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ADVANCED COPULA-BASED METHODS FOR NONPARAMETRIC DETECTION AND CHARACTERIZATION OF WIDEBAND RADAR SIGNALS

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Abstract**—***This paper introduces advanced copula-based methods for the nonparametric detection and characterization of wideband radar signals. The research focuses on developing signal detection algorithms that are invariant to changes in the probability density function of the sounding or reflected signals, employing multiscale analysis techniques and copula-based statistics. Two primary approaches are explored: multiscale analysis using wavelet transforms and rank-based signal detection with copula-based ambiguity functions. Simulation results confirm the effectiveness of the proposed approaches. The research demonstrates that integrating rank-based methods with copula-based statistics significantly improves the detection and analysis of wideband radar signals, particularly in complex scenarios where signals exhibit intricate dependency structures. This comprehensive detection framework is well-suited for handling high-dimensional radar signal data, enhancing accuracy and reliability under varied conditions. Future work will focus on optimizing copula selection and permutation strategies to further improve performance.*

Index Terms—Ambiguity function; rank; copula; detection; radar signal; noise radar.

I. INTRODUCTION

Designing a statistic that is invariant to all possible changes in the probability density function (PDF) of the sounding or reflected signals is a challenging task. In statistical signal processing, invariance to changes in the PDF typically involves creating a statistic that depends only on the structure of the signal rather than its distribution. One common approach is to use non-parametric methods or statistics that rely on the ranks of the partial likelihood ratios [1] rather than their actual values. Another approach is to use generalization of the radar ambiguity function, Copula ambiguity function. The approach has been broadly described in paper [3].

In this paper it is developed and tested two approaches to the synthesis of signal detection algorithms invariant to changes of the probability density function of the sounding or reflected signals.

The multiscale analysis approach is dealing with wideband radar signals which often exhibit characteristics at multiple scales. Also time-varying nature of wideband signals requires joint time-frequency analysis. To solve these problems it is proposed to apply multiscale analysis techniques (e.g., wavelet transform) to decompose the signals into different frequency components. And compute copula-based statistics at each scale to capture more detailed dependencies between sounding and reflected signals.

Another approach researched in this paper is rank-based signal detection algorithms with copula-based ambiguity function. It is proposed further development and enhancement of the generalized copula ambiguity function for better detection and analysis of wideband radar signals. Developed algorithms combine the strengths of rank-based signal detection algorithms with copula-based ambiguity functions.

It was researched how permutation tests and partial likelihood ratios used in rank-based methods can be integrated into copula-based frameworks to enhance signal detection accuracy under uncertain conditions.

By using such a rank-based approach, we can create statistics that are invariant to the underlying PDF of the signal, focusing instead on the structure and relative positions of the data points.

II. MULTISCALE ANALYSIS

Wideband radar signals often exhibit characteristics at multiple scales. In order to solve this problem we apply multiscale analysis technique to decompose the signals into different frequency components. For capturing more detailed dependencies between sounding and reflected signals we compute copula-based statistics at each scale.

To address the problem of decomposing wideband radar signals into different frequency components and analyzing the dependencies between sounding and reflected signals, we will follow the next approach.

At first we will do a signal decomposition using wavelet transform. We apply a wavelet transform to decompose the wideband radar signals into multiple scales. This will help to separate the signal into different frequency components.

Then we will compute copula-based statistics. At each scale, we compute such statistics to capture the dependencies between the sounding and reflected signals. This involves modeling the joint distribution of the sounding and reflected signals using copulas and computing measures such as mutual information or rank correlation [2].

Signal Decomposition using Wavelet Transform. The wavelet transform decomposes a signal $x(t)$ into different scales. This can be expressed as:

$$W_x(a,b)=\int_{-\infty}^{\infty}x(t)\psi^*\left(\frac{t-b}{a}\right)dt, \tag{1}$$

where $\psi(t)$ is the mother wavelet; a is the scale parameter; b is the translation parameter; ψ^* denotes the complex conjugate of ψ.

For discrete signals, the wavelet decomposition can be performed using a discrete wavelet transform (DWT), which results in approximation and detail coefficients at various levels.

