!" 621.396.4

[1] !"$&'(* ., ., #.$.%,
[2]-/12&2&/ 3.5 ., &$.%,
[2]5/7/8$&'(3.: .

5<:*> ,5-3@B@3-*C,> DE5FC,<5* * <G<GH355<I
@*G<-J : K*L*M*N <B-,P* R5<S< EB@*: 35,>
P* <S*G*@,-5JP, G3UB, <-5JP, 3-*-3 R5JP,
BB@*-*P,

[1]')+,-%)/0%12 $35%,738&,2%,<3=8,$3$ &=),%1 «"?A »
[2]')+,-%)/0%12)<,)+,-%%12 :%,<3=8,3

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"$&'(*$+, +(./ 0+2$"4"$2,60/ 78+:60(+,', (;(;<$++(= ",;(2> . ?,&,@,A(420 - C,'D+(E(84",.'$+0/ C,'(E,;,"02+>C0 ;$F40'(2+>C0 '$2,2$'D+>C0 ,44,",2,C0

FWX1$Y!$ "W(Y/: -B$,C)/0%-3 :B=)</3%,3, #,%)C,738&)D8,8$3C), =3&:==3%$%1,2E- - =,$C

"#%#&'#

F %)8$-DH33<=3CD1,=-&-3 B=,C3-%3%,3B-/:7,/, J38B,/-$%13 /3$)$3/0-%13)BB)=)$1 (K?LM), O$-C: 8B-8-J8$-<:Q$ %-<32I,3 #-8$,R3%,D < -J/)8$, O/3&$=-%,&, <17,8/,$3/0%-2 $35%,&, 8=3#8$<8<DS, 8,8$3C:B=)</3%,D.

A%$3%8,<%#1D)S=)J-$&) K?LM C%-E-&=)$%-E-B=,C3%3%,D1D</D3$8D5)=)&-$3=%-2 $3%#3%+,32=)S<,$,D C,=-<-E-)<,)8$=-3%,D< %)8$-DH33<=3CD, < J/, -R)2I32 B3=8B3&$,<-3

?3=8B3&$,<%1C,-J/)8$DC, B=,C3-%3%,B K?LM D</DQ$8D-J8/3#-<)%,3 $3=-=,$-=,2 #/D B-,8&) -7)E-< B-R)=-<, S)-E=DS%3%&2)=$-E=)T,=-<)%,3 C38$%-8$, B-,8& /Q#32 %)8:I3 , %)<-#3, B=,C3%3%,3 < <-3%%15J/)8$D5, B=, O$-C%),J- -/33 OTT3&$,<%1C3 $-7&, S=3%,DS)$=)$ D</D3$8D B=,C3%3%,3 C)/-E)J)=,$%15 J38B,/-$%15 /3$)$3/0%15)BB)=)$-< (UK?LM).

?=3,C:H38$<)C, C)/-E)J)=,$%15 K?LM D</DQ$8D<18-&)D C-J,/0%-8$0, C%-E-T:%&+,-%)/0%-8$0 -$%-8,$3/0%)D #3I3<,S%) &-%8$=:&+,, ?=, O$-C& J-=-$-<-C: -J-=:#-<)%,Q $)&,5 K?LM B=3#WD</DQ$8D8$&,3C)88-E)J)-=,$%13 $=3J-<)%,D &-$-=13 -J:8/)</, -<)Q$ B=,C3%3%,3C3$-#-< -B$,C)/0%-E-:B=)</3%,D, -J38B37,<)QH,3 <18-&-3 &)738$<-,5 T:%&+,-%,=-<)%,D B=, O&-%-C,, =38:=8-<, B-<1I3%,, J18$=-#32-8$<,D, %)/-R3%,, =)S/,7%-E- =-#) -E=)-%,73%,2

X)S<,$,3 C)$3C)$,738&-E-)BB)=)$) #/D =)S=)J-$&, -B$,C)/0%15 B- J18$=--#328$<,Q <17,8/,$3/0%15)/E-=,$C-<B=, O$-C8$=-,$8D%) -8%-<3,8B-/0S-<) -%,D&/)88,738&,5 &=,$3=,3<-B$,C)/0%--8$,.

