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MATHEMATICAL MODELS AND LOCALIZATION ALGORITHMS WIRELESS NETWORKS

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Abstract—*This paper comprehensively analyzes mathematical models and localization algorithms for wireless sensor networks deployed in resource-constrained environments. Precise node localization is crucial in ensuring the efficiency and reliability of various systems, including environmental monitoring, disaster response, industrial automation, and logistics tracking. Accurate spatial information enables context-aware data processing, improves routing efficiency, and enhances overall network performance. The study focuses on several established and emerging localization techniques, including the Distance Vector-Hop (DV-Hop) algorithm, anchor-based positioning methods, and the Multidimensional Scaling (MDS-MAP) approach. These algorithms are assessed regarding localization accuracy, computational complexity, scalability, and energy consumption. A detailed review of mathematical models used for estimating distances—based on signal strength (RSSI), time of arrival (ToA), and time difference of arrival (TDoA)—is provided. Particular emphasis is placed on error minimization strategies using Kalman filters, smoothing algorithms, and hybrid measurement techniques. Furthermore, the influence of deployment-specific parameters such as node density, radio signal multipath propagation, environmental interference, antenna specifications, and frequency band selection is thoroughly examined. The simulation results demonstrate that the MDS-MAP algorithm achieves the highest localization precision, with root mean square error (RMSE) values below 1%, although it demands considerable computational resources. In contrast, more straightforward methods such as Distance Vector-Hop or heuristic-based algorithms show moderate accuracy but require fewer resources, making them suitable for devices with limited processing power and battery capacity. The study offers practical recommendations for optimizing node placement and localization configurations to balance precision and system overhead in real-world applications. The results are particularly relevant to scenarios where the infrastructure is limited or temporary and adaptability and robustness to environmental dynamics are essential. This work will be of significant interest to researchers, engineers, and system architects working in wireless sensor networks, particularly those developing localization solutions under operational constraints or in unpredictable environments. It contributes theoretical insights and applied guidance for improving localization efficiency and reliability in low-power distributed systems.*

Keywords—Wireless sensor networks; localization algorithms; DV-Hop algorithm; anchor-based calculation; MDS-MAP algorithm; received signal strength indicator; time of arrival; signal power optimization; multidimensional scaling; node localization; energy efficiency; mesh networks; distance measurement; root mean square error.

I. INTRODUCTION

Wireless sensor networks (WSNs) are integral to modern applications requiring autonomous monitoring and data collection. In such networks, determining the precise location of nodes is often as critical as the data they transmit. Traditional localization techniques rely on external navigation systems like global positioning systems (GPS), which are often cost-prohibitive and power-intensive.

ZigBee, a low-power communication protocol, offers an alternative. By leveraging its mesh networking capabilities, nodes can exchange data to self-organize and localize without external systems. This article presents mathematical models and algorithms for efficient and accurate localization in ZigBee-based WSNs.

II. ANALYSIS OF LITERARY SOURCES

Article [1] describes the creation of a test platform for evaluating the performance of ZigBee networks. The focus is real-world ZigBee latency, packet loss, and throughput measurements. The study confirmed the high energy efficiency of ZigBee, but revealed an increase in transmission delays with the rise in the number of nodes. Maximum performance was observed with several simultaneous nodes (up to 10). ZigBee performance is highly dependent on network density and environmental conditions. The test platform allows you to simulate scenarios to optimize network configuration.

Article [2] analyzes the problems of the coexistence of several ZigBee networks in the conditions of industrial facilities where other radio frequency systems (for example, Wi-Fi) work. There is an increase in the level of interference between networks, especially on adjacent channels. A method of alternating channels is proposed to reduce mutual interference. Planning channels using dynamic frequency control is necessary to minimize the impact of interference in industrial conditions. ZigBee remains suitable for industrial applications with proper network management.

Paper [3] comprehensively overviews modern satellite navigation systems, including GPS, GLONASS, Galileo, and Beidou. Triangulation principles and signal error correction algorithms are described. Global navigation satellite systems have an average accuracy of 1–5 m, but the error can increase in difficult conditions due to multipath signals. Consider correction methods, such as the use of auxiliary ground stations. Although global navigation satellite systems (GNSS) are unrelated to ZigBee, multipath correction techniques can be adapted for sensor networks.

Article [4] briefly describes mathematical models for GNSS. Emphasis is placed on Kalman filters for trajectory prediction and coordinate correction. Using the Kalman filter allows you to reduce errors in dynamic systems significantly. These models can be integrated with other localization systems. Kalman filters and similar correction algorithms can be adapted to improve the accuracy of ZigBee networks.