Let x_s and x_r be the sounding and reflected radar signals, respectively. Using DWT, we decompose these signals into approximation A and detail D coefficients at multiple levels:

$$A_j, D_j = DWT(x,\psi,j), \tag{2}$$

where j represents the decomposition level.

In our case, for wavelet decomposition, the signals are decomposed using a Dobechy wavelet with four vanishing moments (db4) to level five.

Each level represents a different frequency component of the signal, starting from the low-frequency approximation (level 0) to the high-frequency details (higher levels). This multiscale decomposition allows us to analyze the signals at various frequency bands.

III. COPULA-BASED APPROACH

Copulas are used to model the dependency between two random variables. Let u and v be the marginal distributions of the approximation and detail coefficients of the sounding and reflected signals, respectively. The copula $C(u,v)$ captures the joint distribution of these marginals.

To fit a gaussian copula and compute dependency measures, we follow these steps:

A) Computing Rank Transforms

Convert the data to uniform marginals using rank transforms.

$$u_i=\frac{R_{A_i}}{n+1},\qquad v_i=\frac{R_{D_i}}{n+1}, \tag{3}$$

where R_{A_i} and R_{D_i} are the ranks of A_i and D_i respectively, and n is the sample size.

B) Fitting the Gaussian Copula. (Fig. 1)

The gaussian copula [4] is used to model the joint distribution of the coefficients from the sounding and reflected signals at each scale (Figure 1).

The gaussian copula $C_\rho(u,\upsilon)$ is defined as:

$$C_\rho(u,\upsilon)=\Phi_\rho\left(\Phi^{-1}(u),\Phi^{-1}(\upsilon)\right), \tag{4}$$

where Φ is CDF of the standard normal distribution, Φ^{-1} is its inverse, and Φ_ρ is the bivariate normal CDF with correlation parameter ρ.

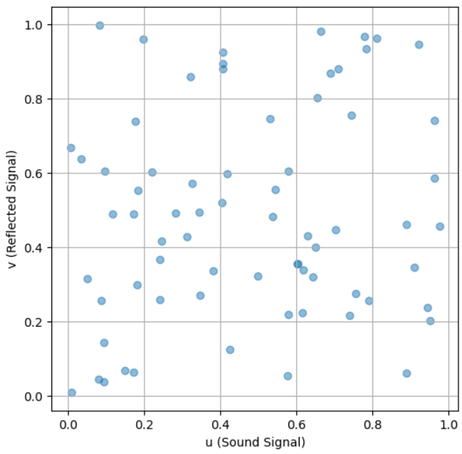

Fig. 1. Gaussian Copula

C) Compute Kendall's Tau and Spearman's Rho

Kendall's Tau and Spearman's Rho are computed to measure the dependencies between the signals at each scale.

These are rank-based measures of dependency:

$$\tau=\frac{2}{n(n-1)}\sum_{i<j}\operatorname{sign}\left((u_i-u_j)(\upsilon_i-\upsilon_i)\right),$$
$$\rho=1-\frac{6\sum d_i^2}{n(n^2-1)}, \tag{5}$$

where $d_i = R_{u_i} - R_{\upsilon_i}$ is the difference between the ranks.

On Figure 2 it is showed obtained plots of Kendall's Tau and Spearman's Rho versus the decomposition scale for the radar signals. The left plot shows the variation of Kendall's Tau with the scale. The right plot shows the variation of Spearman's Rho with the scale.

These plots help visualize the dependencies between the sounding and reflected signals across different frequency components, as analyzed through the wavelet decomposition.

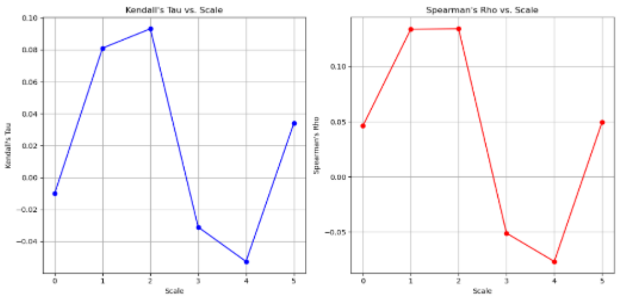

Fig. 2. The dependency measures variety across different scales of the wavelet decomposition

The results are obtained to show how the dependency measures vary across different scales of the wavelet decomposition.

