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SELF-BASIS OPERATOR AND ORTHOGONAL STOCHASTIC BASIS APPLICATION FOR INFORMATION PROCESSING

Introducing the presented self-basis operator either in real or complex form can be a convenient way to get an autocorrelated signal for different fields of applied mathematics, for example for computerized composition of music, involving "AQUARIUS" software."White-noise" orthogonal stochastic basis can be a useful tool for digital signal processing.

Key words: matrix operator, orthogonal stochastic basis, cross-correlated signal, autocorrelated signal, correlation index, phasor.

Introduction. Let`s take any 3D-object, described by some function of geometrical coordinates x_1, x_2 and x_3 [1]:

$$s(x_1, x_2, x_3) = \iiint f(x_1, x_2, x_3)\, dx_1\, dx_2\, dx_3$$

Function $f(x_1, x_2, x_3)$ can describe, for example distribution of objects` temperature, colour, porosity etc.

Let there be an arbitrary signal $s(t) = s(x_i)$, $(i = 1, 2, 3)$, got by 1-dimensioning of $s(x_1, x_2, x_3)$.

Statement of the problem. How to find a signal $\overset{*}{s}(t)$ resembling initial signal $s(t)$ in the sense of being autocorrelated.

Research and publications analysis. A lot of important results about the synthesis and use of Walsh-Cooley basis has been presented in [2]. Stochastic basises were thoroughly investigated and developed in [3 – 6].

Stochastic white-noise basis realization cut as a matrix operator for modifying of initial signal. Let it be at the input of some device, aimed on obtaining an orthogonal stochastic basis, a "white noise" signal. Structural scheme of such a device is given in fig. 1.

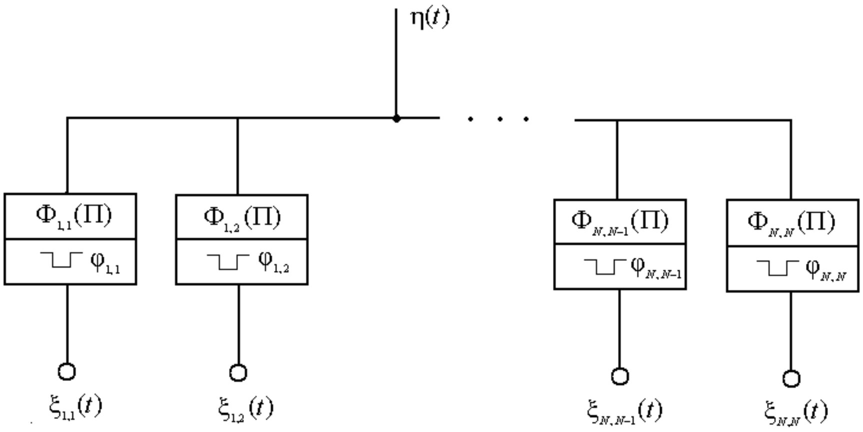

Fig. 1. Device for obtaining a stochastic basis

As it is seen from above, it is composited of infinite number of filters $\Phi_{i,j}(\Pi)\,(N \to \infty)$, having same bandwidth $\Pi_i = f^{\,i}_{top} - f^{\,i}_{bottom}$ and rectangularly shaped amplitude-frequency characteristic. The filters embrace the whole spectrum of initial signal $\eta(t)$, without being mutually covered, and $f^{\,i}_{top} = f^{\,i+1}_{bottom}$.

It is proven in mathematics [6], that a sequence of stochastic processes $\xi(t)$

$$\xi(t)=\begin{vmatrix}\xi_{1,1}(t) & \xi_{1,2}(t) & \ldots & \xi_{1,N}(t)\\ \xi_{2,1}(t) & \xi_{2,2}(t) & \ldots & \xi_{2,N}(t)\\ \ldots & \ldots & .. & \ldots\\ \xi_{N,1}(t) & \xi_{N,2}(t) & \ldots & \xi_{N,N}(t)\end{vmatrix} \quad (1)$$

makes an orthogonal basis in space L_2 if $N\to\infty$. Here

$$\begin{aligned}&\xi_{1,1}(t)=\int\varphi_{1,1}(t-\tau)d\eta(\tau),\\ &\xi_{1,2}(t)=\int\varphi_{1,2}(t-\tau)d\eta(\tau),\\ &\ldots,\\ &\xi_{N,N}(t)=\int\varphi_{N,N}(t-\tau)d\eta(\tau),\end{aligned}$$

or

$$\begin{aligned}&\xi_{i,j}(t)=\int\varphi_{i,j}(t-\tau)d\eta(\tau),\\ &i,j=1,2,\ldots\end{aligned}$$

