems.

In our case, the subject of research is such a subtle substance as an imaginary utility function that arises in the mind of the decision maker when solving a specific multi-criteria problem. If it exists, then each decision maker has his own utility function. Nevertheless, it is possible to obtain the prerequisites for setting a single type of meaningful model of the objective function if we identify and analyze some general patterns observed in the process of making multi-criteria decisions by different decision makers in different situations.

Formalization

Under some external influences, a situation may arise when one or more partial criteria approach their limitations. It is logical to consider the difference between the current value of the criterion and its maximum allowable value as a measure of the intensity of the situation:

$$\rho_k(x) = A_k - y_k(x), \rho_k \in [0, A_k], k \in [1, s],$$

where $A = \{A_k\}_{k=1}^{s}$ is the vector of maximum admissible minimized criteria.

If a multi-criteria decision is made in a tense situation, then this means that under given external conditions $r^o \in R$, one or more partial criteria $y_k(x), k \in [1, s]$, as a result of solving x, may be dangerously close to their limit values $A_k, k \in [1, s]$, that is, $\rho_k(x) \to 0$. And if one of them reaches the limit (or goes beyond it), then this event is not compensated by the possible low level of the remaining criteria – usually violation of any of the restrictions is not allowed.

In this situation, it is necessary to prevent in every possible way the dangerous increase of the most unfavorable (ie, closest to its limit) partial criterion, not taking much into account the behavior of the others at this time. And in a very tense situation (the first polar case: $\rho_k(x) \approx 0$) the decision maker generally leaves in the field of view only this one, the most unfavorable partial criterion, not paying attention to the rest.

Therefore, an adequate expression of the trade-off scheme in the case of a tense situation is the minimax (Chebyshev) model:

$$x^* = \arg\min_{x \in X} \max_{k \in [1,s]} \frac{y_k(x)}{A_k}. \quad (1)$$

In less stressful situations, it is necessary to return to the simultaneous satisfaction of other criteria, considering the contradictory unity of all the interests and goals of the system. At the same time, the decision maker varies his assessment of gain according to one criteria and loss according to others, depending on the situation. In intermediate cases, trade-off schemes are chosen that give different degrees of partial alignment of relative partial criteria. With a decrease in the intensity of the situation, preferences for individual criteria are aligned.

And, finally, in the second polar case ($\rho_k(x) \approx 1$), the situation is so calm that the partial criteria are small and there is no threat of violation of the constraints. The decision maker here believes that a unit of worsening of any of the relative partial criteria is fully compensated by an equivalent unit of improvement of any of the others. This case corresponds to an economical compromise scheme that provides the minimum total losses for given conditions according to relative partial criteria. Such a scheme is expressed by the integral optimality model:

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} \frac{y_k(x)}{A_k}. \quad (2)$$

The analysis shows that compromise schemes are grouped at two poles, reflecting different principles of optimality: 1) egalitarian – the principle of uniformity and 2) utilitarian – the principle of economy.

The application of the principle of uniformity expresses the desire uniformly, i.e., equally reduce the level of all relative criteria during the functioning of the system under study. An important implementation of the principle of uniformity is the Chebyshev model (1), the polar scheme of this group. This scheme forces us to minimize the worst (the largest under the minimized criteria), reducing it to the level of the others, i.e., leveling all relative partial criteria. The disadvantages of egalitarian schemes of uniformity include their "diseconomy". Ensuring the level of relative criteria that is closest to each other is often achieved by significantly increasing their total level. In addition, sometimes even a slight deviation from the principle of uniformity can significantly reduce one or more important criteria.

The principle of economy, which is based on the possibility of compensating for some deterioration in quality according to some criteria by a certain improvement in others, is devoid of these shortcomings. The polar scheme of this group is realized by the integral optimality model (2). The utility scheme provides a minimum summary level of relative criteria. A common disadvantage of schemes of the principle of economy is the possibility of a sharp differentiation of the level of individual criteria.

