AVIATION TRANSPORT

UDC 656.7.052.002.5:681.32(045)
DOI:10.18372/1990-5548.80.18692

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COMPLEMENTARY FILTER IN INERTIAL-DOPPLER SYSTEM OF NAVIGATION

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Abstract—*The article is devoted to filtering algorithms that are used to process flight speed data from an inertial navigation system and from a Doppler speed and angle demolition meter. It is shown that the compensation scheme and the filtering scheme, the more common name of which is the complementary filter scheme, are the most common for processing flight and navigational information in inertial-Doppler navigation systems. The paper proves the equivalence of the compensation scheme and the complementary filter scheme, and also substantiates that these schemes, unlike Kalman filtering, are simple to implement, and for design, they require the selection and setting of only one parameter - the filter time constant. A technique for choosing the optimal value of the filter time constant is proposed, which ensures the maximum accuracy of the estimated parameter, namely: the aircraft's cruising speed. The methodology is based on the analysis of error spectra of integrated systems: the inertial system - with a low-frequency error spectrum, and the Doppler system with a high-frequency spectrum. Studies of synthesized filtering schemes have been conducted.*

Index Terms—Inertial navigation system; Doppler speed meter; Kalman filter; mutual compensation and filtering; complementary filter; frequency characteristics; spectral characteristics; filter's time constant.

I. INTRODUCTION

The high efficiency of the use of information available on board aircraft is ensured by the use of various methods of its processing.

The best results of increasing the accuracy characteristics of measuring complexes are achieved in systems with structural redundancy. Structural redundancy in flight management and guidance systems (FMGS) is understood as the possibility of determining flight and navigation information in parallel in several ways using signals from measurement systems built on different physical principles. The information obtained in this way is integrated.

In FMGS of modern aircraft, such methods of joint processing of homogeneous information as the method of mutual compensation and filtering of errors of measuring devices measuring the same navigation parameter, and methods of optimal estimation of the state vector have become widespread. The latter include the least squares method, the maximum likelihood method and the method of optimal estimation of the state vector using a priori information about the controlled process and current measurements using a Kalman filter.

The compensation scheme has become the most common for processing flight navigation information in inertial-Doppler navigation systems, since it has the same parameter, namely: the components of ground speed are measured by two meters – an inertial navigation system (INS) and a Doppler speed meter and angle demolition (DSMD), the operation of which is based on different physical principles and their error spectra are in different frequency ranges.

The block diagram that implements the compensation method in inertial-Doppler navigation systems is shown in Fig. 1.

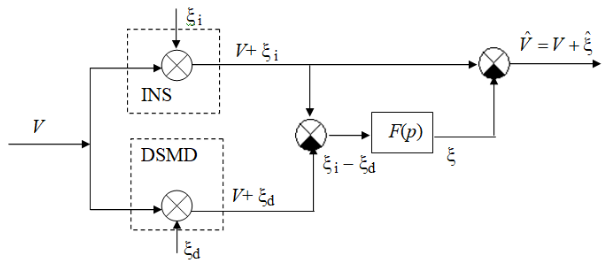

Fig. 1. The block diagram of the compensation method in inertial-Doppler navigation systems

The output signal of such a system (the evaluation signal) is described by the equation:

$$\hat{V} = V + \xi_i - F(p)(\xi_i - \xi_d),$$

or

$$\hat{V} = V + [1 - F(p)]\xi_i + F(p)\xi_d = V + \hat{\xi}, \quad (1)$$

where $\hat{\xi} = [1 - F(p)]\xi_i + F(p)\xi_d$ is the error of the complex system.

Another scheme for joint processing of homogeneous information – a filtering scheme (the more common name is the scheme of complementary filter) has the form shown in Fig. 2.