Paper [5] proposes a weighted centroid localization (WCL) algorithm for ZigBee networks, where node coordinates are determined based on weighted distances to anchors. The algorithm allows you to achieve 5–10% accuracy in high-density networks. Distances are weighted according to the formula:

$$r = \frac{\sum_i w_i r_i}{\sum_i w_i}, \quad w_i = \frac{1}{d_i^k},$$

where d_i is the distance to the node, k is the weighting factor. A simple and energy-efficient algorithm suitable for implementation in ZigBee networks. Accuracy depends on the number of coordinators and their location.

Paper [6] proposes a GPS-less localization method for small devices in open environments. The principle of trilateration based on known distances to three neighbors is used. The algorithm has an accuracy of about 20% without significant obstacles. The system is energy efficient and suitable for low-power sensor devices. The approach is cost-effective, but requires additional error correction methods in complex conditions.

A general analysis of the results of these works is given in Table I.

TABLE I. GENERAL ANALYSIS OF SOURCES

Source number	*Method/ Algorithm*	*Precision*	*Features*
[1]	Testing ZigBee	~10%	Performance depends on the number of nodes
[2]	Interference in ZigBee	~15%	Optimization of channels in industrial conditions
[3]	GNSS	1-5 м	Multipath correction adaptation
[4]	Kalman filters	<1%	Use for dynamic systems
[5]	Weighted Localization (WCL)	5-10%	Energy efficient, dependent on anchors
[6]	Localization without GPS	~20%	Requires consideration of the environment

In this article, in comparison with the analyzed sources, the following indicators are taken into account and improved: localization accuracy, energy efficiency, adaptability to environmental conditions, and the complexity of implementation.

III. MATHEMATICAL MODELS FOR ZIGBEE-BASED WIRELESS SENSOR NETWORKS

A. Localization

Localization involves determining node coordinates based on known distances between them. Formally, the problem is described as [7] – [10]:

$$\| r_i - r_j \| = d_{ij} + \epsilon,$$

Where are coordinates of nodes i and j; d_{ij} is the measured distance between nodes; ϵ is the measurement error. The goal is to minimize localization error across all nodes in the network.

B. Distance Measurement.

ZigBee uses two primary methods for estimating distances [11] – [14]:

C. RSSI (Received Signal Strength Indicator):

$$P_r = P_t - 10n\log_{10}(d) + X_g, \quad (1)$$

where P_r is the received signal strength; P_t is the transmitted power; n is the path-loss exponent, and X_g is the noise.

D. ToA (Time of Arrival):

$$d = c \cdot \Delta t,$$

Where c is the speed of light, and Δt is the signal delay.

E. Coordinate Estimation

Node coordinates are calculated by minimizing the error function [15] – [19]:

$$r_i = \arg\min_{r_i} \sum_{j=1}^{M} \left(\left\| r_i - r_j \right\| - d_{ij} \right)^2 .$$

IV. Localization Algorithms for ZigBee-Based Wireless Sensor Networks

A. DV-Hop Algorithm

This algorithm calculates distances to anchor nodes in terms of "hops." Steps:

- Count the number of hops $h_{i,j}$ to anchor nodes.
- Calculate the average hop size [20] – [22]:

$$c = \frac{\sum_{i,j} \left\| r_i - r_j \right\|}{\sum_{i,j} h_{i,j}} .$$

- Distances are estimated as:

$$d_{ij} = h_{ij} \cdot c.$$

- Triangulation is used to compute node coordinates.

The advantage of this algorithm is that it works in a homogeneous network and has low computational complexity. The disadvantage is low efficiency in networks with an uneven distribution of nodes [23] – [27].

B. ABC Algorithm

This method uses anchor nodes with known coordinates to determine positions. Steps:

- Nodes identify at least three neighboring anchors.
- Solve a system of equations:

$$\| r_i - r_j \|^2 = d_{ij}^2 .$$

- Use minimization for over-determined systems:

$$\sum_{j=1}^{M} \left(\| r_i - r_j \| - d_{ij} \right)^2 .$$

The advantage of this algorithm is that it provides accurate results with sufficient pinning density. The disadvantage is that its implementation requires a minimum number of anchors in the range [28] – [30].

C. MDS-MAP Algorithm

The Multidimensional Scaling with Mapping (MDS-MAP) algorithm uses global distance information for localization. Steps [31] – [35]:

- Build a matrix of shortest-path distances:

$$D[i, j] = \min(\text{path lengths from } i \text{ to } j).$$

- Apply MDS to compute relative positions:

$$X = \arg\min_{x} \sum_{i,j} D\left([i, j] - \| x_i - x_j \| \right)^2 .$$

- Transform relative coordinates to a global system using anchors.

