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FEATURES OF MATHEMATICAL MODELLING OF GIMBALLED INERTIAL NAVIGATION SYSTEM FOR MARINE MOVING VEHICLE

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Abstract*—This article represents the features of creating the mathematical model and carrying out modelling of the gimballed inertial navigation system assigned for operation on marine moving vehicles. To increase the accuracy of the system, some modes of operation are introduced. Features of correction for every mode are described. The characteristic of the integral correction is given. The control moments for levelling and gyrocompassing modes are represented. The expressions for projections of the gyro-stabilized platform angular rates are created. The simulation results of stabilization and navigation processes are represented. The advantages of the integral correction are shown. The obtained results can be useful for the high-precision navigation systems and gyroscopic stabilization systems with payload. The proposed approaches can be applied for moving objects of the wide class.*

Keywords—Gimballed inertial navigation system; gyroscopic stabilization; gyroscopic devices; integral correction; mathematical modelling; multi-mode system.

I. INTRODUCTION AND PROBLEM STATEMENT

There are two independently developing directions in modern ways to create inertial navigation systems. The first direction deals with strapdown inertial navigation systems. The second direction deals with gimballed ones. Nowadays, strapdown inertial navigation systems are widespread in most applications despite some disadvantages. The basic disadvantage is the presence of errors caused by inaccuracy in the initial alignment of a system. Nevertheless, the high-precision autonomous navigation of different moving vehicles can be realized using only gimballed inertial navigation systems. To ensure the high precision of gimballed inertial navigation systems, it is possible using correction or special operating modes, which allow us to take into consideration maximally the drifts of the gyrostabilized platform.

The current stage of transportation development is characterized by increasing traffic intensity and requires improved approaches to creating navigation systems for various classes of mobile objects. Heading determination is of great importance for the navigation of mobile objects. Recently, with the advent of high-precision miniature navigation sensors and high-speed computing equipment, there has been a trend toward the development of Attitude and Heading Reference Systems (AHRS). These systems are similar in their capabilities to inertial navigation systems, but are simpler and less expensive. Typically, such systems include a vertical gyro and a heading gyroscope, as well as accelerometers that provide correction for the gyroscopic instruments and obtain information about the linear velocity and distance travelled by a moving object.

It is widely recognized that in marine navigation, significant attention is given to gimballed inertial navigation systems, where angular position and linear acceleration sensors are installed on a gyro-stabilized platform. These systems offer the advantage of simplified navigation data processing, as the gyro-stabilized platform provides more favourable operating conditions.

This article aims to analyse the process of mathematically modelling and describing a platform-type course determination system. The purpose of the research is to examine the characteristics involved in developing a mathematical model and description for a platform-type course determination system.

II. REVIEW OF PUBLICATIONS

The process of developing and modelling gimballed stabilization systems has been extensively discussed in the scientific literature [1] – [4]. Basics of mathematical description of a gimballed navigation system are given in the publications [5], [6]. Nevertheless, this study overlooks an important feature of modern heading determination systems – the existence of numerous operating modes that differ in sensor configurations and, consequently, in the mathematical formulations and modelling approaches required. General approaches to the development of complex engineering systems are represented in textbooks [7] – [9]. Information about gyroscopic devices, as important constituents of the gimballed inertial navigation systems, is presented in [10] – [12]. The full mathematical description of the precision dynamically tuned gyroscopes is represented in [11]. The detailed description of approaches to the correction of gyroscopic devices used for the measurement of angles, which define a position of the moving object in the inertial space, is given in [12]. This textbook also includes the description of the integral correction that makes the gyroscopic device mounted on the gyro-stabilized platform into a non-disturbed gyro vertical. Features of studying stochastic gimballed stabilization systems are represented in the paper [13]. The very useful information about the possibility of carrying out modelling of complex engineering systems in MATLAB is described in [14], [15].