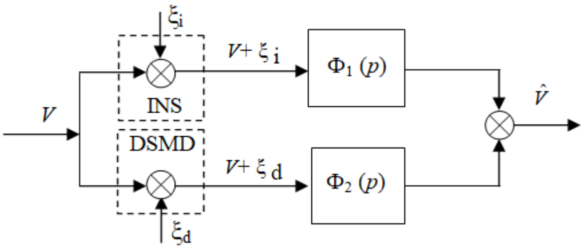

Fig. 2. The block diagram of the of complementary filter in inertial-Doppler navigation systems

The signal at the output of such a system is described by the equation:

$$\hat{V} = \Phi_1(p)(V+\xi_i) + \Phi_2(p)(V+\xi_d),$$

or

$$\hat{V} = \left[\Phi_1(p) + \Phi_2(p)\right]V + \Phi_1(p)\xi_i + \Phi_2(p)\xi_d.$$

In order for the system not to introduce dynamic errors, it is necessary to fulfill the condition:

$$\Phi_1(p) + \Phi_2(p) = 1.$$

In this case, the output signal will have the form:

$$\hat{V} = V + \left[1 - \Phi_2(p)\right]\xi_i + \Phi_2(p)\xi_d = V + \hat{\xi}, \qquad (2)$$

where $\hat{\xi} = \left[1 - \Phi_2(p)\right]\xi_i + \Phi_2(p)\xi_d$.

Thus, when $\Phi_2(p) = F(p)$, expression (2) turns into expression (1), which indicates the equivalence of the compensation scheme and the scheme of complementary filter.

The error $\hat{\xi}$ will be smaller, the greater the difference in the spectral characteristics of the interference (Fig. 3).

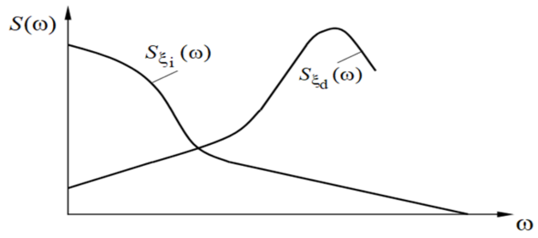

Fig. 3. Spectral characteristics of interference ξ_i and ξ_d

If you choose a filter so that it passes low-frequency interference ξ_i with minimum distortions and does not pass high-frequency interference ξ_d, then the interference ξ_i will be completely reproduced at the output of the filter $F(p)$ of the compensation scheme. In this case, a more accurate value of the measured parameter V would be reproduced at the output of the compensation scheme.

However, real filters passes part of the interference energy ξ_i and does not completely suppress the interference ξ_d. As a result, in the output signal, in addition to the desired value, there will be an error $\hat{\xi}$ in the output signal. The desired frequency characteristics of the filters are shown in Fig. 4.

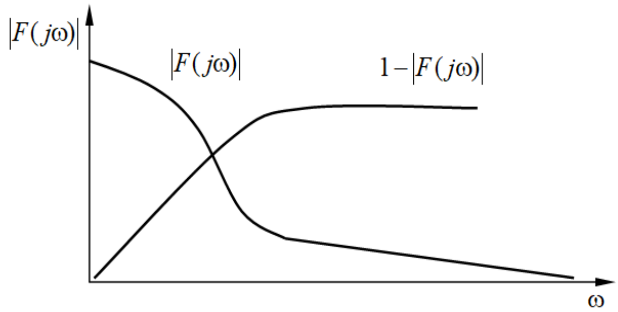

Fig. 4. Frequency characteristics of filters

The filter should be a low-pass filter, which in the simplest case could be implemented as a real aperiodic link with a transfer function

$$F(p) = \frac{1}{Tp+1},$$

where T is the time constant of the filter.

Then $[1 - F(j\omega)]$ will be a high-pass filter with a transfer function

$$[1 - F(p)] = \frac{TP}{TP+1}$$

that is, a real differential link.

Thus, the complementary filter, as well as the compensation circuit, in contrast to Kalman filtering, are easy to implement, and for design they require the selection and adjustment of only one parameter – the filter time constant.

II. Problem Statement

Taking into account the nature of the spectral characteristics, it is possible to plot graphs of the spectral characteristics of the errors ξ_i and ξ_d (Fig. 5), obtained as a result of passing the signals through the appropriate filters.