The performed analysis reveals a regularity due to which the decision maker varies his choice from the integral optimality model (2) in calm situations to the minimax model (1) in tense ones. In intermediate cases, the decision maker chooses compromise schemes that give different degrees of satisfaction of individual criteria, in accordance with the given situation. If we accept the conclusions from the above analysis as a logical basis for formalizing the choice of a trade-off scheme, then various constructive concepts can be proposed, one of which is the concept of a nonlinear trade-off scheme.

The Concept of a Nonlinear Compromise Scheme

In contrast to the concept of Charnes-Cooper based on the principle "closer to the utopian point", we will consider such an approach to the formalization of solutions to multi-criteria problems, in which the principle "away from restrictions" is fulfilled.

From the standpoint of a systematic approach, it is advisable to replace the problem of choosing a compromise scheme with an equivalent problem of synthesizing a certain unified scalar convolution of partial criteria, which in various situations would automatically express adequate principles of optimality. Separate models of trade-off schemes are combined into a single integral model, the structure of which is adapted to the situation of making a multi-criteria decision. Requirements for the synthesized function $Y[y(x)]$:

- it must be smooth and differentiable;
- in tense situations, it should express the minimax principle;
- in calm conditions – the principle of integral optimality;
- in intermediate cases should lead to Pareto-optimal solutions, giving different measures of partial satisfaction of the criteria.

In other words, such a universal convolution should be the expression of a trade-off scheme that adapts to the situation. It can be said that adaptation and the ability to adapt are the main content essence of the study of multi-criteria systems. In order for the principle "away from restrictions" to be fulfilled for any$r \in R$, it is necessary that the expression for scalar convolution explicitly include the characteristics of the intensity of the situation. Several functions can be considered that satisfy the above requirements. The simplest of them in the case of minimized criteria is the scalar convolution

$$Y(y) = \sum_{k=1}^{s} A_k \left[A_k - y_k(x)\right]^{-1}.$$

Thus, a nonlinear trade-off scheme is proposed, which corresponds to the vector optimization model, which explicitly depends on the characteristics of the intensity of the situation:

$$x^* = \arg\min_{x \in X} \sum_{k=1}^{s} A_k \left[A_k - y_k(x)\right]^{-1}. \quad (3)$$

It can be seen from this expression that if any of the partial criteria, for example yi(x), starts to come close to its limit A_i, i.e. the situation becomes tense, then the corresponding term $Y_i = \frac{A_i}{A_i - y_i(x)}$ in the sum being minimized will increase so much that the problem of minimizing the entire sum will be reduced to minimizing only the given worst term, i.e., ultimately, the criterion $y_i(x)$. This is equivalent to the action of the minimax model (1). If the partial criteria are far from their limits A_i , i.e., the situation is calm, then model (3) acts equivalent to the integral optimality model (2). In intermediate situations, various degrees of partial alignment of the criteria are obtained.

The non-linear trade-off scheme has the property of continuous adaptation to the situation of making a multi-criteria decision. It has been repeatedly emphasized above that the choice of a compromise scheme is the prerogative of a person, a reflection of his subjective utility function in solving a specific multi-criteria problem. Nevertheless, we managed to identify some general patterns and, on this objective basis, construct a universal scalar convolution of criteria, the form of which follows from meaningful ideas about the essence of the phenomenon under study.

The solution of a multi-criteria problem according to a nonlinear scheme of compromises is carried out in a formalized manner, without the direct participation of the decision maker. This solution is basic and intended for general use. If such a task is solved in the interests of a particular person, then the basic solution can only be adjusted in accordance with the informal preferences of the decision maker.

The above analysis refers to the case of minimized criteria, where "better" means "less". For maximizable criteria, the unified scalar convolution has the form:

$$Y(y) = \sum_{k=1}^{s} B_k [y_k(x) - B_k]^{-1},$$

where B_k are the minimum allowable values of the criteria to be maximized.