III. Features of Mathematical Modelling

Features of creating a mathematical model are considered on the example of a gimballed inertial navigation system assigned for operation on a marine moving object. We will study the above-mentioned AHRS systems. The studied system uses such precision inertial navigation instruments as dynamically tuned gyroscopes and pendulum accelerometers. To ensure high navigation accuracy, the multi-mode approach was applied. This allows us to form corrections, which eliminate the influence of the parametric and coordinate disturbances and also the measuring noise [1].

The modelling characteristics of a gyroscopic system designed to determine the heading of a moving object are analyzed using a platform-based system as an example. This system includes dynamically adjustable gyroscopes functioning as a vertical gyro and a heading gyro, along with accelerometers that provide data on the object's linear velocity and distance travelled. The chosen combination of sensors enables the determination of key navigation parameters, including: the heading relative to the geographic meridian in gyrocompass mode or the angular deviation from a specified direction in gyro-azimuth mode; platform tilt angles relative to the horizontal plane; as well as linear accelerations, velocities, and travelled distance.

The heading determination system operates in several modes that differ in their functions, sensor configurations, and corresponding types of correction moments. The primary modes include preliminary levelling, precise levelling, gyro-compassing, and operation in either gyroscopic compass or gyro-azimuth mode.

Key factors influencing the choice of model type are the presence of integral correction and a computing device. The first main modelling direction involves selecting the parameters of the integral correction, which requires simulating the system's operation over extended periods. This type of model primarily focuses on the development of control laws.

The second modelling direction examines how the discreteness of information processing affects the system's accuracy in steady-state conditions and the quality of transient responses. In this case, a detailed mathematical representation of the heading determination system is used, incorporating models of electronic components, which substantially increases simulation time.

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The heading determination system in question is distinguished by its separation of control functions. Platform stabilization is achieved through stabilization motors installed along the axes of the platform's gimbal mount, with control commands computed from the signals of the vertical gyro angle sensors. Meanwhile, corrective inputs applied to the vertical gyro torque sensors are derived from accelerometer and log data. Division of controls enables high stabilization precision [1].

Correction torques are produced using a complex algorithm. To maintain system stability, integral correction based on accelerometer signals is employed. Additionally, damping – implemented using information about the object's relative velocity obtained from the log – enhances control performance.

A key feature of this system is the use of dynamically tuned gyroscopes, which considerably increase the complexity of the system model. The equations of motion for a dynamically tuned gyroscope, functioning as a vertical gyro, are [11].

$$
\begin{aligned}
&J\ddot{\gamma}_g - H\dot{\beta}_g + c\gamma_g + d\dot{\gamma}_g - \frac{H}{T}\beta_g = H_1\omega_{xp} + M^g_{corx},\\
&J\ddot{\beta}_g + H\dot{\gamma}_g + c\beta_g + d\dot{\beta}_g + \frac{H}{T}\gamma_g = -H_1\omega_{yp} + M^g_{cory},
\end{aligned}
\tag{1}
$$

here H is the kinetic moment of the gyroscope's rotor; c is the gimbals' residuary stiffness; d is the damping factor; T is the gyroscopic precession time factor; $H_1 = H(1-s)$; $s = 10^{-3}$; ω_x, ω_y are projections of the platform's horizontal rate; β_g, γ_g are angles defining the space position of the platform with payload relative to the Resal axes; M^g_{corx}, M^g_{cory} are moments for the correction.

The following equations describe the motion of a dynamically tuned gyroscope functioning as a course gyroscope [11]

$$
\begin{aligned}
&J\ddot{\alpha}_A + H\dot{\beta}_A + c\alpha_A + d\dot{\alpha}_A + \frac{H}{T}\beta_A = -H_1\omega_{xp} + M^A_{corx},\\
&J\ddot{\beta}_A - H\dot{\alpha}_A + c\beta_A + d\dot{\beta}_A - \frac{H}{T}\alpha_A = H_1\omega_{zp} + M^A_{corz},
\end{aligned}
\tag{2}
$$

here α_A is an angle, which defines an arrangement of the course gyroscope relative to the meridian; β_A is an angle, which defines an arrangement of the course gyroscope relative to the horizon plane; M^A_{corx}, M^A_{corz} are moments, which implement correction.