This approach provides a detailed multiscale analysis of the dependencies between the sounding and reflected radar signals, capturing the relationships at different frequency components.

IV. ESTIMATES

The gaussian copula ambiguity function can be estimated using kernel estimates [3]. The example of such function, which is obtained with the help of the noise acoustic radar, is presented on Fig. 3. The copula kernel estimate, calculated for the signals of the acoustic noise radar, which has been obtained using copula-based approach.

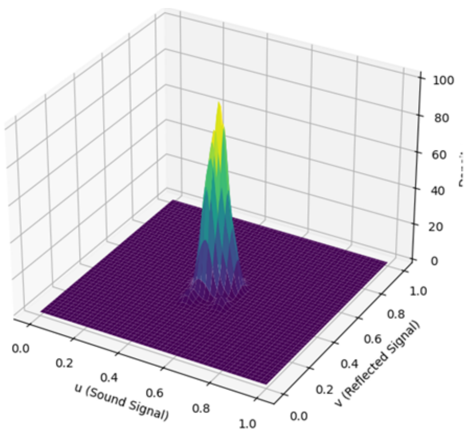

Fig. 3. Kernel Estimate of Gaussian Copula Ambiguity Function

V. ADVANCED COPULA-BASED AMBIGUITY FUNCTION

With the help of the noise acoustic radar, designed and constructed by author [8], the gaussian copula ambiguity function was measured for real signals. The acoustic radar sounding signal is a wideband random signal with a normal distribution. The signal reflected from the solid object at the distance equal to 70 m from the radar. The signals are presented on Fig. 4. For these signals the ambiguity function (6) and the gaussian copula ambiguity function (7) were calculated. The results are presented on Figs 5 and 6.

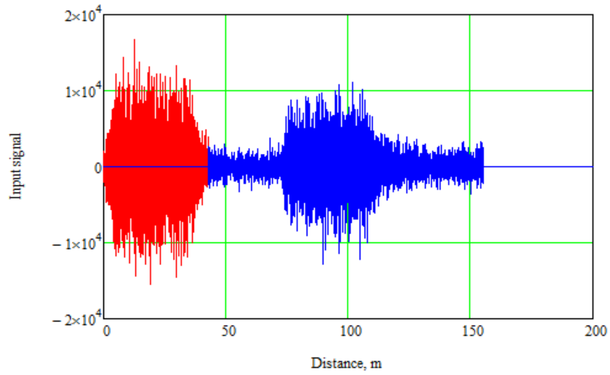

Fig. 4. The souding (red) and reflected signal (blue). Input signal in ADC samples, which are equal to 15.258789 µV

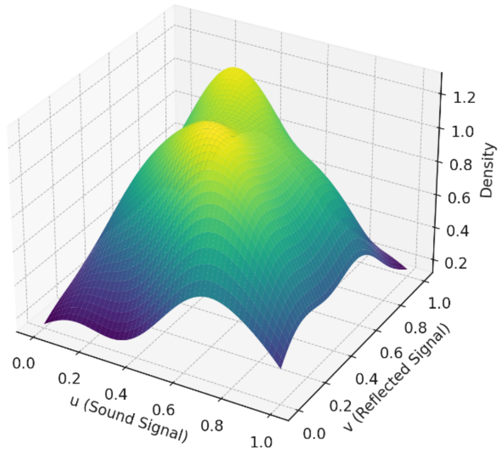

Fig. 5. Kernel estimate of probability density function for acoustic radar signal

$$\chi(\tau,\nu) = \sqrt{|\nu|} \int_{-\infty}^{\infty} \left(s_1(t) - m_{s_1}\right)\left(s_2\left(\nu(t-\tau)\right) - m_{s_2}\right) dt, \quad (6)$$

where, τ is the time delay; ν is the Doppler frequency shift; s_1 is the original signal; s_2 is the reflected signal. Sampling rate f_s = 1 MHz. Distance to object d = 70 m.