Let us have now a signal $s(t)$, digitized in such a way, that it consists of N readouts, and our stochastic white-noise basis realization cut, taken at the time moment t_k, to be given at a multiplier. Then at the output of multiplier we'll get a signal $\overset{*}{s}(t)$, cross-correlated with both $s(t)$ and $\xi(t)$:

$$\overset{*}{s}(t)=\begin{vmatrix}\xi_{1,1}(t)_k & \xi_{1,2}(t)_k & \ldots & \xi_{1,N}(t)_k\\ \xi_{2,1}(t)_k & \xi_{2,2}(t)_k & \ldots & \xi_{2,N}(t)_k\\ \ldots & \ldots & .. & \ldots\\ \xi_{N,1}(t)_k & \xi_{N,2}(t)_k & \ldots & \xi_{N,N}(t)_k\end{vmatrix}\bullet\begin{vmatrix}s(t)_1\\ s(t)_2\\ \ldots\\ s(t)_N\end{vmatrix}=\begin{vmatrix}\overset{*}{s}(t)_1\\ \overset{*}{s}(t)_2\\ \ldots\\ \overset{*}{s}(t)_N\end{vmatrix}.$$

So, *stochastic white-noise basis realization cut* $\xi(t)_k$ *can be used as a matrix operator for modifying initial signal* $s(t)$, what could have become a very good thing, improving functional-aesthetics possibilities of "AQUARIUS" [1], solving in such a way the problem of investigation.

Notion of self-basis operator. Let`s introduce a notion of self-basis operator.

A self-basis operator is a matrix, consisting of elements of initial signal, by multiplying which times the digitized initial signal we obtain an autocorrelated (self-resembling) signal.

We could give it a following form [2]:

$$\Psi=\begin{vmatrix}\psi_{1,1}(t) & \psi_{1,2}(t) & \ldots & \psi_{1,N}(t)\\ \psi_{2,1}(t) & \psi_{2,2}(t) & \ldots & \psi_{2,N}(t)\\ \ldots & \ldots & \ldots & \ldots\\ \psi_{N,1}(t) & \psi_{N,2}(t) & \ldots & \psi_{N,N}(t)\end{vmatrix}, \quad (2)$$

where $\psi_{i,j}$ – self-basis operator functions for signal $s(t)$, got in a way presented in fig. 2, *a*:

After such a presentation, we could give (2) in a more obvious way (fig. 2, *b*).

As it can be seen from fig. 2 those self-basis functions $\psi_{i,j}$ are nothing more than little "wavelets", having, of course all signs of orthogonality. So we can say, that our *self-basis operator is a real and orthogonal one.*

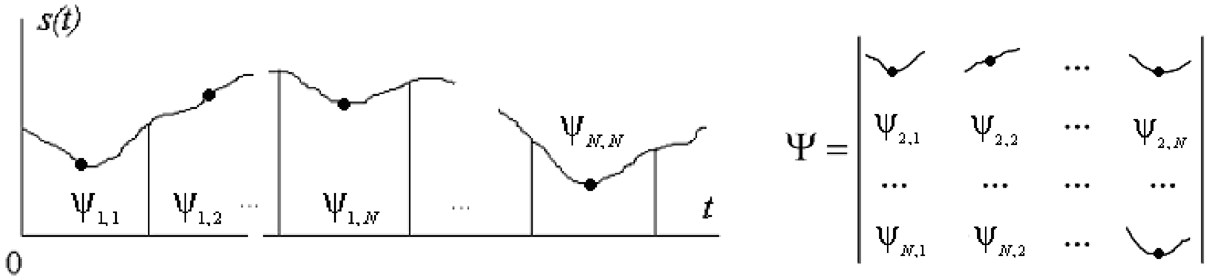

Fig. 2. Self-basis functions: *a* – getting; *b* – matrix representation

Now, after constructing the concept of self-basis operator, let`s move to making some “expantion” involving it.

In order to do this, let`s take some $N$ equidistant points on the initial signal $s(t)$ (fig. 3, *a*). There will be as many coefficients, as the number of self-basis functions in a row or column of a square matrix (1). Then, let`s shift their positions on t axis to ψ (fig. 3, *b*):

