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MATHEMATICAL MODEL OF GYROSCOPE WITH CONTACTLESS SUSPENDED ROTOR AND THREE-COMPONENT ACCELEROMETER FOR SOLVING PROBLEMS OF AUTONOMOUS NAVIGATION

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Abstract—The problem of creating a mathematical model of a free three-degree gyroscope with a contactless suspended rotor and a three-component accelerometer for solving problems of high-precision autonomous navigation is solved. The problem under consideration includes determination of kinematic relations of the gyro device and solution of a direct problem for finding navigation parameters for a stationary base. The suspension system of these gyroscopes is practically indifferent to the environment in which the gyro operates, but in order to reduce braking moments, the gyro rotor is placed in a vacuum chamber. The given structure of vector relationships between coordinate systems and model parameters allows us to write down several groups of equations regarding the angular positions of the gyro rotor and its angular velocities.

Keywords—Non-contact gyroscope; three-component accelerometer; kinematic relations; mathematical model; autonomous navigation.

I. INTRODUCTION

Ukraine has successfully tested and created samples of gyroscopes based on a three-stage gyroscope with a magnetic rotor suspension. The suspension system of these gyroscopes is practically indifferent to the environment in which the gyroscope operates, but in order to reduce braking moments, the gyroscope rotor is placed in a vacuum chamber.

The basic gyro device used a magneto-resonant suspension [1].

The dependence of the electric current in the working windings of the magneto-resonance suspension on the displacement of the rotor relative to the gyroscope casing, which, in turn, has a displacement due to the linear acceleration of the object on which the gyro device is installed, makes it possible to use the magneto-resonance suspension as a three-component accelerometer.

Another feature of these gyroscopes is the use of a two-axis sensor located on the gyro device casing. The sensor provides two-axis tracking of the casing along the rotor rotation axis. In these conditions, the two-axis angle sensor and the casing position control motors are required to perform their functions in a small angular range.

II. PROBLEM STATEMENT

Creation of a mathematical model of a non-contact gyroscope with a combined three-

component accelerometer as which the rotor suspension system is used is of paramount importance for solving the inverse problem. The solution of the inverse problem is supposed to be carried out step by step. At the first stage, the problem of determining the azimuthal orientation of an object on a fixed base using a single gyro device is solved. At the second stage, with the use of two gyros, the problem of full geographic reference of the stationary object is solved. At the third stage, the coordinates of the mobile object are determined.

III. PROBLEM SOLUTION

The initial data for the construction of a mathematical model of a free gyroscope with a contactlessly suspended rotor are

- electrokinematic scheme of the gyro device based on a free three-stage gyroscope with a two-axis sensor for tracking the position of the rotor relative to the casing (inner frame) of the gyroscope;
- use of the gyroscope rotor suspension system as a three-component accelerometer;
- systematic drifts along the two axes of the gyroscope rotor cutter are certified;
- a mathematical model is developed for a fixed base for its further use to determine the azimuthal orientation of the gyro device and for full geographic reference when using two gyro devices.

The basic gyroscope used a magnetoresonant suspension [1]. The dependence of the electric current in the working windings of the magnetoresonant suspension on the displacement of the rotor relative to the gyroscope casing, which, in turn, has a displacement due to the linear acceleration of the object on which the gyro device is installed, makes it possible to use the magnetoresonant suspension as a three-component accelerometer.

The electrokinematic structure of the gyro device based on a free gyroscope with a two-axis rotor position sensor and a three-component accelerometer can be represented as (Fig. 1).