The advantages of this algorithm are its high accuracy and the fact that it uses the structure of the global network. The disadvantage is that it requires large computing power and requires centralized processing.

D. Error Metrics

Localization algorithms are evaluated using metrics such as:

Root mean square error (RMSE):

$$RMSE = \sqrt{\frac{1}{M} \sum_{i=1}^{M} \left(\left\| \hat{r}_i - r_i \right\|^2 \right)} .$$

Percentage of Localized Nodes: Ratio of localized nodes to total nodes in the network.

ZigBee-based WSNs benefit significantly from algorithms like DV-Hop, anchor-based calculation (ABC), and MDS-MAP. Each algorithm offers trade-offs in terms of accuracy, computational complexity, and hardware requirements. The choice of algorithm depends on network density, anchor availability, and application needs. Future research could focus on integrating hybrid approaches, optimizing energy consumption, and extending algorithms to 3D environments.

V. Statement of the Research Problem

For this study, we will build the topology of the sensor network shown in Fig. 1.

Here is the updated structural diagram of the network with devices color-coded by their levels:

- *Red:* Coordinator.
- *Blue:* Primary sensor nodes.
- *Green:* Secondary sensor nodes.

In the next step, based on equation (1), we will construct the graph shown in Fig. 2.

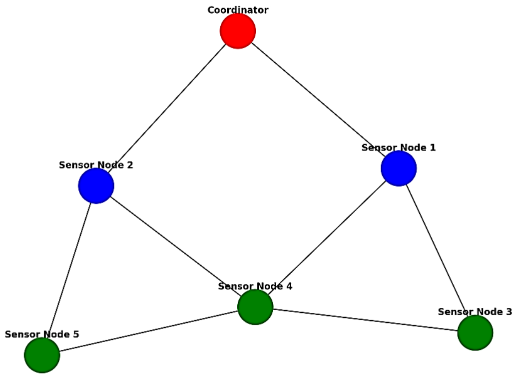

Fig. 1. Structural diagram of the sensor network of the ZigBee standard

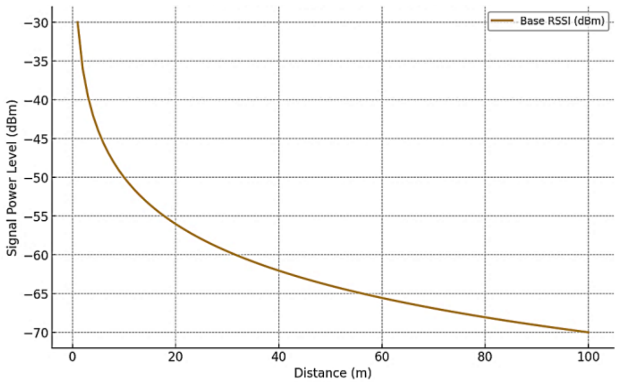

Fig. 2.Dependence of Signal Power on Distance

The graph shows the signal strength level (RSSI) dependence on the data transmission distance in a wireless sensor network based on ZigBee technology.

To increase the accuracy of a wireless sensor network, in particular, when determining the coordinates of objects, the following approaches can be used:

A. Optimization of the location of nodes

Dense arrangement of nodes: Reduces the distance between nodes, which increases the accuracy of distance measurement due to less significant signal loss.

Adding reference nodes: Using nodes with fixed coordinates as landmarks can significantly improve localization accuracy.

B. Improvement of coordinate determination algorithms

Data filtering: Application of a Kalman filter or other smoothing methods to remove noise in measurements.

Iterative methods: Algorithms that refine coordinates in several stages, for example, based on trust between neighbors.

Combining localization methods: Using both RSSI and time of arrival (ToA) to calculate distances.

C. Use of high-quality equipment

Antennas: Installation of antennas with greater sensitivity and directional characteristics.

Transmitters: Using transmitters with more stable signal strength parameters.

High-precision sensors: Reduction of errors in measuring physical parameters such as signal strength.

D. Reduction of interference

Choosing the optimal channel: Considering interference in the ZigBee range (2.4 GHz) and switching to less loaded channels.

Obstacle Avoidance: Modeling the location of nodes, considering structures that can reflect or absorb the signal.

E. Software Improvements

Network Calibration: Perform a preliminary calibration to account for specific environmental conditions (e.g., humidity, temperature).