X3I3%,3 S)#)7 -B$,C)/0%-E- :B=)<-/3%,D8 ,8B-/0S-<)%,3C &/)88,738&,5 C3-$-#-< <)=,)+,-%%-E- ,87,8/3%,D, B=,%+,B) ?-%$=DE,%)1.Z. [7], #,%)C,738&-E-B=-E=)CC,=-<)%,DK3//C)%) [1], 7,8/3%-%15 C3$-#-< C)$3C)$,738&-E- B=-E=)GC,=-<)%,D, 8<DS)%8 =3I3%,3C #<:5$--737%-2 &=)3<-2 S)#)7, [2], , #/D %3/,-%32%15,8,8$3C$=3J:3$<18-&,5 <17,8/, -$3/0%15 S)$=)$ B=3BD8<:QH,5=3)/, -S)+,, O$,5 C3$-#-< < B=-+3883B-/3$). \$- D</D3$8D9:H38$<3%%1Q%3#-8$)$&-C8,%$3S)B=)</3%,D. F =3S:/0$)$3,8B-/0 -S-<)%,D O$,5 C3$-#-< B-/:7)3$8D B=--E=)CC%-3:B=)</3%,3, %38B-8-J%-3 OTT3&$,<%- =3I)$0 S)#)7: B=,<3#3%,D UK?LM < S)#)%%-3-8$-D%,3J3S&-==3&$,=-<&, 8)C-E-)/E-=,$C) :B=)</3%,D B=, ,SC3%3%,B)=)C3$=-<,85-#%-2 8,8-$3C1, :8/-<,2 33T:%&+,-%,=-<)%,D.

F 8<DS,8 O$,C, #/D)<$-C)$,738&-E- :B=)</3%,D UK?LM B=3#B-7$,$3/0%1)#)B$,<%13)/E-=,$C1 8 C)/1C -JW3C-C <17,8/3%,2, 8B-8-J%1 T-=C,=-<)$0 :B=)</3%,3 < ,SC3%DQH,58D:8/-<,D5 B-/3$). " $)&,C C3$-#)C < $3-=,, -B$,-C)/0%-E- :B=)</3%,D -$%-8,$8D C3$-# -B$,C,S)+,, B- T:%&+,-%)/: -J-JH3%-%-2=)J-$1 "=)8-<8&-E- M.M. ^#380S)#)-%,3 $=3J-<)%,2 %) &-%37%-38-8$-D%,3 <1B-/%D3$8D:B:$3C B-#J-=) 8--$<3$8$ <:QH32 7)8$, &)738$<), B=,<-#,$ & &-CB=-C,88%-C:=3S:/0$)$:.

!/D T-=C,=-<)%,D -B$,C)/0%-E- :B=)</3%,D K?LM %3-J5-#,C- < =3)/0-%-CC)8I$)J3 <=3C3%B-/:7)$0 #)%%13 B)=)C3$=)5 #<,R3%,D A8B-/0S-<)%,3)#)B$,<%-2 C-#3/, #/D =)87_$) -B$,-C)/0%-2 $=)3&$-=,, -8-J-)&$:)/0%-, B--8&-/0&: B-S<-/D3$T-=C,=-<)$0 $=)3&$-=,, B-/_$) 8 B=,3C/3C-2 $-7%-8$0Q < $-C 7,8/3 B=, 8-&=)H3%,, -JW_C) #)%-%%15- $3&:H3CB-/-R3%,, K?LM.

F #)%%-28$)$03=)88C)$=,<)3$8D:S)-#)7) T-=C,=-<)%,D -B$,C)/0%-E- :B=)<-/3%,D B-/_$-C UK?LM &)& $<_=#1C$3-/-C < B=-8$=)%8$<B=, -#%-<=3C3%%-C <1B-/%3%,, $=3J-<)%,2 & C,%,C,S)+,, O%3=E3$,738&:5)$=)$%) :B=)</3%,3 B--/3$-C.