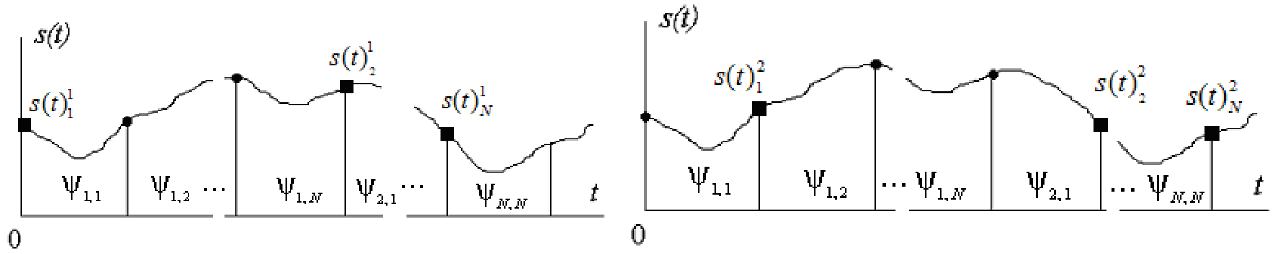

Fig. 3. Expansion coefficients: *a* – introducing; *b* – shifting

Making the multiplication will yield:

$$s(t)_1=\begin{vmatrix}\psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,N}\\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,N}\\ \cdots & \cdots & \cdots & \cdots\\ \psi_{N,1} & \psi_{N,2} & \cdots & \psi_{N,N}\end{vmatrix}\bullet\begin{vmatrix}s(t)_1^1\\ s(t)_2^1\\ \cdots\\ s(t)_N^1\end{vmatrix}=\begin{vmatrix}\overset{*}{s}(t)_1^1\\ \overset{*}{s}(t)_2^1\\ \cdots\\ \overset{*}{s}(t)_N^1\end{vmatrix},\quad \ldots\quad ,s(t)_N=\begin{vmatrix}\psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,N}\\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,N}\\ \cdots & \cdots & \cdots & \cdots\\ \psi_{N,1} & \psi_{N,2} & \cdots & \psi_{N,N}\end{vmatrix}\bullet\begin{vmatrix}s(t)_1^N\\ s(t)_2^N\\ \cdots\\ s(t)_N^N\end{vmatrix}=\begin{vmatrix}\overset{*}{s}(t)_1^N\\ \overset{*}{s}(t)_2^N\\ \cdots\\ \overset{*}{s}(t)_N^N\end{vmatrix}. \qquad (3)$$

Or, by simple collecting from (3):

$$\left|\overset{*}{s}(t)\right|=\begin{vmatrix}\overset{*}{s}(t)_1^1 & \overset{*}{s}(t)_2^1 & & \overset{*}{s}(t)_N^1\\ \overset{*}{s}(t)_1^2 & \overset{*}{s}(t)_2^2 & \cdots & \overset{*}{s}(t)_N^2\\ \cdots & \cdots & & \cdots\\ \overset{*}{s}(t)_1^N & \overset{*}{s}(t)_2^N & & \overset{*}{s}(t)_N^N\end{vmatrix}.$$

In such a way we get N^2 points on t axis representing our target signal $\overset{*}{s}(t)$, composed from N^2 points of initial signal $s(t)$, and autocorellated with the help of self-basis operator Ψ proposed.

Notion of self-basis operator for case of complex signal. In this case $s(*)=S\,e^{i\varphi}$. The idea of self-basis operator remains the same, but for angle φ let’s suppose taking a phasor $\varphi=\dfrac{180^{\circ}}{\pi}k$, $k=1,2,...N^2$. Then it will result in

$$\overset{*}{s}(*)_1 = \begin{vmatrix} \psi_{1,1}e^{i\frac{180^\circ}{\pi}} & \psi_{1,2}e^{2i\frac{180^\circ}{\pi}} & \dots & \psi_{1,N}e^{Ni\frac{180^\circ}{\pi}} \\ \psi_{2,1}e^{(N+1)i\frac{180^\circ}{\pi}} & \psi_{2,2}e^{(N+2)i\frac{180^\circ}{\pi}} & \dots & \psi_{2,N}e^{2Ni\frac{180^\circ}{\pi}} \\ \dots & \dots & \dots & \dots \\ \psi_{N,1}e^{(N\times N-(N-1))i\frac{180^\circ}{\pi}} & \psi_{N,2}e^{(N\times N-(N-2))i\frac{180^\circ}{\pi}} & \dots & \psi_{N,N}e^{N\times Ni\frac{180^\circ}{\pi}} \end{vmatrix} \bullet \begin{vmatrix} s(t)_1^1 \\ s(t)_2^1 \\ \dots \\ s(t)_N^1 \end{vmatrix} = \begin{vmatrix} s(*)_1^1 \\ s(*)_2^1 \\ \dots \\ s(*)_N^1 \end{vmatrix},$$