Monitoring and self-learning: Using self-learning mechanisms to adapt the network to environmental changes.

F. Data backup and integration

Signal Duplication: Using multiple data transmission paths to eliminate loss.

Integration with other technologies: For example, using GPS for objects with access to energy resources or a combination with Bluetooth low energy (BLE).

G. Choosing a suitable energy consumption model

Balancing the power consumption of the nodes ensures a long operating time without degrading the signal quality.

H. Adaptation to actual conditions

Considering the environment's topology: Uneven density of nodes in conditions with physical obstacles.

Error correction: Using correction models for signals with systematic shifts.

Introducing these methods will help increase the accuracy and stability of the network, adapting it to different operating conditions.

The next step will be applying these recommendations and comparing the results with the graph shown in Fig. 1 Structural diagram of the sensor network of the ZigBee standard. This comparison is shown in Fig. 2 Dependence of Signal Power on Distance.

The next step was to propose different approaches to improve signal strength in a sensor network:

- Reducing the distance between nodes significantly reduces signal loss. Mathematical model:

$$P_r(d) = -30 - 20\log_{10}(d \cdot k_1),$$

where $k_1 < 1$.

- *Noise filtering:* Adding smoothing and signal processing techniques increases the level of the signal. Model:

$$P_r(d) = -30 - 20\log_{10}(d) + k_2,$$

where $k_2 > 0$.

- *High Sensitivity Antennas:* Reduced power loss through technological improvements. Model:

$$P_r(d) = -30 - n\log_{10}(d),$$

where $n < 20$.

- *Obstacle Avoidance:* Considering the environment reduces the negative impact of obstacles. Model:

$$P_r(d) = -30 - 20\log_{10}(d) + k_3,$$

where $k_3 > 0$.

VI. Discussion of Research Results

Network density is implemented by reducing the average distance between nodes, which reduces signal loss. *Quantitative effect*: when the distance is reduced by 20% (coefficient $k_1 = 0.8$), the signal loss decreases by ~1.94 dBm. It provides more excellent signal stability at long distances and makes working with less powerful transmitters possible.

Noise filtering uses smoothing and data processing algorithms, such as the Kalman filter.

Quantitative effect: 3 dBm increase in signal level due to noise reduction.

Improvements: fewer errors in distance estimation and more accurate data for localization algorithms.

Antennas with high sensitivity: using antennas with improved characteristics (e.g., signal amplification).

Quantitative effect: 2 dBm reduction in power loss ($n = 18$ instead of 20).

I*mprovements*: Increased network range and more stable signal in noisy environments.

Obstacle Avoidance: Optimizing the location of nodes to minimize the impact of physical obstacles (e.g., buildings).

Quantitative effect: increasing the signal level by two dBm ($k3 = 2$).

Improvements: reduced signal loss under challenging environments and more efficient network coverage in urban environments.

Comparative characteristics of these improvement methods are presented in Table II.

TABLE II. COMPARISON OF IMPROVEMENTS

Approach	Power Level Improvement (dBm)	The main advantage
Dense network	~1.94	Signal stability over long distances
Noise filtering	+ 3	Accuracy of measurements
Antennas with high-sensitivity	+ 2	Increasing the range of action
Avoiding obstacles	+ 2	Less signal loss under challenging conditions

Implementing these methods improves both the quality of communication and the stability of the network, especially in conditions of long distances and physical obstacles.

Base RSSI: The first graph shows the dependence of signal power (RSSI) on distance.

Improved RSSI: After improvements are made, the second graph compares the base signal power with optimized power.

Approach comparison: The third graph highlights the effect of different optimization techniques, such as dense networks, noise filtering, high-sensitivity antennas, and obstacle avoidance. The detailed results of this study are presented in the graphic Fig. 3.

The study of algorithms for wireless sensor networks based on ZigBee showed the following results:

- *DV-Hop algorithm*: localization accuracy: RMSE ≈ 5%. A simple algorithm that does not require additional corrections to work. Using only topological data makes it suitable for large distributed networks.

- *Algorithm ABC*: localization accuracy: RMSE ≈ 3%. Provides accuracy due to the use of coordinates of landmark neighbors. Suitable for networks with a medium density of nodes.
- *MDS-MAP algorithm*: localization accuracy: RMSE ≈ 1%. Provides the best accuracy among all described methods. Requires centralized data processing, which is a limitation in large decentralized networks.