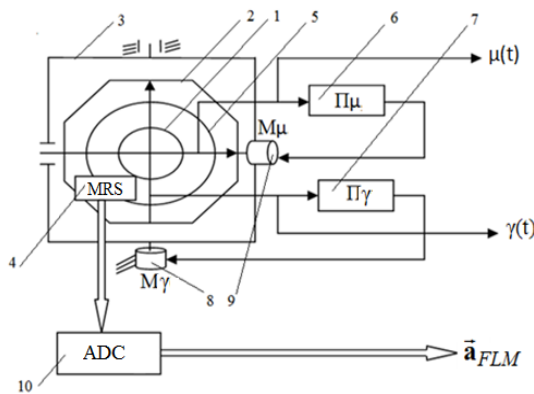


Fig. 1. The electrokinematic structure of the gyro device: 1 is the spherical rotor of the gyro unit; 2 is the body of the gyro unit; 3 is the outer frame of the gyro device connected to the object; 4 is a magnetic resonance suspension (MRS) placed in the body of the gyro unit; 5 is the two-axis angular position sensor (APS) of the gyro unit; 6, 7 are P_μ, P_γ signal amplifiers from the APS via channels μ and γ ; 8, 9 are M_μ, M_γ motors for controlling the position of the gyro assembly and the outer frame; 10 is a analog-to-digital converter whose input receives the rotor position control voltage from the MPP $u_i, i = 1-6$, and whose output measures the digitalized acceleration values along the three Rezal axes $\vec{a}_{FLM}, \mu, \gamma$ – output signals from the ACS

A gyro device with an uncorrected gyro has a number of undeniable advantages, including:

- no moments generating gyro drifts due to the influence of gimbal frames;
- absence of moments that generate gyro drifts due to the presence of cross-links in the gimbal frames.

At the same time, the use of a magnetoresonant rotor suspension in the basic gyro device [1] leads to its inhibition even in vacuum due to rotational hysteresis. Therefore, the electric drive in the magnetic suspension gyroscope operates continuously, although the power required for it is insignificant [3].

The mathematical model of the gyro device provides for its output to obtain the angular deviations of the gyroscope casing $\mu(t)$ and $\gamma(t)$ relative to the rotor from a two-axis sensor and signals from a three-component accelerometer \vec{a}_{FLM} in the axes of the Rezal trihedron, which in this task is a device for determining the position of the rotor in inertial space.

Let us consider the following coordinate systems (CS):

- $01\xi\eta\zeta$, inertial with the origin at the center of the Earth, 01ξ axis directed along the polar axis of the Earth to the North Pole, η, ζ axes lying in a plane parallel to the equatorial plane and not participating in the daily rotation of the Earth.

The position of the gyro device kinetic moment vector in this SC is determined by the inertial coordinates $\alpha, \Delta, \varepsilon$ and the corresponding matrices of guide and inverse guide cosines $\mathbf{B}(\alpha), \mathbf{B}(\Delta), \mathbf{B}(\varepsilon), \mathbf{B}^{-1}(\alpha), \mathbf{B}^{-1}(\Delta), \mathbf{B}^{-1}(\varepsilon)$. Angle transition graphs and transition matrices for each case of rotation in the coordinate system $01\xi\eta\zeta$ (Fig. 2):

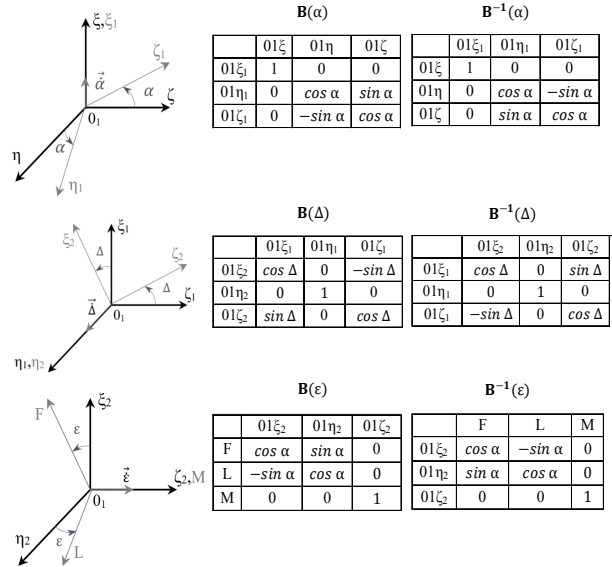


Fig. 2. Angle transition graphs and transition matrices for each case of rotation in the coordinate system $01\xi\eta\zeta$