If the errors are stationary and statistically independent random functions of time with spectral densities $S_{\xi_i}(\omega)$ and $S_{\xi_d}(\omega)$, then the variances of the errors of the output signals can be determined by the expressions:

$$D_{\xi i} = \sigma^2_{\xi i} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\xi\, i}(\omega) d\omega,$$

$$D_{\xi_d} = \sigma^2_{\xi d} = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{\xi d}(\omega) d\omega.$$

Dispersion of the system error during the implementation of the compensation method

$$D_{\xi} = \sigma^2_{\xi} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\Big\{\left[1-F(j\omega)\right]^2 \cdot S_{\xi\, i}(j\omega) + \left[F(j\omega)\right]^2 \cdot S_{\xi d}\Big\} d\omega.$$

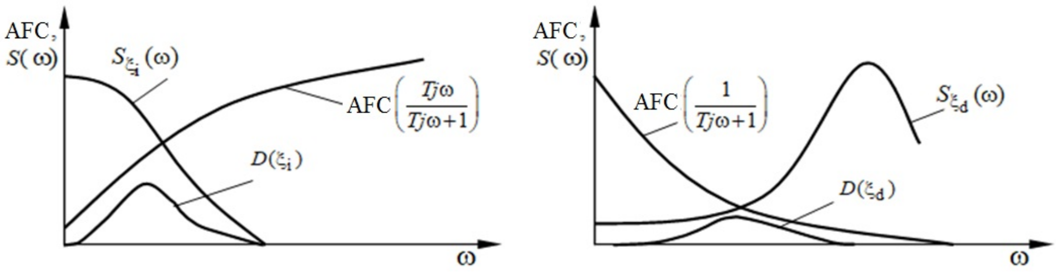

Fig. 5. Spectral characteristics of interference ξ_i and ξ_d

Based on Fig. 5, it can be concluded that the dispersion of the system error σ^2_{ξ} is smaller, the more the spectral densities of the errors of the output signals differ in frequency.

Therefore, the complexation of two meters consists in choosing such a frequency characteristic of the filter that after the summation of the signals passed through this filter, the parameter at the output of the circuit is close to the measured parameter.

And the problem statement can be formulated as the development of a methodology for selecting the optimal value of the filter time constant, ensuring maximum accuracy of the estimated parameter, namely: the ground speed of the aircraft. The error spectrum of the inertial system is low-frequency, and that of the Doppler meter is high-frequency.

III. PROBLEM SOLUTION

When designing a filter, it must be taken into account that the spectral error density is related to the correlation function as follows:

$$S(\omega) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j\omega\tau} d\tau.$$

As shown by the evidence of testing and research, the correlation functions of errors of the inertial and Doppler meters are described by the following expressions:

$$R_{\xi_i}(\tau) = \sigma_i^2 e^{-\alpha_i|\tau|}, \qquad R_{\xi d}(\tau) = \sigma_d^2 e^{-\alpha_d|\tau|}$$

The spectral density $S_i(\omega)$ of the error signal of the inertial meter can be calculated by the formula:

$$S_i(\omega) = \int_{-\infty}^{\infty} \sigma_i^2 e^{-\alpha_i|\tau|} e^{-j\omega\tau} d\tau = \sigma_i^2 \int_{-\infty}^{\infty} e^{-\alpha_i|\tau| - j\omega\tau} d\tau = \sigma_i^2 \left\{ \int_{-\infty}^{0} e^{(\alpha_i - j\omega)\tau} d\tau + \int_{0}^{\infty} e^{-(\alpha_i - j\omega)\tau} d\tau \right\}$$

$$= \sigma_i^2 \left[\frac{1}{\alpha_i - j\omega} + \frac{1}{\alpha_i + j\omega} \right] = \sigma_i^2 \frac{\alpha_i + j\omega + \alpha_i - j\omega}{\alpha_i^2 + \omega^2} = \sigma_i^2 \frac{2\alpha_i}{\alpha_i^2 + \omega^2}.$$