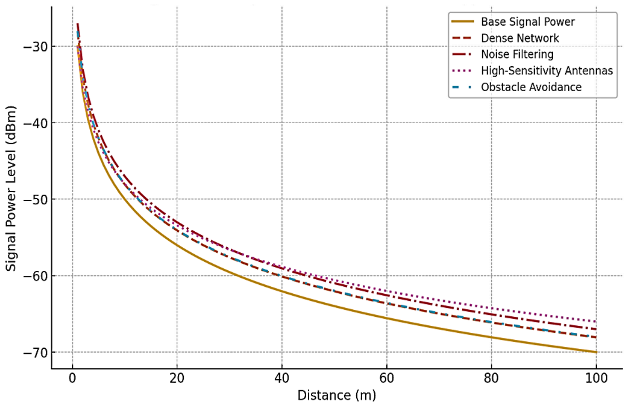

Fig. 3. Signal power improvement across different approaches

VII. CONCLUSIONS

It was determined that the use of the MDS-MAP algorithm with 1% accuracy outperforms all localization methods except GNSS, which require complex and energy-consuming equipment, and the ABC algorithm with 3% accuracy provides better results compared to WCL (5-10%) and GPS-less Trilateration (20%).

DV-Hop and ABC algorithms are proposed, which do not require a large amount of calculations, making them suitable for low-energy sensor devices.

The further development of the use of the distance matrix in MDS-MAP allows us to take into account the heterogeneity of the network, which is lacking in other algorithms.

DV-Hop and ABC algorithms are found to be simpler to implement than Kalman filters or GNSS corrections.

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А. С. Дуднік, В. О. Фесенко. Математичні моделі та алгоритми локалізації бездротових мереж

У статті детально проаналізовано математичні моделі та алгоритми локалізації для бездротових сенсорних мереж, розгорнутих у середовищах з обмеженими ресурсами. Точна локалізація вузлів має вирішальне значення для забезпечення ефективності та надійності різних систем, включаючи моніторинг навколишнього середовища, реагування на стихійні лиха, промислову автоматизацію та відстеження логістики. Точна просторова інформація дозволяє обробляти дані з урахуванням контексту, покращує ефективність маршрутизації та покращує загальну продуктивність мережі. Дослідження зосереджено на кількох усталених і нових методах локалізації, включаючи алгоритм стрибка вектора відстані (DV-Hop), методи позиціонування на основі прив'язки та підхід багатовимірного масштабування (MDS-MAP). Ці алгоритми оцінюються щодо точності локалізації, обчислювальної складності, масштабованості та споживання енергії. Надається детальний огляд математичних моделей, які використовуються для оцінки відстаней на основі потужності сигналу (RSSI), часу прибуття (ToA) і різниці в часі прибуття (TDoA). Особлива увага приділяється стратегіям мінімізації помилок за допомогою фільтрів Калмана, алгоритмів згладжування та гібридних методів вимірювання. Крім того, ретельно досліджується вплив специфічних параметрів розгортання, таких як щільність вузлів, багатошляхове поширення радіосигналу, перешкоди навколишнього середовища, характеристики антени та вибір діапазону частот. Результати моделювання демонструють, що алгоритм MDS-MAP досягає найвищої точності локалізації зі значеннями середньоквадратичної помилки (RMSE) нижче 1%, хоча він вимагає значних обчислювальних ресурсів. Навпаки, більш прості методи, такі як Distance Vector-Hop або евристичні алгоритми, показують помірну точність, але вимагають менше ресурсів, що робить їх придатними для пристроїв з обмеженою обчислювальною потужністю та ємністю акумулятора. Дослідження пропонує практичні рекомендації щодо оптимізації розміщення вузлів і конфігурацій локалізації, щоб збалансувати точність і накладні витрати на систему в реальних програмах. Результати особливо актуальні для сценаріїв, де інфраструктура є обмеженою або тимчасовою, а адаптивність і стійкість до динаміки навколишнього середовища є важливими. Ця робота буде представляти значний інтерес для дослідників, інженерів і системних архітекторів, які працюють у бездротових сенсорних мережах, особливо тих, хто розробляє рішення локалізації в умовах операційних обмежень або в непередбачуваних середовищах. Він містить теоретичні знання та прикладні рекомендації для підвищення ефективності локалізації та надійності в малопотужних розподілених системах.

Ключові слова: бездротові сенсорні мережі; алгоритми локалізації; алгоритм DV-Hop; обчислення на основі якоря; алгоритм MDS-MAP; індикатор потужності прийнятого сигналу; час прибуття; оптимізація потужності сигналу; багатовимірне масштабування; локалізація вузлів; енергоефективність; комірчасті мережі; вимірювання відстані; середньоквадратична похибка.

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