$$\overset{*}{s}(*)_2 = \begin{vmatrix} \psi_{1,1}e^{i\frac{180^\circ}{\pi}} & \psi_{1,2}e^{2i\frac{180^\circ}{\pi}} & \dots & \psi_{1,N}e^{Ni\frac{180^\circ}{\pi}} \\ \psi_{2,1}e^{(N+1)i\frac{180^\circ}{\pi}} & \psi_{2,2}e^{(N+2)i\frac{180^\circ}{\pi}} & \dots & \psi_{2,N}e^{2Ni\frac{180^\circ}{\pi}} \\ \dots & \dots & \dots & \dots \\ \psi_{N,1}e^{(N\times N-(N-1))i\frac{180^\circ}{\pi}} & \psi_{N,2}e^{(N\times N-(N-2))i\frac{180^\circ}{\pi}} & \dots & \psi_{N,N}e^{N\times Ni\frac{180^\circ}{\pi}} \end{vmatrix} \bullet \begin{vmatrix} s(t)_1^2 \\ s(t)_2^2 \\ \dots \\ s(t)_N^2 \end{vmatrix} = \begin{vmatrix} s(*)_1^2 \\ s(*)_2^2 \\ \dots \\ s(*)_N^2 \end{vmatrix},$$

...

$$\overset{*}{s}(*)_N = \begin{vmatrix} \psi_{1,1}e^{i\frac{180^\circ}{\pi}} & \psi_{1,2}e^{2i\frac{180^\circ}{\pi}} & \dots & \psi_{1,N}e^{Ni\frac{180^\circ}{\pi}} \\ \psi_{2,1}e^{(N+1)i\frac{180^\circ}{\pi}} & \psi_{2,2}e^{(N+2)i\frac{180^\circ}{\pi}} & \dots & \psi_{2,N}e^{2Ni\frac{180^\circ}{\pi}} \\ \dots & \dots & \dots & \dots \\ \psi_{N,1}e^{(N\times N-(N-1))i\frac{180^\circ}{\pi}} & \psi_{N,2}e^{(N\times N-(N-2))i\frac{180^\circ}{\pi}} & \dots & \psi_{N,N}e^{N\times Ni\frac{180^\circ}{\pi}} \end{vmatrix} \bullet \begin{vmatrix} s(t)_1^N \\ s(t)_2^N \\ \dots \\ s(t)_N^N \end{vmatrix} = \begin{vmatrix} s(*)_1^N \\ s(*)_2^N \\ \dots \\ s(*)_N^N \end{vmatrix}$$

or:

$$\left|\overset{*}{s}(*)\right| = \begin{vmatrix} s(*)_1^1 & s(*)_2^1 & & s(*)_N^1 \\ s(*)_1^2 & s(*)_2^2 & \dots & s(*)_N^2 \\ \dots & \dots & & \dots \\ s(*)_1^N & s(*)_2^N & & s(*)_N^N \end{vmatrix}.$$

Conclusions

1. Introducing the presented "self-basis operator" either in real or complex form can be a convenient way to get an autocorrelated (self-resembling [7]) signal for different fields of applied mathematics, for example for computerized composition of music, involving "AQUARIUS" [1] software.

2. A concept of orthogonality, put into realization of self-basis operator is not obligatory (because we can shift and superimpose self-basis operator functions $\psi_{i,j}$). This gives a wide variety of ways to use as basis operator any imaginable structures and structural complexes either from real field of physics, biophysics and bioinformatics, – or artificial substances taken from author`s wits` end.

3. A "white-noise" orthogonal stochastic basis realization "cut" can be applied as a matrix operator for fractional-autocorrelated [7] signal change in the fields of digital signal processing in different artificial intelligence and expert systems applications including the computerized composition of music

4. In future investigations a correlation index must be set for estimation of input and target signals similarity.

5. For case of complex signal there could be many possible variants of phasors implementation.

6. If one takes as a self-basis operator some alien structure, which can differ in nature, that`s a way to result in a mutual-correlation input-target signal, that could be reffered to as a “shifted” one.

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Автобазисний оператор та застосування ортогонального стохастичного базису для завдань оброблення інформації

Подання цього автобазисного оператора в дійсній або комплексній формі може бути зручним способом отримання автокорельованого сигналу для різних галузей прикладної математики, наприклад для комп’ютеризованої композиції музики з використанням програмного забезпечення «AQUARIUS». Ортогональний стохастичний базис на основі білого шуму може бути корисним інструментом для оброблення цифрових сигналів.

А. В. Вишневский

Автобазисный оператор и применение ортогонального стохастического базиса для задач обработки информации

Представление данного автобазисного оператора в действительной или комплексной форме может быть удобным способом получить автокоррелированный сигнал для различных областей прикладной математики, например для компьютеризированной композиции музики с использованием программного обеспечения «AQUARIUS». Ортогональный стохастический базис на основе белого шума может быть полезным инструментом для обработки цифровых сигналов.