That is, we get:

$$S_i(\omega) = \frac{2\sigma_i^2 \alpha_i}{\alpha_i^2 + \omega^2}.$$

Similarly, the spectral density of the signal ξ_d of the Doppler meter is equal to:

$$S_d(\omega) = \frac{2\sigma_d^2 \alpha_d}{\alpha_d^2 + \omega^2}.$$

Since $\alpha_i << \alpha_d$, the spectrum of errors of the Doppler meter in the passband of the low-pass filter is almost constant and is equal to the value of the spectral density at $\omega = 0$:

$$S_d(\omega) = S_d(0) = \frac{2\sigma_d^2}{\alpha_d}$$

The dispersion of the signal $\hat{\xi}$ of the integrated system consists of two components – the dispersion of the inertial meter signal and the dispersion of the DSMD signal:

$$\sigma_{\hat{\xi}}^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left\{\left|\frac{Tj\omega}{Tj\omega+1}\right|^2 S_{\text{i}}(\omega)+\left|\frac{1}{Tj\omega+1}\right|^2 S_{\text{d}}(\omega)\right\}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{Tj\omega}{Tj\omega+1}\right|^2 S_{\text{i}}(\omega)d\omega + \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{Tj\omega+1}\right|^2 S_{\text{d}}(\omega)d\omega = \sigma_{\hat{\xi}_{\text{i}}}^2 + \sigma_{\hat{\xi}_{\text{d}}}^2 .$$

The dispersion of the signal of the Doppler meter

$$\sigma_{\hat{\xi}_{\text{d}}}^2 = \int_{-\infty}^{\infty}\left|\frac{1}{Tj\omega+1}\right|^2 \frac{\sigma_{\text{d}}^2}{\pi\alpha_{\text{d}}}d\omega = \frac{\sigma_{\text{d}}^2}{\pi\alpha_{\text{d}}}\int_{-\infty}^{\infty}\left|\frac{Tj\omega-1}{(Tj\omega+1)(Tj\omega-1)}\right|^2 d\omega = \frac{\sigma_{\text{d}}^2}{\pi\alpha_{\text{d}}}\int_{-\infty}^{\infty}\left|\frac{1-jT\omega}{1+(Tj\omega)^2}\right|^2 d\omega = \frac{\sigma_{\text{d}}^2}{\alpha_{\text{d}}T}.$$

The dispersion of the inertial system signal:

$$\sigma_{\hat{\xi}_{\text{i}}}^2 = \int_{-\infty}^{\infty}\left|\frac{jT\omega}{1+jT\omega}\right|^2 \frac{1}{\pi}\frac{\sigma_{\text{i}}^2\alpha_{\text{i}}}{\alpha_{\text{i}}^2+\omega^2}d\omega = \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}}{\pi}\int_{-\infty}^{\infty}\left|\frac{jT\omega(1-jT\omega)}{1+(T\omega)^2}\right|^2 \frac{1}{\alpha_{\text{i}}^2+\omega^2}d\omega$$

$$= \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}}{\pi}\int_{-\infty}^{\infty}\frac{(T\omega)^2\left[1+(T\omega)^2\right]}{\left[1+(T\omega)^2\right]^2}\frac{1}{\alpha_{\text{i}}^2+\omega^2}d\omega = \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T^2}{\pi}\int_{-\infty}^{\infty}\frac{\omega^2}{1+(T\omega)^2}\frac{1}{\alpha_{\text{i}}^2+\omega^2}d\omega.$$

After calculating this integral, we obtain the expression for the dispersion of the inertial system signal:

$$\sigma_{\hat{\xi}_{\text{i}}}^2 = \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T^2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\alpha_{\text{i}}^2}{T^2\alpha_{\text{i}}^2-1}\frac{1}{\omega^2+\alpha_{\text{i}}^2}+\frac{1}{1-T^2\alpha_{\text{i}}^2}\frac{1}{T^2\omega^2+1}\right)d\omega$$

$$= \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T^2}{\pi}\left(\frac{\alpha_{\text{i}}^2}{T^2\alpha_{\text{i}}^2-1}\frac{1}{\alpha_{\text{i}}}\operatorname{arctg}\frac{\omega}{\alpha_{\text{i}}}\Bigg|_{-\infty}^{\infty}+\frac{1}{1-T^2\alpha_{\text{i}}^2}\operatorname{arctg}(T\omega)\frac{1}{T}\Bigg|_{-\infty}^{\infty}\right)$$

$$= \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T^2}{\pi}\left(\frac{\pi\alpha_{\text{i}}}{T^2\alpha_{\text{i}}^2-1}-\frac{\pi}{T(T^2\alpha_{\text{i}}^2-1)}\right) = \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T^2(\alpha_{\text{i}}T-1)}{T\left(T^2\alpha_{\text{i}}^2-1\right)} = \frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T}{T\alpha_{\text{i}}+1}.$$