$01XYZg$, a geocentric spherical SC in which the coordinates of the gyro device on the Earth's surface are given by the parameter $q = \lambda \text{ost} + Sgr$, determined by the east longitude of point 0 and the hour angle of the point Aries γ in Greenwich and the north latitude ϕ of point 0, where point 0 is the point of suspension of the gyro device rotor fixed relative to the Earth. The transformation of the vector components from SC $01\xi\eta\zeta$ to $01XYZg$ is provided by the matrices of guide and reverse guide cosines $\mathbf{B}(Sgr), \mathbf{B}(\lambda \text{ost}), \mathbf{B}(\phi), \mathbf{B}^{-1}(Sgr), \mathbf{B}^{-1}(\lambda \text{ost}), \mathbf{B}^{-1}(\phi)$;

- $0ENH$ is the accompanying trihedron. The origin of the trihedron is at the point where the gyro device is located, the $0H$ axis is directed along the local vertical, the $0N$ axis coincides with the direction to the north. The position of the $0FLM$ trihedron in this SC is determined by the azimuth angle A , height h , angle β and the corresponding matrices of directional and reverse directional cosine guides $\mathbf{B}(A)$, $\mathbf{B}(h)$, $\mathbf{B}(\beta)$, $\mathbf{B}^{-1}(A)$, $\mathbf{B}^{-1}(h)$, $\mathbf{B}^{-1}(\beta)$;

- aircraft – centered SC $0XYZI$. The position of the axes $0XYZI$ relative to the accompanying tech-grid $0ENH$ is determined by the true heading, pitch, roll angles and the corresponding matrices of guide and reverse guide cosines $\mathbf{B}(\psi)$, $\mathbf{B}(\vartheta)$, $\mathbf{B}(\gamma\mathbf{B}^{-1}(\psi)$, $\mathbf{B}^{-1}(\vartheta)$, $\mathbf{B}^{-1}(\gamma)$;

- $0XYZvr$, the SC is associated with the internal frame of the gyroscope. The two axes of the coordinate system coincide with the axes of the two-axis rotor position sensor of the gyro device. The angular position of the Resal axes relative to the SC is determined by the angles μ and γ and the corresponding matrices of guide and backward guide cosines $\mathbf{B}(\mu)$, $\mathbf{B}(\gamma)$, $\mathbf{B}^{-1}(\mu)$, $\mathbf{B}^{-1}(\gamma)$.

Thus, taking into account the above mentioned SCs, the conversion of an arbitrary vector from one SC to another can be represented as follows (Fig. 3).

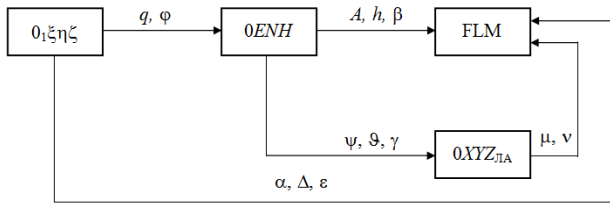


Fig. 3. Structure of vector relations between coordinate systems

The given structure of vector relations between coordinate systems and model parameters allows us to write down several groups of equations for the angular positions of the gyro device rotor and its angular velocities.

The position of the Resalem axes (FLM) relative to the inertial geocentric equatorial SC for the vector of readings of the three-component accelerometer along its sensitivity axes is as follows [2].

$$\bar{\mathbf{a}}_{FLM} = \mathbf{B}(\varepsilon)\mathbf{B}(\Delta)\mathbf{B}(\alpha)\mathbf{B}^{-1}(q)\mathbf{B}^{-1}(\varphi) \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}. \quad (1)$$