)*,.0&*"20 30%04'

UKLM &)& -JW3&$B=)</3%,D B=3#8$)</D3$8-J-2 8/-R%:Q #,%)C,738&:Q 8,8$3C:. X)88C-$=,C S)#)7: -B$,C)/0%--E- :B=)</3%,D #,%)C,738&-2 8,8$3C-2 8:7_$-C -E=)%,73%,2%) :B=)</3%,3 B=,-#%-<=3C3%%-C1B-/%3%,, $=3J-<)%,2& C,%,C,S)+,, O%3=E3$,738&:5)$=)$

^)#)7:)<$-C)$,738&-E- :B=)</3%,D UK?LM , =)88C)$=,<)3C:Q %,R3 C-R%-%)S<)$0#<:5:=-<%3<-2 Z:H%-8$0 O$-2 S)#)7, 8-8$-,$ < 8/3#:QH3C.

AC33$8D#,%)C,738&)D8,8$3C)

$$M = f(x, u, W_1, t), \qquad (1)$$

$$M \in E^n, u \in E^{m_1}, W_1 \in E^{m_2},$$

E#3 x - <3&$-=T)S-<-E- 8-8$-D%,D u, W_1- <3&$-=%13B=)</3%,D, &-$-=:Q %3-J5-#,C- B3=3<38$,S 8-8$-D%,D

$$(M(t_0), t_0) \in Q_0 \qquad (2)$$

< 8-8$-D%,3

$$(M(t_f), t_f) \in Q_f \qquad (3)$$

$)&, 7$-J1 C,%,C,S,=-<)$0 &=,$3=,2

$$I = S(M(t_f), t_f) + \int_{t_0}^{t_f} L(x, u, W_1, W_2, t)\, dt \to \min_{u(\cdot)} \qquad (4)$$

E#3 W_2-< -JH3C 8/:7)3 C)$=,7%)DT:%&+,D :B=)</3%,D=)SC3=)$m_2 \times m_2^*$,

B=, :8/-<,, , 7$- $W_1 = W_1(x, u, t)$, $W_2 = W_2(x, u, t)$,S<38$%13T:%&+,, <=3C3%, , -B$,C)/0%15 S%)73%,2T)S-<15 &--=#,%)$, :B=)</3%,2, $.3.

$$I = I(M(t_0), t_0; M(t_f), t_f); u(\cdot), x(\cdot))\big|_{W_i(x,u,t)(i=1,2)}?$$

-8$)</3%%:QS)#)7: %)S-<3CS)#)732B3=<-E- :=-<%D b3 =3I3%,3 C3$-#-C C,%,-C:C) #)Q$ 8/3#:QH,3 :=)<%3%,D[3,4,5]:

$$\partial H / \partial u\big|_{\wedge} = 0, \qquad (5)$$

$$\dot{\lambda} + \partial H / \partial x\big|_{\wedge} = 0, \qquad (6)$$

$$\dot{x} = f(x, u, W_1(x, u, t), t), \qquad (7)$$

$$(M(t_0), t_0) \in Q_0, \ (M(t_f), t_f) \in Q_f, \qquad (8)$$

E#3

$$H(x, u, \lambda, t) = \left[\begin{matrix} L(x, u, W_1, W_2, t) + \\ + \lambda^T f(x, u, W_1, t) \end{matrix} \right]\Big|_{W_i(x,u,t)(i=1,2)}$$

, C3$-#-C #,%)C,738&-E- B=-E=)CC,=-<)%,D=3I3%,3 -B,81<)3$8D:=)<%3%,DC,

$$\frac{\partial}{\partial u}\left[L(x, u, W_1, W_2, t) + \left(\frac{\partial V}{\partial x}\right)^T \Big|_{\wedge} f(x, u, W_1, t) \right] = 0, \qquad (9)$$

$$-\frac{\partial V}{\partial t} = L(x, u, W_1, W_2, t) + \left(\frac{\partial V}{\partial x}\right)^T \Big|_{\wedge} f(x, u, W_1, t), \qquad (10)$$