However, it is essential to note that developing a detailed mathematical model of a dynamically tuned gyroscope is crucial for designing a precision gyroscope. The developers of the heading determination system require such a model to evaluate the overall system errors and to optimize the control parameters.

To simplify the system's mathematical models (1), (2), it can be assumed that, apart from the stabilization system errors and drift, the equations of motion of dynamically tuned gyroscopes coincide with those of the platform. In other words, the stabilization system is considered ideal [1].

Under this assumption, the mathematical model for determining the heading and angular position in precise leveling mode can be formulated based on equation (1), which describes the angular motion of the vertical gyroscope, and the second equation of system (2), which represents the motion of the system in azimuth

$$
\begin{aligned}
&H_1\omega_{xp} = -M^g_{corx} + H\Delta\omega_x,\\
&H_1\omega_{zp} = M^g_{cory} - H\Delta\omega_y,\\
&H_1\omega_{zp} = -M^A_{corz} + H\Delta\omega_z.
\end{aligned}
\tag{3}
$$

A mathematical model of the system used to determine heading and angular position in gyroscopic compass mode can be developed based on equations (2), which describe the angular motion of the heading gyroscope, and the second equation of system (1), which defines the angular deviation of the system from the horizontal plane

$$
\begin{aligned}
&H_1\omega_{xp} = M^A_{corx} - H\Delta\omega_x,\\
&H_1\omega_{yp} = M^g_{cory} - H\Delta\omega_y,\\
&H_1\omega_{zp} = -M^A_{corz} + H\Delta\omega_z.
\end{aligned}
\tag{4}
$$

The position of the gimballed system for determining the course and position of a moving object is determined in the coordinate system $O\xi_1\eta_1\zeta_1$, arranged according to the geographical reference frame $O\xi\eta\zeta$ on the angle A_0.

At the initial instant of time, the angle between the diametrical plane and the platform's longitudinal axis is equal to $(k_0 - A_0)$, here k_0 is a heading of the marine moving object; A_0 is the platform's azimuth. The location of the reference frame $Ox_py_pz_p$ connected with a platform relative to the reference frame $O\xi_1\eta_1\zeta_1$ can be determined by rotations of small angles α, β, γ. In this case, it is assumed that, in the initial position, the platform's axes are aligned with the sensitivity axes of the accelerometers mounted on it. The platform levelling accuracy is determined by the angular deviations β and γ. The deviation of the platform from the meridian plane or initial platform azimuth (A_0) is defined by an angle α taking into consideration the smallness of angles β and γ. Projections of the platform's angular rates onto proper axes for small angles α, β, γ become

$$
\begin{aligned}
&\omega_{xp} = \dot{\beta} - \dot{\alpha}\gamma + \omega_{\xi 1}\cos\alpha + \omega_{\eta 1}\sin\alpha - \omega_{\zeta_1}\gamma,\\
&\omega_{yp} = \dot{\gamma} + \dot{\alpha}\beta - \omega_{\xi 1}\sin\alpha + \omega_{\eta 1}\cos\alpha - \omega_{\zeta_1}\beta,\\
&\omega_{zp} = \dot{\alpha} + \dot{\gamma}\beta + \omega_{\xi 1}\gamma - \omega_{\eta 1}\beta + \omega_{\zeta 1},
\end{aligned}
\tag{5}
$$

here $\omega_{\xi 1}, \omega_{\eta 1}, \omega_{\zeta_1}$ represent projections of the platform's angular rates on the axes of the reference frame $O\xi_1\eta_1\zeta_1$. Projections of angular rates $\omega_{\xi 1}, \omega_{\eta 1}, \omega_{\zeta_1}$ can be determined by angular rates in the geographical reference frame $O\xi\eta\zeta$ as follows

$$\begin{aligned} &\omega_{\xi 1} = \omega_\xi \cos A_0 - \omega_\eta \sin A_0, \\ &\omega_{\eta_1} = \omega_\eta \cos A_0 + \omega_\xi \sin A_0, \quad \omega_{\zeta 1} = \omega_\zeta. \end{aligned} \tag{6}$$