The vector of readings of the three-component accelerometer through the parameters and matrices of the guide cosines of the aircraft – centered SC $0XYZI$ and the SC associated with the inner frame and through the horizontal coordinates of the vector

of the kinetic moment of the gyro device can be represented as

$$\bar{\mathbf{a}}_{FLM} = \mathbf{B}(\mu)\mathbf{B}(\nu)\mathbf{B}(\gamma)\mathbf{B}^{-1}(\vartheta)\mathbf{B}^{-1}(\varphi) \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}, \quad (2)$$

$$\bar{\mathbf{a}}_{FLM} = \mathbf{B}(\mu)\mathbf{B}(h)\mathbf{B}(A) \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}. \quad (3)$$

The vector equation for the angular velocity of the Earth's rotation in the Resal axes of the gyro device through the inertial coordinates α , Δ , ε is

$$\bar{\boldsymbol{\Omega}}_{FLM} = \mathbf{B}(\varepsilon)\mathbf{B}(\Delta)\mathbf{B}(\alpha) \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix}. \quad (4)$$

The angular velocity of the gyroscope's kinetic momentum vector in projections on the Resal axes in inertial space is determined by the gyroscope's drifts in the F and L axes, which leads to the appearance of output signals from the two-axis sensor $\mu(t)$ and $\gamma(t)$. Based on the theorem of the addition of rotations about intersecting axes, the angular velocity of the Resal axes can be represented as

$$\bar{\boldsymbol{\omega}}_{FLM} = \begin{bmatrix} -\frac{ML}{H} \\ \frac{MF}{H} \\ \omega M \end{bmatrix} = \begin{bmatrix} -\dot{\mu} \\ 0 \\ 0 \end{bmatrix} + \mathbf{B}(\mu) \begin{bmatrix} 0 \\ 0 \\ \dot{\gamma} \end{bmatrix} + \mathbf{B}(\nu)\mathbf{B}(\psi)\mathbf{B}(\vartheta)\mathbf{B}(\gamma)\mathbf{B}(\varphi)\mathbf{B}(q) \begin{bmatrix} 0 \\ \Omega \\ 0 \end{bmatrix}. \quad (5)$$

The position of the kinetic momentum vector of the gyroscope in the inertial coordinate system $0i\xi\eta\zeta$ is as follows:

$$\bar{\mathbf{H}}_{0i\xi\eta\zeta} = \mathbf{B}^{-1}(\alpha)\mathbf{B}^{-1}(\Delta)\mathbf{B}^{-1}(\varepsilon) \begin{bmatrix} 0 \\ 0 \\ -g \end{bmatrix} = \begin{bmatrix} H \sin \alpha \\ H \cos \alpha \\ H \cos \alpha \sin \alpha \end{bmatrix}, \quad (6)$$

$$\begin{bmatrix} H \sin \alpha \\ H \cos \Delta \\ H \cos \alpha \sin \Delta \end{bmatrix} = \mathbf{B}^{-1}(q)\mathbf{B}^{-1}(\psi)\mathbf{B}^{-1}(A)\mathbf{B}^{-1}(h)\mathbf{B}^{-1}(\beta) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (7)$$

$$\begin{bmatrix} H \sin \alpha \\ H \cos \Delta \\ H \cos \alpha \sin \Delta \end{bmatrix} = \mathbf{B}^{-1}(q)\mathbf{B}^{-1}(\varphi)\mathbf{B}^{-1}(\psi) \times \mathbf{B}^{-1}(\vartheta)\mathbf{B}^{-1}(\nu)\mathbf{B}^{-1}(\gamma)\mathbf{B}^{-1}(\mu) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (8)$$

The gyroscope operation, proceeding from natural assumptions, we will divide it into two sub-modes: before and after gyroscope deployment [4].

After turning on the gyroscope in the work begins sub-mode number 1 in which the rotor of the gyroscope spins, gyroscope warms up, the vector of kinetic moment of the gyroscope H takes its initial position, which corresponds to the values of angles from the two-coordinate sensor $\gamma_0 = \mu_0 = 0$ for t is less than equal to t_0 , where t_0 – moment of disassembly. Due to corrective moments applied to the rotor it is kept in this position during the whole first sub-mode.

