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PARAMETRIC OPTIMIZATION OF THE HIERARCHICAL FUZZY MODEL OF CONTROL WITH TRANSFER OF FUZZY VALUES OF INTERMEDIATE DATA

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Abstract—*The subject of the study is the intellectualization of the technological process of controlling complex objects in order to intellectualize and replace the labor of a human operator. In conditions that are difficult to describe by mathematical methods due to incompleteness and uncertainty, a hybrid neuro-fuzzy model with a hierarchical structure is used to control the process. The aim of the article is to study and develop a learning algorithm for the Mamdani→Sugeno model with the transfer of fuzzy intermediate data between hierarchical levels, implemented by an adaptive neural network. To ensure the accuracy of real-time forecasting, an algorithm for parametric adaptation to operating conditions with the adjustment of the parameters of antecedents and consequences at two levels has been defined. When studying the methods of data transfer between levels, fuzzy logic and artificial neural networks methods, the gradient descent method, Mamdani and Takagi–Sugeno–Kang algorithms, etc. were used. The study confirms the possibility of using hybrid models to intellectualize the process of controlling complex objects. The scientific innovation of the obtained results is the construction of a neural network of a hierarchical control system and the development of a learning algorithm for the transfer of fuzzy intermediate variables with parametric model adaptation based on the gradient descent algorithm.*

Keywords—Neural network; fuzzy algorithm; gradient descent; model training; parameter adaptation.

I. INTRODUCTION

The use of new methods and modern information technologies to improve the quality of technological process control and reduce the negative impact of the "human factor" is extremely relevant. The introduction of computer-integrated tools, having a nature of functioning close to human mental activity, should replace automatic and mechanical technical means that are hopelessly outdated. Achievements in the field of artificial intelligence allow for high reliability of control systems at a relatively low cost for development and implementation. The development of systems based on intelligent technologies allows for the improvement of the control process, solving a number of tasks:

- control of linear or nonlinear objects;
- the ability to modify the parameters of the control system;
- recognition and classification of object states in real time;
- prompt decision-making;
- support for operator decision-making when choosing optimal control parameters.

The use of classical theoretical approaches and the latest information capabilities make it possible to intellectualize the control of technological processes.

II. CONCEPT ANALYSIS OF LITERARY DATA AND PROBLEM STATEMENT

The world experience of creating and using intelligent systems based on various approaches that allow to increase efficiency by combining different methods, testifies to their advantages. Good results for overcoming the problems of incompleteness of the set of fuzzy rules and inconsistency of membership functions of input variables were obtained by the study of hybrid algorithms. They allow to simplify the construction of fuzzy models and optimization due to the ability to adapt parameters when using neural networks.

The combination of fuzzy logic and neural networks was studied by Takagi and Hayashi [1]. The authors of [2] proved the actual equivalence of the functional behavior of radial basis function networks and fuzzy inference systems, which built the basis for the study and application of hybrid models for many tasks.

Jyh-Shing Roger Jang proposed the architecture and training procedure underlying adaptive ANFIS networks [3]. The system is based on the Takagi-Sugeno fuzzy inference algorithm and is capable of tuning parameters by classical optimization methods. Backpropagation of the error and the Least Squares Method are used for error estimation. The ANFIS architecture is used for modeling, identification of nonlinear components in on-line mode in control systems.

Chang Shu-Chieh considered the problem of adaptive control of nonlinear dynamic systems with unknown parameters [4]. A hybrid adaptive fuzzy logic network (FLAN) is proposed, which combines the structure of the controller and training. FLAN is capable of both structural learning and gradient descent-based parameter learning.

A widely used fuzzy reasoning algorithm is also the Mamdani fuzzy inference system. The implementation of this algorithm by an adaptive neural network is considered in [5] and the structure of the M-ANFIS model is proposed.

Abraham A. considered the integration of neural networks and fuzzy inference systems, formulating three main categories: cooperative, simultaneous and integrated neuro-fuzzy models [6]. Different types of cooperative neuro-fuzzy models are presented: fuzzy associative memory, fuzzy rule mining using self-organization, capable of learning parameters of fuzzy sets. The author presents various integrated neuro-fuzzy systems of the Mamdani and Takagi-Sugeno types, features and advantages of different types of integrated neuro-fuzzy models.