By substituting expressions (6) into relations (5) and assuming the angles α, β, γ are small, the projections of the platform's angular velocities onto its coordinate axes can be expressed as follows

$$\begin{aligned} \omega_{xp} &= \dot\beta - \dot\alpha\gamma + \omega_\xi \cos A + \omega_\eta \sin A - \omega_\zeta \gamma, \\ \omega_{yp} &= \dot\gamma + \dot\alpha\beta - \omega_\xi \sin A + \omega_\eta \cos A - \omega_\zeta \beta, \\ \omega_{zp} &= \dot\alpha + \dot\gamma\beta + \gamma(\omega_\xi \cos A_0 - \omega_\eta \sin A_0) \\ &\quad - \beta(\omega_\eta \cos A_0 + \omega_\xi \sin A_0) + \omega_\zeta. \end{aligned} \tag{7}$$

Expressions (3), (4), and (7) represent the mathematical description of the gimballed inertial navigation system.

To meet the accuracy requirements for the inertial navigation system operating in precision leveling mode, integral control grounded on accelerometer data and damping grounded on log data can be applied. The integral correction moments applied to the gimballed platform axes are defined by the relationships

$$M_{iy} = \frac{H_1}{R_1}\int_0^t (\dot V_n + \Delta W_y + W_\zeta \beta - g\beta + \Delta a_y)dt, \tag{8}$$

here $\dot V_n$ is the northern constituent of the system's acceleration; ΔW_y is a correction value taking into consideration translation and Coriolis accelerations; $W_\zeta\beta$ is a correction value taking into consideration a vertical constituent on the vertical acceleration; Δa_y is the accelerometer's error.

$$M_{ix} = \frac{H_1}{R_2}\int_0^t (\dot V_e + \Delta W_x - W_\zeta \gamma - g\gamma + \Delta a_x)dt, \tag{9}$$

here $\dot V_e$ is the eastern constituent of the acceleration; ΔW_x is a correction value on translation acceleration, Coriolis acceleration and the earth non-sphericity; $W_\zeta\gamma$ is a correction value on the vertical acceleration; Δa_x is the accelerometer's error.

The corrections applied for translational and Coriolis accelerations, Earth's non-sphericity, and the vertical constituent of acceleration are as follows

$$\begin{aligned} \Delta W_x = {} & \frac{\Omega V_n e^2 \sin\varphi \cos^2\varphi}{1-e^2} - \Omega V_n \sin\varphi\left(1+\frac{R_1}{R_2}\right) - \frac{V_n V_e}{R_1}\mathrm{tg}\varphi + V_h \Omega \cos\varphi + \frac{V_n V_e}{R_1} \\ & - \left(\dot V_h - \frac{V_e^2}{R_1} - \frac{V_n^2}{R_2} - R_1\Omega^2\cos\varphi - 2\Omega V_e \cos\varphi\right)\gamma, \end{aligned} \tag{10}$$

here $\dot V_e$ is the eastern acceleration constituent; ΔW_x is a correction value on translational acceleration, Coriolis acceleration, and the Earth's non-sphericity; $W_\zeta\gamma$ is a correction value on the vertical acceleration; Δa_x id the accelerometer's error.