Horizontal coordinates of Rezal axes up to t less than equal to t_0 have

$$A_0 = \Psi_0, \quad h_0 = \vartheta_0, \quad \beta_0 = \gamma_0.$$

The equatorial coordinates $\alpha_0, \Delta_0, \varepsilon_0$ are determined from Eqs.

$$\begin{aligned} & \mathbf{B}^{-1}(\alpha_0)\mathbf{B}^{-1}(\Delta_0)\mathbf{B}^{-1}(\varepsilon_0) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \\ &= \mathbf{B}^{-1}(q)\mathbf{B}^{-1}(\varphi)\mathbf{B}^{-1}(A_0)\mathbf{B}^{-1}(h_0)\mathbf{B}^{-1}(\beta_0) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad (9) \end{aligned}$$

$$\begin{aligned} & \mathbf{B}^{-1}(\alpha_0)\mathbf{B}^{-1}(\Delta_0)\mathbf{B}^{-1}(\varepsilon_0) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \\ &= \mathbf{B}^{-1}(q)\mathbf{B}^{-1}(\varphi)\mathbf{B}^{-1}(A_0)\mathbf{B}^{-1}(h_0)\mathbf{B}^{-1}(\beta_0) \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad (10) \end{aligned}$$

obtained on the basis of the fourth group of equations for the position of the gyroscope kinetic momentum vector in the inertial coordinate system (9) (10).

Sub-mode No. 1 ends with the determination of $\alpha_0, \Delta_0, \varepsilon_0$ at $t = t_0$.

Sub-mode No.2 is reduced to determining the angular orientation of the FLM trihedron with respect to the inertial equatorial coordinate system, obtaining the measured $\mu(t), \gamma(t), \vec{a}_{FLM}$, FLM, and drift angular velocity compensation.

The angular velocity of the Rezal triangle after the gyro device disassembly is caused by the rotation of the OM axis relative to OF and OL due to the components of the gyro drift along these axes ω_F, ω_L and the motion of the FLM triangle together with the Earth, i.e. the following equation can be written down

$$\vec{\omega}_{FLM} = \vec{\omega}_g(\omega_F, \omega_L) + \vec{\omega}(\Omega). \quad (11)$$

However

$$\vec{\omega}_{FLM} = \vec{\dot{\alpha}} + \vec{\dot{\Delta}} + \vec{\dot{\varepsilon}} = \begin{bmatrix} 0 \\ 0 \\ \dot{\varepsilon} \end{bmatrix} + \mathbf{B}(\varepsilon) \begin{bmatrix} 0 \\ \dot{\Delta} \\ 0 \end{bmatrix} + \mathbf{B}(\varepsilon)\mathbf{B}(\Delta) \begin{bmatrix} 0 \\ 0 \\ \dot{\alpha} \end{bmatrix}. \quad (12)$$

We transform this vector equation (12) to a system of three scalar equations and solve them with respect to $\dot{\alpha}, \dot{\Delta}, \dot{\varepsilon}$

$$\alpha = \frac{\omega_L \sin \varepsilon - \omega_F \cos \varepsilon}{\sin \Delta}, \quad (13)$$

$$\dot{\Delta} = \omega_L \cos \varepsilon + \omega_F \sin \varepsilon, \quad (14)$$

$$\dot{\varepsilon} = \Omega M = \Omega \cos \Delta. \quad (15)$$

Given the initial conditions $\alpha(t_0) = \alpha_0, \Delta(t_0) = \Delta_0, \varepsilon(t_0) = \varepsilon_0$, found from equations (9) and (10), we define the current $\alpha(t), \Delta(t), \varepsilon(t)$ as

$$\alpha(t) = \alpha_0 + \int_0^t \dot{\alpha} dt = \alpha_0 + \int_0^t \frac{\omega_L \sin \varepsilon - \omega_F \cos \varepsilon}{\sin \Delta} dt, \quad (16)$$

$$\Delta(t) = \Delta_0 + \int_0^t (\omega_L \cos \varepsilon - \omega_F \sin \varepsilon) dt, \quad (17)$$

$$\varepsilon(t) = \varepsilon_0 + \Omega \int_0^t \cos \Delta dt. \quad (18)$$

To calculate the gyro output parameters $\mu(t), \gamma(t), \vec{a}_{FLM}$ FLM we will use equations (9) and (10).