Variants of connecting different types of elements when building hierarchical fuzzy knowledge bases are considered in [7]. The hierarchical hybrid Mamdani→Sugeno system can have two combinations: with defuzzification of the results of the intermediate hierarchical level and with the transfer of fuzzy values in the form of fuzzy sets.

The difficulties in constructing the rule base are one of the main disadvantages of fuzzy logic systems. To eliminate this, an algorithm is proposed in the work [8], which allows to refine the rule base during the operation of the control system.

III. PURPOSE AND OBJECTIVES OF THE RESEARCH

The purpose of the research is to apply fuzzy algorithms in combination with adaptive neural networks for open real-time systems. The task is to create an algorithm for training a hierarchical neuro-fuzzy model with parameter adaptation under conditions of incomplete information certainty.

IV. RESEARCH OF FUNCTIONAL STRUCTURE OF A FUZZY MODEL

When implementing a hierarchical model based on hybrid algorithms [9], the results from the lower-level output can be transmitted as clear or fuzzy data. As noted in [8], the experience of transmitting intermediate results when applying the Mamdani→Sugeno model with defuzzification of intermediate results has difficulties with tuning, while transmitting fuzzy values contributes to good tuning during parametric identification of a fuzzy model. Consider building a fuzzy neural network with fuzzy intermediate variables.

Forward signal propagation. According to the fuzzy algorithm, each neuron, as an element of a layer, performs a certain function on the incoming data. Using the classical Mamdani and Takagi–Sugeno–Kang algorithms, the neural network for a fuzzy system has the following structure:

Layer L1: Input vector $X=\{x_1\ x_2\ ...\ x_{n_x}\}$, represented by numerical and linguistic variables. According to the defined membership function, the output signal is defined as $y_i^{[1]}=f(x_i)$.

Layer L2: Determination of the weight coefficient w_j of the j-rule for the current vector X of input variables

$$y_j^{[2]}=\mu_{A_1}(x_1)\cdot\mu_{A_2}(x_2)\cdot...\cdot\mu_{A_{n_x}}(x_{n_x})=w_j.$$

Layer L3: Implication with determination of the consequences of fuzzy rules of first level

$$y_j^{[3]}=w_j\cdot\mu_{B_i}=\mu_y,$$

where μ_y is a fuzzy number that is an integral indicator of y_i.

Layer L4: Determination of the membership of the current vector $Y=\{y_1\ y_2\ ...\ y_{n_y}\}$ to the fuzzy set $\mu_{B_i}(y_i)$. When determining the T-norm by the product, the value at the output of this layer is the expression

$$z_j^{[4]}=\mu_y\cdot\mu_{B_i}$$

Layer L5: Determination of the strength w_j of the j-rule for vector Y of input data. When modeling the T-norm by the product operation, the value at the output is

$$z^{[5]}=\mu_{B_i}(y_1)\cdot...\cdot\mu_{B_{n_y}}(y_{n_y})=w_j.$$

Layer L6: Normalization of the strength of the j-rule by the formula

$$z^{[6]}=\frac{w_j}{w_1+w_2+...+w_k}=\overline{w}_j,\quad j=1,2,...,k.$$

Layer L7: Calculation of the consequence of the j-rule as a linear function of the input variables

$$z^{[7]}=\overline{w}_j\cdot f^{(k)}=z_j,$$

$$f^{(k)}=q_0+\sum_{i=1}^{n_y}q_iy_i,\ i=1,2,...,n_y.$$

Layer L8: Aggregation of the results of individual rules and determination of the output value

$$z^{[8]} = \sum_{j=1}^{k} z_j^{[9]}, \; j = 1,2,...,k,$$

$$\hat{z}(Y) = \sum_{j=1}^{k} \overline{w}_j \cdot \left(q_0 + \sum_{i=1}^{n_y} q_i y_i \right).$$

The functional structure of the hierarchical model implemented by the adaptive neural network is presented in Fig. 1.