Corrections are applied for translational and Coriolis accelerations, Earth's non-sphericity, and vertical acceleration components as follows

$$\begin{aligned} \Delta W_x = {} & \frac{\Omega V_n e^2 \sin\varphi \cos^2\varphi}{1-e^2} - \Omega V_n \sin\varphi\left(1+\frac{R_1}{R_2}\right) - \frac{V_n V_e}{R_1} tg\varphi + V_h \Omega \cos\varphi \\ & + \frac{V_n V_e}{R_1} - \left(\dot V_h - \frac{V_e^2}{R_1} - \frac{V_n^2}{R_2} - R_1\Omega^2\cos\varphi - 2\Omega V_e \cos\varphi\right)\gamma. \end{aligned} \tag{11}$$

The corrective damping moments based on the information from the lag are determined by the expressions

$$M_{dx} = k_2 \frac{H_1}{R_2}\int_0^t (V_{ly} - V_y)dt, \; M_{dy} = k_3 \frac{H_1}{R_1}\int_0^t (V_{lx} - V_x)dt, \tag{12}$$

here $V_{ly} = V_l \cos(k_0 - A_0)$; $V_{lx} = V_l \sin(k_0 - A_0)$; V_l is the translation speed measured by a log

$$\begin{aligned} V_y &= \int_0^t (\dot V_n + \Delta W_y + W_\zeta \beta - g\beta + \Delta a_y)dt + k_2(V_{ly} - V_y), \\ V_x &= \int_0^t (\dot V_e + \Delta W_x - W_\zeta \gamma - g\gamma + \Delta a_x)dt + k_3(V_{lx} - V_x). \end{aligned} \tag{13}$$

Expressions (8) – (13) represent control moments. Namely, such an approach to creating the gimballed inertial navigation systems ensures its high precision.

The model of the heading and attitude determination system operating in gyroscopic compass mode is characterized by the following main features:

1) In contrast to other modes, the system's kinematics are governed by the primary device for this mode – the heading gyroscope.

2) The control torques are implemented according to the conventional design of a corrected gyrocompass [12].

In this mode, the platform of the heading and attitude determination system is controlled using accelerometer signals, which are employed to generate the following torques:

1) A compensating torque that offsets the apparent deviation of the platform from the meridian plane caused by arbitrary azimuthal displacement.

2) A damping torque that suppresses platform oscillations, derived from a filtered (attenuated) accelerometer signal.

3) A correction torque proportional to the velocity of the geographic coordinate system, acting on the outer ring of the dynamically tuned gyroscope.

4) A correction torque proportional to the velocity of the geographic coordinate system, acting on the inner ring of the dynamically tuned gyroscope.

5) An integral correction torque that maintains the system's invariance to external accelerations.

To establish the kinematic relationships for the model of the heading and angular position determination system operating in gyroscopic compass mode, the coordinate system $O\xi_1\eta_1\zeta_1$ arranged relative the geographical reference frame $O\xi\eta\zeta$ on the angle A_0 is taken as the initial reference frame.

For the chosen coordinate axes, the expressions used to determine the platform's angular velocities can be written as follows

$$\begin{aligned}
\omega_{x_p} &= \dot\beta\cos\gamma - \dot A\cos\beta\sin\gamma + (\cos A\cos\gamma - \sin A\sin\beta\sin\gamma)\omega_{\xi_1} + (\sin A\cos\gamma + \cos A\sin\beta\sin\gamma)\omega_{\eta_1} - \cos\beta\sin\gamma\omega_{\zeta_1},\\
\omega_{y_p} &= \dot\gamma + \dot A\sin\beta - \sin A\cos\beta\omega_{\xi_1} + \cos A\cos\beta\omega_{\eta_1} + \sin\beta\omega_{\zeta_1},\\
\omega_{z_p} &= \dot A\cos\beta\cos\gamma + \dot\beta\sin\gamma + (\cos A\sin\gamma + \sin A\sin\beta\cos\gamma)\omega_{\xi_1} + (\sin A\sin\gamma - \cos A\sin\beta\cos\gamma)\omega_{\eta_1} + \cos\beta\cos\gamma\omega_{\zeta_1}.
\end{aligned} \tag{14}$$