The algorithm for determining the current gyro readings taking into account (16), (17) and (18) represents its mathematical model (Fig. 4).

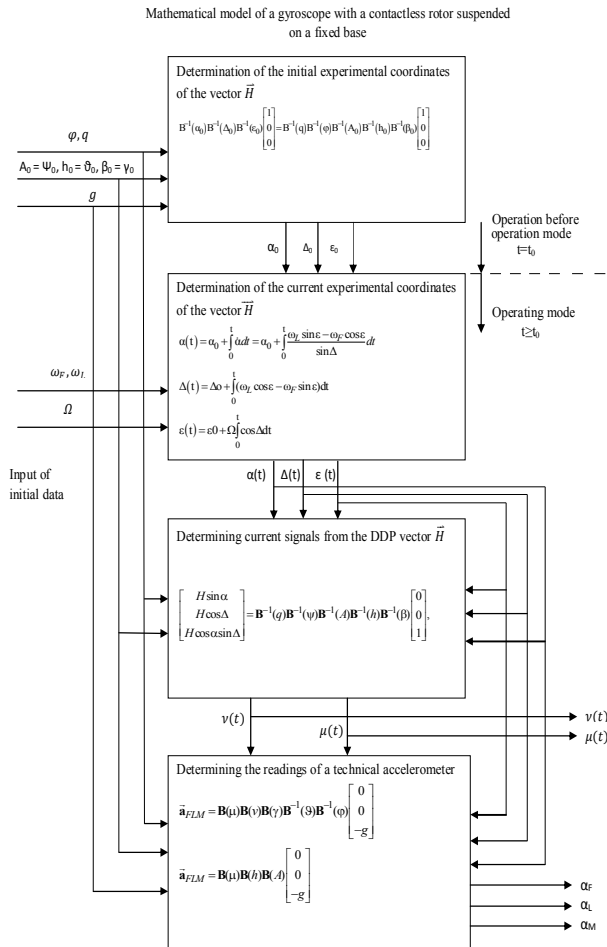


Fig. 4. Model of a gyroscope with a non-contact rotor

IV. CONCLUSIONS

The obtained kinematic dependencies of a gyro device with a contactlessly suspended rotor and the equations of its parameters (Eqs 1–8) are

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О. І. Смірнов, Ю. М. Кемєняш. Математична модель гіроскопа з безконтактним підвісним ротором та трикомпонентним акселерометром для вирішення задач автономної навігації

Розв'язано задачу створення математичної моделі вільного триградусного гіроскопа з безконтактним підвісним ротором та трикомпонентним акселерометром для вирішення задач високоточної автономної навігації. Розглянута задача включає визначення кінематичних співвідношень гіроскопічного пристрою та розв'язання

information for constructing its mathematical model and an algorithm for determining the azimuthal reference of a fixed base. In the case of using two gyro devices, it is assumed that they will be fully georeferenced, and hence the location where they are installed will be determined.

A mathematical model of a gyro device based on a free three-stage gyroscope with a contactlessly suspended rotor with a random component of the angular drift velocity of 0.01 – 0.001 g/h and a three-component accelerometer for algorithmic support of navigation systems has been created. The proposed mathematical model of the gyro device allows, on the basis of the inverse problem, to solve the creation of algorithms for a moving and a fixed base.

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прямої задачі знаходження навігаційних параметрів для стаціонарної основи. Система підвіски цих гіроскопів практично байдужа до середовища, в якому працює гіроскоп, але для зменшення гальмівних моментів ротор гіроскопа розміщено у вакуумній камері. Задана структура векторних співвідношень між системами координат та параметрами моделі дозволяє записати кілька груп рівнянь щодо кутових положень ротора гіроскопа та його кутових швидкостей.

Ключові слова: безконтактний гіроскоп; трикомпонентний акселерометр; кінематичні зв'язки; математична модель; автономна навігація.

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