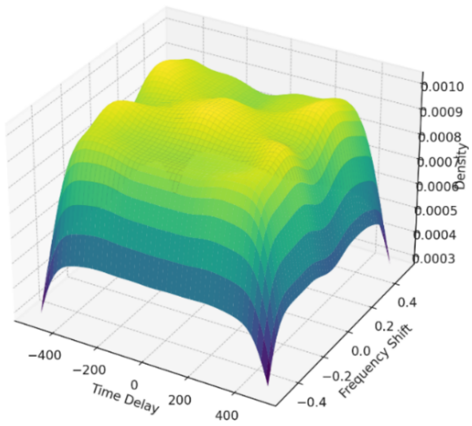

Fig. 6. Kernel Estimate of Gaussian Copula Ambiguity Function

Distance conversion factor Δd = 0.0038820862 meters per ADC sample. Signal length N = 1000 samples. Maximum delay $\tau_{\max}$ = 200 samples. Maximum Doppler shift $\nu_{\max}$ = 50 Hz.

$$\chi(\tau,\nu)=\sqrt{|\nu|}\int_{-\infty}^{\infty}\left(\hat{E}_{s_1}\left(s_1(t)\right)-m_{s_1}\right)\left(\hat{E}_{s_2}s_2\left(\nu(t-\tau)\right)-m_{s_2}\right)dt, \quad (7)$$

where $\hat{E}_{s_1}, \hat{E}_{s_2}$ estimates of cumulative distribution functions

The shape of the the ambiguity function does not depend on the PDFs of the sounding and reflected signals [5]. That is why signal detection algorithms, which are based on this notion, are distribution free.

Figure 6 shows the kernel estimate of the gaussian copula ambiguity function for the acoustic radar signal.

A) Generated the Sounding and Reflected Signals

The sounding signal is a wideband random signal with a normal distribution. The reflected signal is delayed to simulate a reflection from a solid object at 70 meters. Performed Wavelet Decomposition: Decomposed the signals using the Daubechies wavelet (db4) up to level 5.

B) Converted Data to Uniform Marginals

Transformed the wavelet coefficients to uniform marginals using the CDF of the normal distribution.

Fitted the Gaussian Copula: used kernel density estimation to fit the Gaussian copula to the uniform marginals.

C) Ambiguity Function

Estimate showing the dependencies between the sounding and reflected signals, capturing their joint distribution at the selected wavelet decomposition level. Also it helps to visualize the density function of the copula.

Figure 7 shows the copula cross-ambiguity function cross section in the time (distance) domain. The x-axis represents the distance in meters, and the y-axis represents the magnitude of the copula ambiguity function [6].

This cross-correlation function is effectively a cross-section of the gaussian ambiguity function for zero velocity of the target. The peak in the correlation function corresponds to the time delay (and thus the distance) of the reflected signal, which indicates the presence of the reflecting object at 70 meters distance. This visualization helps in understanding the time (distance) relationship and the correlation between the original and reflected radar signals.

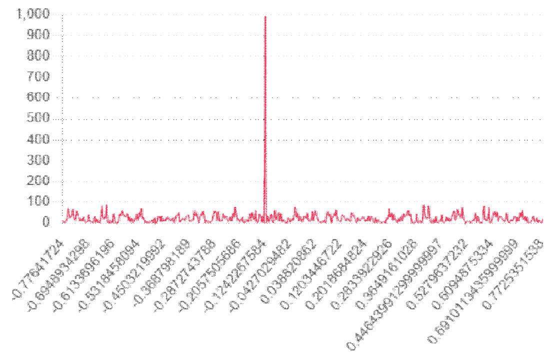

Fig. 7. Estimate of the copula cross-ambiguity function cross section (cross-correlation function) for the acoustic radar

This plot also effectively shows how the correlation between the original and reflected radar signals varies with distance. The peak in the correlation function indicates the distance to the reflecting object, which in this case is around 70 meters. This is consistent with the simulated delay for the reflected signal.