The dispersion of the signal $\hat{\xi}$ of the integrated system

$$\sigma_{\hat{\xi}}^2 = \frac{\sigma_{\text{d}}^2}{\alpha_{\text{d}}T}+\frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T}{T\alpha_{\text{i}}+1}.$$

Having marked

$$\alpha_{\text{i}}T = z, \qquad \frac{\sigma_{\text{i}}^2\alpha_{\text{d}}}{\sigma_{\text{d}}^2\alpha_{\text{i}}} = m^2,$$

where $z = f(T)$, $m^2 = \text{const}$,
we will get

$$\sigma_{\hat{\xi}}^2 = \frac{\sigma_{\text{d}}^2}{\alpha_{\text{d}}T}+\frac{\sigma_{\text{i}}^2\alpha_{\text{i}}T}{T\alpha_{\text{i}}+1} = \sigma_{\text{i}}^2\left(\frac{\sigma_{\text{d}}^2\alpha_{\text{i}}}{\sigma_{\text{i}}^2\alpha_{\text{d}}}\frac{1}{\alpha_{\text{i}}T}+\frac{\alpha_{\text{i}}T}{\alpha_{\text{i}}T+1}\right)$$

$$= \sigma_I^2\left(\frac{1}{m^2z}+\frac{z}{1+z}\right) = \psi(z).$$

The function $\psi(z)$ has a minimum at $\frac{d\psi(z)}{dz}=0$ and $\frac{d^2\psi(z)}{dz^2}>0$:

$$\frac{d\psi(z)}{dz} = \sigma_{\text{i}}^2\left[\frac{(-1)}{m^2z^2}+\frac{1+z-z}{(1+z)^2}\right] = \sigma_{\text{i}}^2\frac{m^2z^2-(z+1)^2}{m^2z^2(z+1)^2}$$

$$= \sigma_{\text{i}}^2\frac{m^2z^2-z^2-2z-1}{m^2z^2(z+1)^2} = \sigma_{\text{i}}^2\frac{(m^2-1)z^2-2z-1}{m^2z^2(z+1)^2}.$$

Analysis of this formula shows that the condition $\frac{d\psi(z)}{dz}=0$ is fulfilled only when

$$(m^2-1)z^2-2z-1=0.$$

The discriminant D of this quadratic equation is equal to

$$D = 4+4(m^2-1) = 4m^2,$$

therefore, as a result of solving the equation $(m^2-1)z^2-2z-1=0$, we get:

$$z_1 = \frac{2+\sqrt{4m^2}}{2(m^2-1)} = \frac{2+2m}{2(m-1)(m+1)} = \frac{1}{m-1},$$

$$z_2 = \frac{2-\sqrt{4m^2}}{2(m^2-1)} = \frac{2-2m}{2(m-1)(m+1)} = -\frac{1}{m-1}.$$

Since $z = \alpha_i T$ is always greater than zero, that the root z_2 is not taken into account in the future, and the condition $\frac{d^2\psi(z)}{dz^2} > 0$ at $z = z_1$ takes the following form:

$$\frac{d^2\psi(z)}{dz^2} = \sigma_i^2\left[\frac{-2}{(1+z)^3} + \frac{2}{m^2 z^3}\right] = \frac{2\sigma_i^2(m-1)^3}{m^2}\left(1-\frac{1}{m}\right)$$

$$= \frac{2\sigma_i^2(m-1)^3}{m^2}\frac{m-1}{m} = \frac{2\sigma_i^2(m-1)^4}{m^3} > 0.$$

Therefore, the found value of z_1 is optimal

$$z_{\text{opt}} = \alpha_i T_{\text{opt}} = \frac{1}{m-1},$$

and the final result looks like

$$T_{\text{opt}} = \frac{1}{\alpha_i(m-1)}.$$

To study the operation of the compensation scheme, the following values of the parameters characterizing the errors of INS and DSMD were adopted: α_i = 1 s^{-1}, α_d = 0.04 s^{-1}, σ_i = 3.6 m/s, σ_d = 0.6 m/s. The value of the time constant of the filter under these conditions $T_{\text{opt}} = 5$ s.