Illustration Example

Consider the problem of distributing a limited global resource of fuel between aircraft when performing flights to different cities. For each flight there is a lower limit, below which it is pointless to allocate fuel, the plane simply will not reach its destination. This is the essence of the lower bound for each partial resource. If this flight receives fuel in excess of the known lower limit, then it has the opportunity to freely maneuver along the echelons, bypass a thunderstorm front, go to an alternate airfield, etc. On the other hand, it is also impossible to increase a partial resource indefinitely, there is an upper limit for it. This is understandable, if only because each aircraft has a certain tank capacity, more than which it cannot physically take fuel on board.

Considering the given set of restrictions, it is required to distribute the global resource of the system between objects in such a way that the most efficient operation of the entire system as a whole is ensured. We will solve this problem within the framework of the concept of a nonlinear trade-off scheme. We represent the objective function in the form:

$$f(p) = \sum_{i=1}^{n} p_{i\min}(p_i - p_{i\min})^{-1},$$

where $p = \{p_i\}_{i=1}^{n}$ is the vector of partial resources, $p \in X_p = [0, P]$; $P_{min} = \{P_{i\ min}\}_{i=1}^{n}$ is the lower constraint vector of partial resources. It is clear that $\sum_{i=1}^{n} p_i = P$, where P is the global resource to be distributed.

The presented objective function is nothing more than an expression of the scalar convolution of the vector of maximized partial criteria $p = \{p_i\}_{i=1}^{n}$ according to the nonlinear compromise scheme (NSC) in the problem of multi-criteria optimization [2].

Indeed, in the problem under consideration, resources $p_i, i \in [1, n]$ have a dual nature. On the one hand, they can be considered as independent variables, arguments of objective function $f(p)$ of optimization. On the other hand, it is logical for each of the objects to strive to maximize its partial resource, to go as far as possible from a dangerous limitation $p_{i\ min}$ in order to increase the efficiency of its functioning.

From this point of view, resources $p_i \geq p_{imin}$ can be considered as partial criteria for the quality of the functioning of the corresponding objects. These criteria are subject to maximization, they are limited from below, non-negative and contradictory (an increase in one resource is possible only at the expense of a decrease in others).

Based on the foregoing, the problem of vector optimization of the distribution of limited resources, taking into account the isoperimetric constraint for the arguments $\sum_{i=1}^{n} p_i = P$, takes the form

$$p^* = \arg\min_{p \in X_p} f(p) = \arg\min_{p \in X_p} \sum_{i=1}^{n} p_{i\min}(p_i - p_{i\min})^{-1}, \sum_{i=1}^{n} p_i = P$$

This problem can be solved both analytically, using the method of indefinite Lagrange multipliers, and numerically, if the analytical solution turns out to be difficult.

The analytical solution provides for the construction of the Lagrange function in the form:

$$L(p, \lambda) = f(p) + \lambda\left(\sum_{i=1}^{n} p_i - P\right),$$

where λ is the indefinite Lagrange multiplier, and the solution of the system of equations

$$\frac{\partial L(p, \lambda)}{\partial p_i} = 0, i \in [1, n]$$

$$\frac{\partial L(p, \lambda)}{\partial \lambda} = \sum_{i=1}^{n} p_i - P = 0.$$

Numerical example

To perform two flights (n =2), the airport has fuel with a total volume of P =12 tons (conditional figures). The minimum requirement for the first run is $p_1 \geq p_{1min}$= 2 tons, for the second run $p_2 \geq p_{2min}$= 5 tons. These are the lower bounds for partial resources.

The task is set: to obtain a solution for a compromise-optimal distribution of fuel between flights.

We solve the problem of vector optimization of the distribution of limited resources analytically using the method of indefinite Lagrange multipliers.