VI. RANK-BASED SIGNAL DETECTION ALGORITHMS WITH COPULA-BASED AMBIGUITY FUNCTION

To combine the strengths of rank-based signal detection algorithms with copula-based ambiguity functions, we will create a hybrid approach that leverages the advantages of both methods. This hybrid approach will involve preprocessing the signals, performing permutation tests to compute partial likelihood ratios, fitting copula models to these ratios, and then combining these components into a single detection statistic.

To integrate permutation tests and partial likelihood ratios used in rank-based methods into copula-based frameworks, we will enhance signal detection accuracy under uncertain conditions [7]. Author propose to use integrated approach which includes the next procedure:

A) Preprocessing and Rank Transformation

We generate two signals, the sounding signal *X* and the reflected signal *Y*, we preprocess the signals by transforming them into their rank equivalents.

Let *N* be the number of samples in each signal:

$$\text{ranked\_sounding\_signal} = \frac{\text{rank}(X)}{N+1},$$

rank (*X*) = ranks of the elements in *X*,

$$\text{ranked\_sounding\_signal} = \frac{\text{rank}(Y)}{N+1},$$

rank (*Y*) = ranks of the elements in *Y*.

B) Permutation Tests and Partial Likelihood Ratios

Permutation tests involves computing likelihood ratios for permutations of the signal.

For a signal *Z* with *P* permutations:

$$Likelihood\_ratio\ (\pi) = \frac{\prod_{i=1}^{P} \pi_i}{binom.pmf\,(\pi, \mu, \sigma)},$$

where π is a permutation of *Z*, μ is the mean, and σ is the standard deviation of the permutation [8].

C) Copula-based Dependency Modeling

Fit a copula model to the partial likelihood ratios and compute the density function.

Let *L* be the partial likelihood ratios:

1) Fit a copula *C* to *L*.

2) Compute the copula density f_C for the data *L*.

D) Hybrid Detection Statistic

Combine the rank-based statistics with the copula-based density estimates to form a hybrid detection statistic.

Ranked signals R_X and R_Y, and their corresponding partial likelihood ratios L_X and L_Y:

1) Combine L_X and L_Y into L_{XY}.

2) Fit a copula to L_{XY} and compute the density $f_C(L_{XY})$.

3) Compute the rank-based statistic S_R as:

$$S_R = \frac{1}{N}\sum_{i=1}^{N} L_{X_i} + \frac{1}{N}\sum_{i=1}^{N} L_{Y_i}.$$

4) Compute the hybrid statistic S_H as:

$$S_H = S_R + \frac{1}{N}\sum_{i=1}^{N} f_C(L_{XY}).$$

In order to implement and test proposed approach the next calculations has been performed:

1) Signal Representation and Preprocessing

$$X = \{x_1, x_2, \ldots, x_n\}.$$

Rank Transformation: convert the signal values into ranks.

$$R(X) = \{r_1, r_2, \ldots, r_n\}$$

2) Copula Modeling

Marginal distribution estimation: estimate the marginal cumulative distribution functions (CDFs) $F_1, F_2, \ldots, F_n$.

Copula fitting: fit a copula *C* to the ranked data

$$U = \{u_1, u_2, \ldots, u_n\}$$

where $u_i = F_i(r_i)$, $\quad C(F_1(r_1), F_2(r_2), \ldots, F_n(r_n))$.

3) Permutation Test within Copula Framework

Generate permutations: Create *m* permutations of the ranked signal *R*(*X*):

$$\{R_1(X), R_2(X), \ldots, R_m(X)\},$$

Compute copula-based ambiguity functions: for each permutation $R_j(X)$, compute the copula-based ambiguity function A_j

$$A_j = A(C, R_j(X)).$$

Null distribution: Construct the null distribution of the test statistic *T* from the ambiguity functions of permutations.