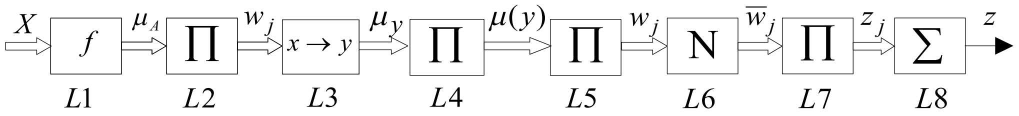

Fig. 1. Functional structure of the hierarchical fuzzy model with the transfer of fuzzy values of the intermediate hierarchical level

V. NEURO-FUZZY MODEL LEARNING ALGORITHM

Backpropagation of error. The loss function is the criterion by which the optimal parameters of the model are formed. The choice of the function should provide a convex surface during optimization. A widely known method for finding the optimal parameters is the least square error (LSE) method. The objective function is to minimize the error.

$$E = \frac{1}{2} \sum_{j=1}^{k} \left(z_j - \hat{z}(Y_j) \right)^2 \to \min,$$

where z_j is the reference value of the output signal for the j-rule; $\hat{z}(Y_j)$ is the actual value predicted by the network.

Transferring fuzzy numbers between levels without converting to scalars avoids the interpretive gap between the data due to the elimination of double transformation.

The Mamdani knowledge base for level 1 has adjustable parameters determined by the selected function for describing fuzzy terms. When using the generalized Gaussian function, the adaptive parameters are a, b, c

$$\mu = \frac{1}{1 + \left(\frac{x-c}{a} \right)^{2b}}.$$

In the Sugeno knowledge base, which implements the second level of the hierarchical system, the conclusions of the rules are a function of the input variables, the parameters of which are adjusted during the training of the model

$$f^{(k)}(X) = q_0^{(k)} + q_1^{(k)} x_1 + ... + q_n^{(k)} x_n.$$

For a fuzzy model, the structure of which is shown in Fig. 1, according to the gradient descent procedure, the following are updated:

- L8: coefficients of the consequents $[q_0^{[8]}, q_1^{[8]}, ..., q_n^{[8]}]$;
- L4: parameters of the bell-functions $[a^{[4]}, b^{[4]}, c^{[4]}]$ of the antecedents of the Sugeno knowledge base rules;
- L3: parameters of the bell-functions $[a^{[3]}, b^{[3]}, c^{[3]}]$ of the consequents of the Mamdani knowledge base rules;
- L1: parameters of the fuzzy antecedent terms $[a^{[1]}, b^{[1]}, c^{[1]}]$ for the input vector $X = \{x_1, x_2, ..., x_n\}$.

The rate of change of the error is the derivative of the loss function $\varepsilon = \frac{\partial E}{\partial \hat{z}} = (z_j - \hat{z}_j)$.

The gradients of the consequents $z_j = f(q_1, q_2, ..., q_k)$ of the second level rules are determined by the formulas:

$$\Delta q_0^{[9]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial q_0} = \varepsilon \cdot \overline{w},$$

$$\Delta q_i^{[9]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial q_i} = \varepsilon \cdot \overline{w} \cdot y_i.$$

The gradients of the antecedents of the nonlinear parameters of the generalized Gaussian function that determine the strength of the fuzzy rules $w = f(a, b, c)$ are calculated as:

$$\Delta a^{[6]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial a},$$

$$\Delta b^{[6]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial b},$$

$$\Delta c^{[6]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial c},$$

where $\frac{\partial \hat{z}}{\partial w} = \frac{z_i - \hat{z}}{\sum_{i=1}^{k} w_j}$, $\frac{dw_j}{d\mu} = \frac{w_j}{\mu}$,

$$\frac{d\mu}{da} = \mu^2 \cdot \frac{2b}{a^2} \cdot (x-c),$$

$$\frac{\partial\mu}{\partial b} = \mu^2 \cdot 2\left(\frac{x-c}{a}\right)^{2b} \ln\left(\frac{x-c}{a}\right),$$

$$\frac{\partial\mu}{\partial c} = \mu^2 \cdot \frac{2b}{a^{2b}} \cdot (x-c)^{2b-1}.$$