After substitution expressions (6) defining values of angular rates $\omega_{\xi_1}, \omega_{\eta_1}, \omega_{\zeta_1}$ in the expression (14), taking into consideration the smallness of angles β, γ and elimination of small terms, we will obtain the following expressions for determining projections of the platform's angular rates

$$\begin{aligned}
\omega_{x_p} &= \dot\beta - \dot A\gamma + \omega_\xi\cos A_0\cos A + \omega_\xi\sin A_0\sin A - \omega_\eta\sin A_0\cos A + \omega_\eta\cos A_0\sin A - \omega_\zeta\gamma,\\
\omega_{y_p} &= \dot\gamma + \dot A\beta - \omega_\xi\cos A_0\sin A + \omega_\xi\sin A_0\cos A + \omega_\eta\sin A_0\sin A + \omega_\eta\cos A_0\cos A + \omega_\zeta\beta,\\
\omega_{z_p} &= \dot A + \dot\beta\gamma + \omega_\xi\cos A_0\cos A\gamma + \omega_\xi\cos A_0\sin A\beta - \omega_\eta\sin A_0\cos A\gamma - \omega_\eta\sin A_o\sin A\beta\\
&\quad + \omega_\eta\cos A_0\sin A_0\gamma - \omega_\eta\cos A_0\cos A\beta + \omega_\xi\sin A_0\sin A\gamma - \omega_\xi\sin A_0\cos A\beta + \omega_\zeta.
\end{aligned} \tag{15}$$

With the selected coordinate axes, the expressions for calculating the platform's angular velocities based on expressions (15) can be formulated as follows

$$\begin{aligned}
\omega_{x_p} &= \dot\beta - \dot A\gamma + \omega_\xi\cos(A_0 - A) - \omega_\eta\sin(A_0 - A) - \omega_\zeta\gamma,\\
\omega_{y_p} &= \dot\gamma + \dot A\beta + \omega_\xi\sin(A_0 - A) + \omega_\eta\cos(A_0 - A) + \omega_\zeta\beta,\\
\omega_{z_p} &= \dot A + \dot\beta\gamma + \omega_\xi\cos A_0\cos A\gamma + \omega_\xi\cos A_0\sin A\beta - \omega_\eta\sin A_0\cos A\gamma - \omega_\eta\sin A_o\sin A\beta\\
&\quad + \omega_\eta\cos A_0\sin A\gamma - \omega_\eta\cos A_0\cos A\beta + \omega_\xi\sin A_0\sin A\gamma - \omega_\xi\sin A_0\cos A\beta + \omega_\zeta.
\end{aligned} \tag{16}$$

The mathematical model, which incorporates the expressions for the platform's angular velocities (16) during an arbitrary azimuthal turn and the corrective and control moments, can be represented as follows

$$
\begin{aligned}
&H_1\omega_{xp} = M^A_{corz1} + M^A_{corz2} + M^A_{corz3} + H\Delta\omega_x,\\
&H_1\omega_{yp} = M^A_{cory1} + M^A_{cory2} + H\Delta\omega_y,\\
&H_1\omega_{zp} = M^A_{corx1} + M^A_{corx2} + M^A_{corx3} + H\Delta\omega_z,\\
&M^A_{corz1} = -k_{12}\frac{W_y}{g}, \quad M^A_{corz2} = -H_1\omega_\xi = \frac{V_N}{R_2},\\
&M^A_{corz3} = -\frac{k_A k_G k_{PWM} k_{TM} k_I}{R}\int_t^{t+\Delta t} W_y dt,\\
&M^A_{cory1} = -k_\gamma \frac{W_x}{g},\\
&M^A_{cory2} = -\frac{k_A k_G k_{PWM} k_{TM} k_I}{R}\int_t^{t+\Delta t} W_x dt,\\
&M^A_{corx1} = k_{13}\frac{W_y}{g}, \quad M^A_{corx2} = H_1\left(\Omega\sin\varphi + \frac{V_E}{R_1}\operatorname{tg}\varphi\right).
\end{aligned}
\tag{17}
$$

here k_A is the accelerometer's transfer factor; k_G is the transfer factor of the pre-amplifier; k_{PWM} is the transfer factor of the pulse-width-modulator; k_{TM} is the transfer factor of the torque motor; k_I is the transfer factor of the integral correction.