$$T = \{A_1, A_2, \ldots, A_m\}.$$

4) Partial Likelihood Ratio Calculation

Compute likelihoods: likelihood under null hypothesis H_0 (no signal):

$$L_0 = P(U|H_0, C).$$

Likelihood under alternative hypothesis H_1 (signal present):

$$L_1 = P(U|H_1, C).$$

Calculate partial likelihood ratio:

$$\Lambda = \frac{L_1}{L_0}.$$

5) Signal Detection

Compare observed statistic to null distribution: Calculate the observed copula-based ambiguity function A_{obs} for the original ranked signal *R(X)*.

$$A_{\text{obs}} = A(C, R(X)).$$

Determine the p-value by comparing A_{obs} to the null distribution T.

$$p-\text{value}=\frac{1}{m}\sum_{j=1}^{m} I\left(A_j \geq A_{\text{obs}}\right).$$

where $I(\cdot)$ is the indicator function.

Decision rule using partial likelihood ratio: set a threshold α for the partial likelihood ratio. Determine signal presence if:

$$\Lambda > \alpha \text{ and } p-\text{value} < 0.05.$$

The results used, have been obtained with acoustic radar wideband random signal with a normal distribution.

Given a ranked signal $R(X)=\{3,1,4,2,5\}$ and a fitted gaussian copula:

1) Permutations: generate permutations of $R(X)$:

$$\left\{R_1(X), R_2(X), \ldots, R_m(X)\right\}$$

2) Ambiguity Functions: compute A_j for each permutation.

$$T=\left\{A_1, A_2, \ldots, A_m\right\}.$$

3) Likelihoods: calculate L_0 and L_1

$$L_0 = P\left(U \middle| H_0, C\right), \qquad L_1 = P\left(U \middle| H_1, C\right).$$

4) Partial Likelihood Ratio:

$$\Lambda = \frac{L_1}{L_0}.$$

5) Signal Detection:

$$A_{\text{obs}} = A\left(C, R(X)\right),$$

$$p-\text{value}=\frac{1}{m}\sum_{j=1}^{m} I\left(A_j \geq A_{\text{obs}}\right)$$

Decision: signal present if

$$\Lambda > \alpha \text{ and } p-\text{value} < 0.05.$$

Figure 8 shows the original signal and its ranked version. The ranked signal is derived by transforming the original signal into ranks, which provides robustness against noise and outliers.

Figure 9 displays the null distribution of the copula-based ambiguity function values obtained from the permutations of the ranked signal. The red dashed line indicates the observed test statistic computed from the original ranked signal. The p-value, which represents the probability of observing such a test statistic under the null hypothesis, is also shown.

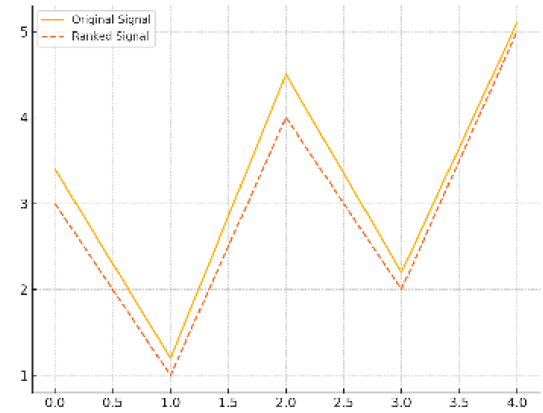

Fig. 8. The original signal and its ranked version

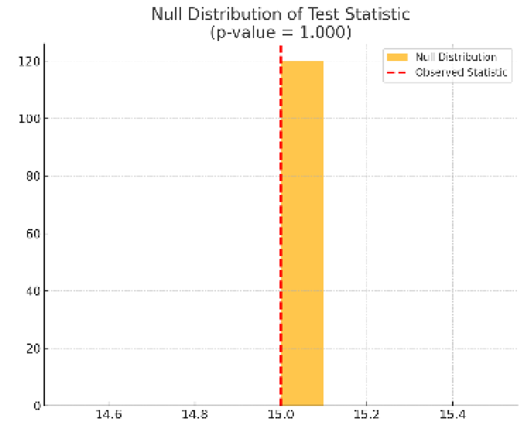

Fig. 9. Null distribution of the copula-based ambiguity function values obtained from the permutations of the ranked signal

Figure 10 shows the integration of permutation tests and partial likelihood ratios into a copula-based framework for signal detection. The histogram represents the null distribution of the copula-based ambiguity function values obtained from permutations of the ranked signal. The red dashed line indicates the observed ambiguity function value, with its associated p-value displayed. Additionally, the partial likelihood ratio is shown in the plot, providing further insight into the signal detection process.