$$V(M(t_f), t_f) = S(M(t_f), t_f); \qquad (11)$$

$$\dot{M} = f(M, u, W_1, t); \qquad (12)$$

$$(M(t_0), t_0) \in Q_0, (M(t_f), t_f) \in Q_f, \qquad (13)$$

E#3

$$V(M(t), t) = \left[\begin{matrix} \min_{u(\cdot)} \int_t^{t_f} L(x, u, W_1, W_2, t)\, dt + \\ + S(M(t_f), t_f) \end{matrix} \right]\Bigg|_{W_i(x,u,t)(i=1,2).}$$

^)#)7: <$-=-E- :=-<%D 8-8$)</DQ$:=)<%3%,D(5-8) ,/, (9-13). b3 =3I3%,3 8-8$-,$ < <1J-=3 :B=)</3%,2 $W_i(x, u, t)$ $(i = 1, 2)$, &-$-=13 J1 -J38B37,/,

:=)<%3%,DC(5-8) ,/, (9-13) <1E-#%13 8<-28$<)

X)88C-$=,C %38&-/0&- B-#5-#-<, B-S<-/DQH,5 #)$0 8-#3=R)$3/0%-3$-/ -&-<)%,3<$-=-C: :=-<%QS)#)7,:

1. ?:8$0 #,%)C,738&)D8,8$3C) (1) -J/)#)3$:B=)</3%,DC, =)S/,7%-2 OTT3& $,<%-8$„ 7)8$0 ,S &-$-=15 C-R3$ J1$0 -$%383%& <3&$-=: U ,) #=:E)D7)8$0 - & <3&$-=: W_1. X3I)D S)#)7: %) <$-=-C :=-<%3 J:#3C ,8&)$0 $)&,3 :B=)</3%,D W_1, &-$-=13 J1 -J38B37,<)/, %),J-/33 J/)E-B=,D$%13:8/-<,D #/D B=,C3%3%,D :B=)</3%,2 B3=<-E-:=-<%DU.

d)&, %)B=,C3=, < &)738<3W_1$ C-E:$ =)88C)$=,<)$08D:B=)</3%,D, -B=3#3/DQH,3 &-%T,E:=)+,Q -JW3&$) F-S%,&)3$ <-SC-R%-8$0B-#8$=),<)$0 -JW3&$8 B--C-H0Q W_1 $)&, 7$-J1 :B=)</3%,3 %,C 8 B-C-H0Q U J1/- %),J-/33 OTT3&$,<%1C

2. B=)</3%,3 W_2 <-S#328$<:3$%) :=)<%3%,D B- &-$-=1C <17,8/D3$8D :B=)</3%,3 U, B=,#)<)D O$-C: :B=)</3-%,Q #-B-/%,$3/0%13 8<-28$<) &-$-=13, < &-%37%-C873$,3 8&)S1<)Q$8D%) 5)=)&$3 =3#<,R3%,D8,8$3C1 (1). A8B-/0S:D &-%+3B+,Q :B=)</3%,D W_2, C-R%- =3I)$0 8 -#%-2 8$-=-%1, B=-J/3C: <1J-=) <38-<15 &-OTT,+,3%$) T:%&+,-%)/) (4), =)88C)$ =,<)D O$, &-OTT,+,3%$1 &)& :B=)</3%,3 W_2,) 8 #=:E-2 8$-=-%1 :B=)</D$0 8/-R -%-8$0Q:=)<%3%,2 B- &-$-=1C <17,8/D-3$8D:B=)</3%,3 U. F 7)8$%-8$„ B-8/3#-%33B=3#/-R3%,3 ,C33$ 8C18/ < 8/:7)3 %3$-7%-E-S%)73%,DB)=)C3$=-< -JW3&$) :B=)</3%,D, <8/3#8$<,373E- %3$%3-J5-#,C-8$,)J8-/Q$%- $-7%-=3I)$0 :=)<%3%,D, -B=3#3/DQH,3 -B$,C)/0%-3 :B=)<-/3%,3.