IV. MODELLING RESULTS

The simulation of the studied system is based on the model (7). The processes of platform stabilization by the angles of the roll (γ) and pitch (β) are represented in Figs 1 and 2. Figures 3 and 4 show the effect caused by the integral correction, which involves forming correcting signals using data entered from accelerometers. Such an approach ensures the resistance of the studied AHRS to disturbing accelerations that sufficiently improve the accuracy of stabilizing the platform with inertial navigation instruments and, correspondingly, navigation.

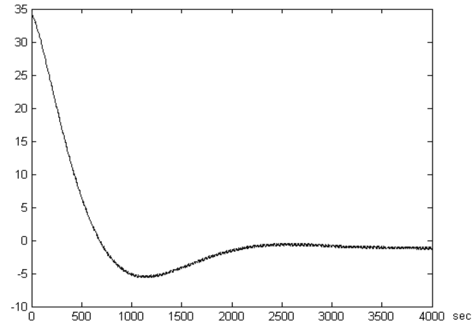

Fig. 1. The process of stabilization by the angle of roll (γ) in conditions of weak sea regular waves

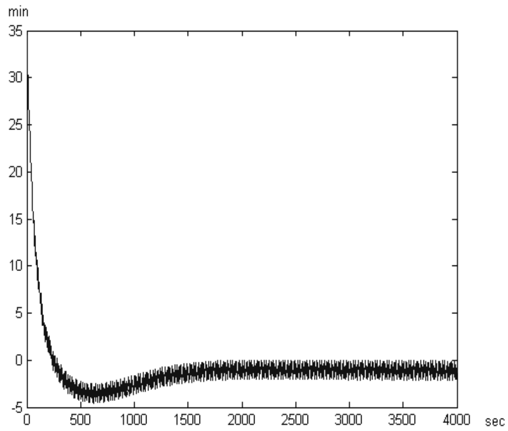

Fig. 2. The process of stabilization by the angle of pitch (β) in conditions of strong sea regular waves

Graphical dependencies represented in Figs 3 and 4 prove the efficiency of the integral correction. The simulation results of the course and spatial position determination system are shown in Figs 5 and 6.

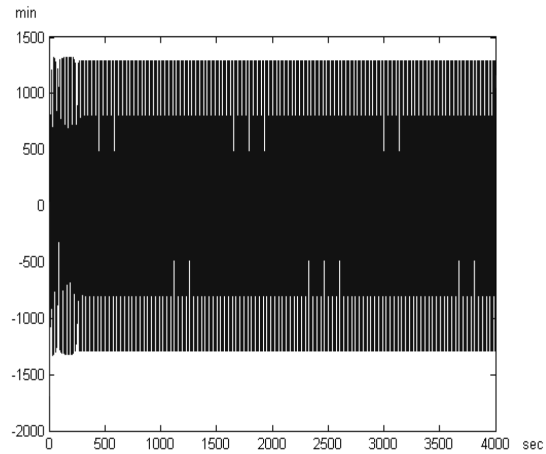

Fig. 3. Control using accelerometers without the integral correction (for demonstration of the efficiency of the integral correction)