We build the Lagrange function:

$$L(p,\lambda) = p_{1\min}(p_1 - p_{1\min})^{-1} +$$
$$+ p_{2\min}(p_2 - p_{2\min})^{-1} + \lambda(p_1 + p_2 - P).$$

We get a system of equations:

$$\frac{\partial L(p,\lambda)}{\partial p_1} = -p_{1\min}(p_1 - p_{1\min})^{-2} + \lambda = 0$$
$$\frac{\partial L(p,\lambda)}{\partial p_2} = -p_{2\min}(p_2 - p_{2\min})^{-2} + \lambda = 0$$
$$p_1 + p_2 - P = 0.$$

Substituting numeric data

$$-2(p_1 - 2)^{-2} + \lambda = 0$$
$$-5(p_2 - 5)^{-2} + \lambda = 0$$
$$p_1 + p_2 - 12 = 0.$$

and solving this system by the Gauss method (successive elimination of variables), we obtain p_1 $*$=3,94 tons, p_2 $*$=8,06 tons.

In more complex cases, numerical methods or a computer program for multi-objective optimization are used [1].

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FORMALIZED METHOD OF THE SOLUTION OF MULTI-CRITERIA PROBLEMS

Multi-criteria (vector) optimization involves finding a set (Pareto area) of acceptable solutions. You usually only need to choose one of them. Since the points of the Pareto set are formally incomparable, in order to solve the problem, it is fundamentally necessary to involve information about the preferences of the person making the decision. When solving a specific problem of vector optimization, the decision-maker creates his own model of the objective function (utility function) according to his preferences. Thus, the solution of multi-criteria problems is subjective in nature. The article proposes a formalized method for solving multi-criteria problems.

A model of multi-criteria optimization is obtained, which allows the object to realize all the goals set in the entire range of possible situations. A systematic approach to the problem of vector optimization made it possible to combine models of individual trade-off schemes into a single integral structure that adapts to the situation of making a multi-criteria decision. The advantage of the concept of a non-linear trade-off scheme is the possibility of making a multi-criteria decision formally, without the direct participation of a person. At the same time, on a single ideological basis, both tasks that are important for general use, and those which main content essence is the satisfaction of individual preferences of decision makers, are solved. The apparatus of the nonlinear trade-off scheme, developed as a formalized tool for studying systems with conflicting criteria, makes it possible to practically solve multi-criteria problems of a wide class.

***Keywords**: optimization, multi-criteria, utility function, scalar convolution, formalization, situation, nonlinear trade-off scheme.*

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ФОРМАЛІЗОВАНИЙ МЕТОД РОЗВ'ЯЗУВАННЯ БАГАТОКРИТЕРІАЛЬНИХ ЗАДАЧ

Багатокритеріальна (векторна) оптимізація передбачає знаходження набору (області Парето) прийнятних рішень. З них, зазвичай, потрібно вибрати лише одне. Оскільки точки множини Парето формально непорівнянні, то для вирішення задачі принципово необхідно залучати інформацію про переваги особи, яка приймає рішення. При вирішенні конкретної задачі векторної оптимізації особа, яка приймає рішення, створює власну модель цільової функції (функції корисності) відповідно до своїх уподобань. Таким чином, рішення багатокритеріальних задач носить суб'єктивний характер. В статті запропоновано формалізований метод розв'язування багатокритеріальних задач.

Отримано модель багатокритеріальної оптимізації, яка дозволяє реалізувати об'єкту всі поставлені цілі у всьому спектрі можливих ситуацій. Системний підхід до проблеми векторної оптимізації дозволив об'єднати моделі окремих компромісних схем в єдину цілісну структуру, що адаптується до ситуації прийняття багатокритеріального рішення. Перевагою концепції нелінійної компромісної схеми є можливість прийняття багатокритеріального рішення формально, без прямої участі людини. Водночас на єдиній світоглядній основі вирішуються як загальнокорисні завдання, так і ті, основною змістовою сутністю яких є задоволення індивідуальних уподобань осіб, які приймають рішення. Апарат нелінійної компромісної схеми, розроблений як формалізований інструмент для дослідження систем із конфліктними критеріями, дає змогу практично розв'язувати багатокритеріальні задачі широкого класу.

***Ключові слова**: оптимізація, багатокритеріальна оптимізація, функція корисності, скалярна згортка, формалізація, ситуація, нелінійна компромісна схема.*