Study of the compensation scheme shown in Fig. 1, was carried out using the Simulink visual modeling program, which is part of the universal mathematical programming package MATLAB. Studies were conducted for the time constant of the filter $T = T_{\text{opt}} = 5$ s and $T < T_{\text{opt}}$. The results of modeling, namely changes in the accuracy of the flight speed of the aircraft over time, are shown in Figs 6 and 7.

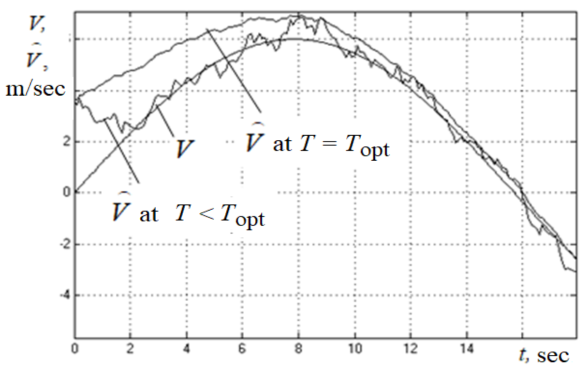

Fig. 6. Transient processes of changes in flight speed V and flight speed estimates $\hat{V}$ for different filter time constants T

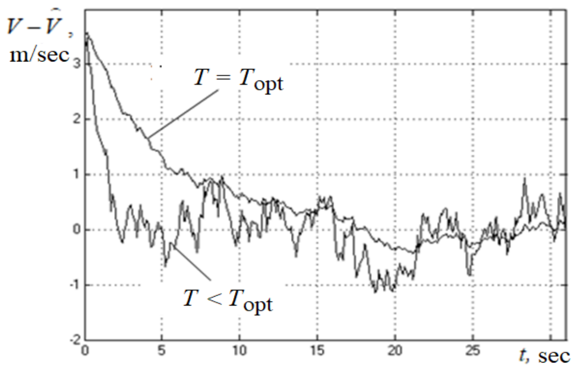

Fig. 7. Transient processes of changes of estimation errors in the flight speed for different time constants of the filter T

IV. CONCLUSIONS

The equivalence of the compensation scheme and the filtering scheme, the more common name of which is the complementary filter scheme, is proved, and it is also substantiated that these schemes, unlike Kalman filtering, are easy to implement, and for design needs to selected and adjusted only one parameter – the filter time constant.

A technique for choosing the optimal value of the filter time constant is proposed. The methodology is based on the analysis of error spectra of integrated systems.

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Received February 16, 2024

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М. К. Філяшкін, Ю. М. Кеменяш. Комплементарний фільтр в інерціально-доплерівській системі навігації

Статтю присвячено алгоритмам фільтрації, які застосовуються для обробки даних про швидкість польоту від інерціальної навігаційної системи та від доплерівського вимірника швидкості та кута знесення. Показано, що найпоширенішими для оброблення пілотажно-навігаційної інформації в інерціально-доплерівських системах навігації стали схема компенсації та схема фільтрації, більш поширена назва якої є схема комплементарного фільтра. У роботі доводиться еквівалентність схеми компенсації та схеми комплементарного фільтра, а також обґрунтовується, що ці схеми на відміну від фільтрації Калмана, прості в реалізації, а при проектуванні вимагають вибору і налаштування тільки одного параметра – сталої часу фільтра. Запропонована методика вибору оптимального значення сталої часу фільтра, що забезпечує максимальну точність оцінюваного параметра, а саме: шляхової швидкості літака. Методика базується на аналізі спектрів похибок систем, що комплексуються: інерціальної системи – з низькочастотним спектром похибок, та доплеровської з високочастотним спектром. Проведено дослідження синтезованих схем фільтрації.

Ключові слова: інерціальна навігаційна система; доплерівський вимірювач швидкості; фільтр Калмана; взаємна компенсація та фільтрація; комплементарний фільтр; частотні характеристики; спектральні характеристики; стала часу фільтру.

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