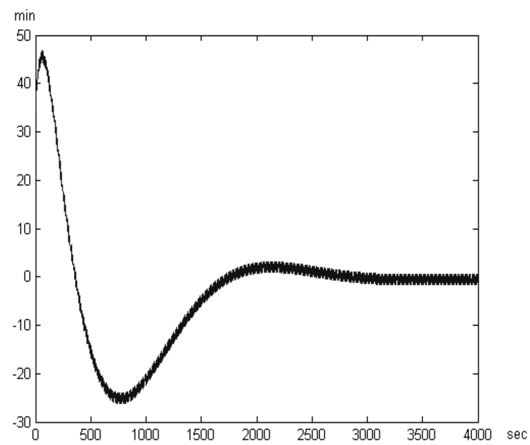

Fig. 4. The control by accelerometers with the integral correction

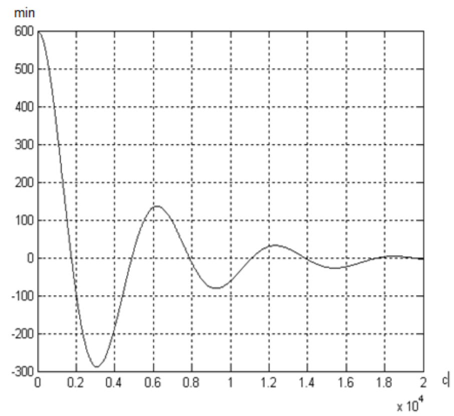

Fig. 5. The transient process on the azimuth α with the initial value α for $A_0 = 10°$

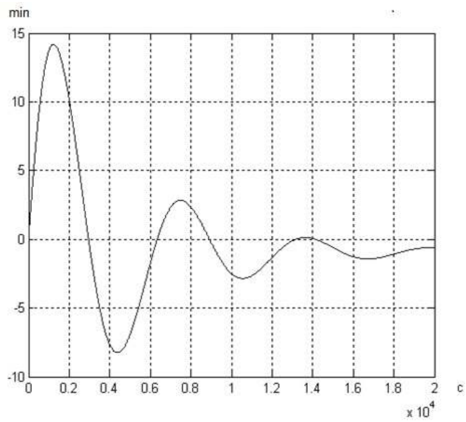

Fig. 6. The transient process on pitch β for initial azimuth $A_0 = 10°$

V. CONCLUSIONS

The full mathematical model of a gimbaled system for determination of a heading and attitude is proposed, emphasizing the analysis of the system's control loops and navigational accuracy. Distinct models are created for the leveling and gyroscopic compass modes.

The approach to the representation of the mathematical model in two different modes is proposed.

Expressions for the determination of the platform's angular rates and control moments are specific to both modes of the system.

Simulation results demonstrate the efficiency of the proposed approach, including high precision of stabilization and navigation processes.

The novelty of the study lies in the development of correction moments directed to the improvement of the accuracy of the gimballed inertial navigation system.

The obtained results can be useful for developing high-precision gyroscopic stabilization systems.

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О. А. Сущенко, Ю. В. Мельник. Особливості математичного моделювання платформної інерціальної навігаційної системи для морського рухомого об'єкту

У статті представлено особливості створення математичної моделі та проведення моделювання інерціальної навігаційної системи з карданним підвісом, призначеної для роботи на морських рухомих об'єктах. Для підвищення точності системи введено деякі режими роботи. Описано особливості корекції для кожного режиму. Наведено характеристику інтегральної корекції. Представлено керуючі моменти для режимів горизонтування та гірокомпасування. Створено вирази для проекцій кутових швидкостей гіростабілізованої платформи. Представлено результати моделювання процесів стабілізації та навігації. Показано переваги інтегральної корекції. Отримані результати можуть бути корисними для високоточних навігаційних систем та гіроскопічних систем стабілізації з корисним навантаженням. Запропоновані підходи можуть бути застосовані для рухомих об'єктів широкого класу.

Ключові слова: інерціальна навігаційна система з карданним підвісом; гіроскопічна стабілізація; гіроскопічні пристрої; інтегральна корекція; математичне моделювання; багато-режимна система.

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