"=-C3 $-E-, $)&-2 B-#5-# C-R%-=)88C)$=,<)$0, &)& -#,% ,S 8B-8-J-< -8:H38$</3%,D=3#:&+,, C)$3C)$,738&,5 C-#3/32 -B$,C)/0%15 B=-+388-<(7) ,/, (8) [4].

3. F 8/:7)3 %3&-==3&$%B-8$)</3% %15 S)#)7 ,C33$ 8C18/ ,8B-/0S-<)$0 :B=)</3%,3 W_2 < &)738$<3 8$)J,/,S, -=:QH32 #-J)<&, & T:%&+,-%)/: , 7$- B--S<-/,$ -=E)%,S-<)$0:8$-27,<:Q B=-+3-#:=: B=,J/,R3%%-E- %)5-R#3%,D-B$,-C)/0%-E-:B=)</3%,D[6].

ZB-8-J -$18&)%,D$W_i(x, u, t)$ $(i = 1, 2)$ 8:H38$<3%%S)<,8,$ -$,85-#%-E- B=3#8$)</3%,D &=,$3=,D , C-#3/, -JW3&$) X)88C-$=,C %3&-$-=-3 &-%8$=:&$,<%13 =3S:/0$)$1, B-/:73%%13B=, ,8B-/0S-<)-%,, <$-=-E- B-#5-#) < 8/:7)3 , &-E#)

$$f(x, u, W_1, t) = \varphi(x, t) + \psi(x, u, W_1, t), \qquad (14)$$

$$L(x, u, W_1, W_2, t) = Q(x, t) + Z(x, u, W_1, W_2, t), \qquad (15)$$

f)C,/0$-%,)% B=,%,C)3$<,#

$$H(x, u, W_1, W_2, \lambda, t) = Q(x, t) + \lambda^T \varphi(x, t) + Z(x, u, W_1, W_2, t) + \lambda^T \psi(x, u, W_1, t) \qquad (16)$$

A88/3#:3C 3E-%, C,%,C:C B- U:

$$\left(\frac{\partial H}{\partial U}\right)^T = \left(\frac{\partial Z}{\partial U}\right)^T + \lambda^T \left(\frac{\partial \psi}{\partial U}\right)^T = 0. \qquad (17)$$

AS8--$%-I3%,D (17) B-/:7)3C , 7$-

$$\lambda^T \psi = -\left(\frac{\partial Z}{\partial U}\right)^T \left(\frac{\partial \psi}{\partial U}\right)^{-1} \psi. \qquad (18)$$

?-8/3 B-#8$)%-<&,$\lambda^T\psi$ < (16) %)-5-#,C <1=)R3%,3#/D C,%,C,S,=-<)%%-E- E)C,/0$-%,)%)

$$H(x, u, W_1, W_2, \lambda, t) = Q(x, t) + Z(x, u, W_1, W_2, t) + \lambda^T \varphi(x, t) - \left(\frac{\partial Z}{\partial U}\right)^T \left(\frac{\partial \psi}{\partial U}\right)^{-1} \psi = Q(x, t) + \lambda^T \varphi(x, t), \qquad (19)$$

E#3:B=)</3%,D W_1 и W_2 <1J=)%1 $)&, 7$-J1 <1B-/%D/-80 =)<3%8$<-

$$Z(x, u, W_1, W_2, t) - \left(\frac{\partial Z}{\partial U}\right)^T \left(\frac{\partial \psi}{\partial U}\right)^{-1} \psi = 0. \qquad (20)$$

7,$1<)D (14-15), (19-20),=3S:/0$)$ =3I3%,D S)#)7, <$-=-E- :=-<%D, S)#)<)3-C-2 :=)<%3%,DC,(5-8) ,/, (9-12), S)B,-I3$8D 8--$<3$8$<3%%<,#3

$$\left(\frac{\partial Z}{\partial U}\right)^T \Bigg|_{\wedge} + \lambda^T \left(\frac{\partial \psi}{\partial U}\right)_{\wedge} = 0, \qquad (21)$$