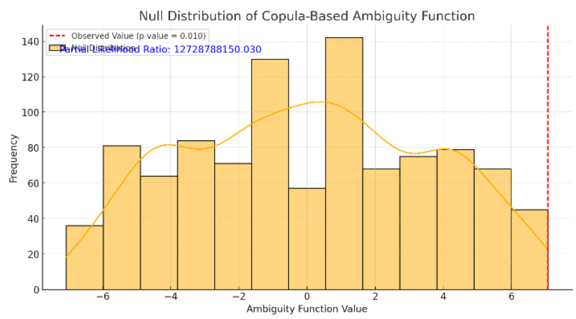

Fig. 10. Null Distribution of Copula-based Ambiguity Function

This visualization demonstrates how the hybrid approach leverages permutation tests and rank-based methods within a copula-based framework to enhance signal detection accuracy under uncertain conditions.

Lets consider advantages of the of such Integrated Approach.

1) Robustness to Noise and Outliers: Rank-based methods and permutation tests are inherently robust to noise and outliers.

2) Modeling Dependencies: Copulas capture complex dependencies between signal components, enhancing the accuracy of the detection framework.

3) Non-Parametric Nature: The combined approach does not rely on strong parametric assumptions, making it versatile for various types of signals and noise conditions.

Conclusion and Future research recommendation.

Integrating permutation tests and partial likelihood ratios into copula-based frameworks creates a powerful hybrid approach for signal detection. This method combines the robustness of rank-based methods with the sophisticated dependency modelling of copulas, providing enhanced detection accuracy under uncertain conditions. Future work can focus on optimizing the copula selection and permutation strategies to further improve performance.

VII. Conclusions

In the paper it has been developed and tested two approaches.

The hybrid approach combines the rank-based signal detection algorithm's robustness to non-parametric data with the copula-based ambiguity function's ability to model dependencies between signals. By leveraging both methods, we can improve detection performance, especially in complex scenarios where signals exhibit intricate dependency structures. The steps outlined ensure that both preprocessing and advanced statistical modeling are integrated into a cohesive detection framework.

The enhanced generalized copula ambiguity function leverages both rank-based signal detection and copula-based dependency modeling to improve the detection and analysis of wideband radar signals. By combining the robustness of rank-based methods with the sophisticated dependency modeling of copulas, this hybrid approach is well-suited for handling complex, high-dimensional radar signal data under uncertain conditions. This method provides a comprehensive framework for signal detection, enhancing accuracy and reliability.

The effectiveness of the proposed approaches is confirmed by the simulation results.

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Ж. М. Бокал. Покращені непараметричні методи виявлення широкосмугових радіолокаційних сигналів
У роботі представлені вдосконалені непараметричні методи на основі копул для виявлення та характеристики широкосмугових радіолокаційних сигналів. Дослідження зосереджено на розробці алгоритмів виявлення сигналів, які є інваріантними до змін у функції щільності ймовірності зондувальних або відбитих сигналів, використовуючи методи багаторівневого аналізу та використання статистик на основі копул. Розглядаються два основні підходи: багаторівневий аналіз за допомогою вейвлет-перетворень та виявлення сигналів на основі рангів з використанням копулярних функцій невизначеності. Результати моделювання підтверджують ефективність запропонованих підходів. Дослідження демонструє, що інтеграція методів на основі рангів та статистик на основі копул значно покращує виявлення та аналіз широкосмугових радіолокаційних сигналів, особливо в складних сценаріях, де сигнали мають складні структурні залежності. Ця комплексна система виявлення добре підходить для обробки багатомірних даних радіолокаційних сигналів, підвищуючи точність та надійність в різних умовах.
Ключові слова: функція невизначеності; ранг; копула; виявлення; радіолокаційний сигнал; шумовий радар.

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