Determining the influence of the parameters of the consequents of fuzzy rules of level 1 on the error through gradients

$$\Delta a^{[4]} = \frac{\partial E}{d\hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial a},$$

$$\Delta b^{[4]} = \frac{\partial E}{d\hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial b},$$

$$\Delta c^{[4]} = \frac{\partial E}{d\hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu_i} \cdot \frac{\partial \mu_i}{\partial c},$$

where $\frac{\partial \hat{z}}{\partial w_j} = w_j \cdot q_{oj}$.

The gradients of the parameters of the membership functions of level 1 antecedents are calculated as

$$\Delta a^{[1]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial a},$$

$$\Delta b^{[1]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial b},$$

$$\Delta c^{[1]} = \frac{\partial E}{\partial \hat{z}} \cdot \frac{\partial \hat{z}}{\partial w_j} \cdot \frac{\partial w_j}{\partial \mu} \cdot \frac{\partial \mu}{\partial c},$$

where $\frac{\partial \hat{z}}{\partial w_j} = \frac{z_j \sum_{j=1}^{k} w_j - w_j z_j}{\left(\sum_{j=1}^{k} w_j\right)^2}$.

According to the stochastic gradient descent procedure, the parameters are updated after each training example

$$q_i^k(n+1) = q_i^k(n) - \eta \frac{\partial E}{\partial q_i^k},$$

$$a_i^k(n+1) = a_i^k(n) - \eta \frac{\partial E}{\partial a_i^k},\ b_i^k(n+1) = b_i^k(n) - \eta \frac{\partial E}{\partial b_i^k},$$

$$c_i^k(n+1) = c_i^k(n) - \eta \frac{\partial E}{\partial c_i^k}.$$

VI. CONCLUSIONS

Expressions for forward and backward propagation of a neural network for the implementation of a fuzzy hierarchical algorithm with the transfer of fuzzy intermediate variables have been obtained. The advantage of such a transfer of fuzzy integral data in conditions of unknown influence of factors on the overall result is the elimination of the need for precise definition of the terms of intermediate variables and avoids the interpretive gap between the data due to the elimination of double transformation.

The use of an adaptive neural network for the implementation of the model allows adjusting the values of parameters and boundaries of fuzzy variables during parametric adaptation. The advantages of such a structure include a reduction in the number of parameters and the training time of the neuro-fuzzy model.

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Н. М. Лазарєва. Параметрична оптимізація ієрархічної нечіткої моделі керування з передаванням нечітких значень проміжних даних

Предметом дослідження є інтелектуалізація технологічного процесу керування складними об’єктами з метою заміни ручного труда людини-оператора. В умовах, що складно описати математичними методами через неповноту та невизначеність, застосовується гібридна нейро-нечітка модель з ієрархічною будовою для керування процесом. Метою статті є дослідження та розробка алгоритму навчання для моделі Мамдані→Сугено з передачею нечітких проміжних даних між ієрархічними рівнями, реалізованої адаптивною нейронною мережею. Для забезпечення точності прогнозування в реальному часі визначено алгоритм параметричної адаптації до умов функціонування з настроюванням параметрів антецедентів та консеквентів на двох рівнях. При дослідженні методів передачі даних між рівнями були застосовані методи нечіткої логіки та штучних нейронних мереж, метод градієнтного спуску, алгоритми Мамдані та Такагі–Сугено–Канга тощо. Дослідження підтверджує можливість використання гібридних моделей для інтелектуалізації процесу керування складними об’єктами. Науковою інновацією отриманих результатів є побудова нейронної мережі ієрархічної системи керування та розробка алгоритму навчання при передачі нечітких проміжних змінних з параметричною адаптацією моделі на основі алгоритму градієнтного спуску.

Ключові слова: нейронна мережа; нечіткий алгоритм; градієнтний спуск; навчання моделі; параметрична адаптація.

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