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RESEARCHING INDICES OF RELIABILITY OF NAVIGATION PARAMETERS METERS CHECKING

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Abstract—*The article deals with the studying process of measuring navigation parameters by means of gyroscopic measurements instruments. The topic of research is connected with studying reliability of checking of gyroscopic measuring instruments. The instrumental and methodical components of the check's reliability are determined. Probabilities of arising both false and indefinite failures are obtained. The relationship, which connects accuracy performances and mass-dimension characteristics, is represented. Graphical dependences, which characterize interconnection between the instrumental component of the check's reliability with accuracy characteristics of navigation measuring instruments, are given. Graphical dependences, which illustrate change of the methodical component for different types of checks, are shown. The analysis of instrumental and methodical components of the reliability of checking is carried out. The obtained results can be useful for grounded assignment of tolerances on measured navigation parameters.*

Index Terms—Reliability of checking; instrumental component; methodical component; probability of false failure; probability of indefinite failure; accuracy and mass-dimension characteristics.

I. INTRODUCTION

The reliability is a basic criterion that allows to estimate the quality of checking instruments for the measurement of navigation parameters. There are different approaches to estimation of indices of the reliability of checks [1] – [3]. It is necessary to notice that the problem of determining correlations between indices of the measuring accuracy and optimal values of checked parameters is well studied by now [4] – [6]. At the same time, the necessity to study correlations between indices of the reliability of checking measuring instruments of navigation parameters and designing and operating characteristics stays relevant by now [7].

The necessity to research correlation between indices of reliability of checking and accuracy characteristics is caused by requirements to decrease of mass-dimension characteristics of perspective instruments for the measurement of navigation parameters. It worth noticing that the above-mentioned characteristics are connected with the measuring accuracy.

II. PROBABILISTIC PRESENTATION OF INSTRUMENTAL COMPONENTS OF RELIABILITY OF CHECKING

The instrumental component of the reliability of checking can be characterized by the conditional probability of making decision "does not correspond" in conditions of checking parameter, value of which in the reality satisfies the given requirements. In other words, this component represents the probability of a false failure P_{ff}. The instrumental component can also be characterized by the conditional probability of making decision "corresponds" in conditions of checking parameter, a value of which in the reality does not correspond to the given requirements. In other words, this characteristic represents the probability of an indefinite failure P_{if}. The above-mentioned probabilities are defined by expressions [5]

$$P_{ff} = \int_{m_x-a}^{m_x+a} f_1(x)\left[\int_{-\infty}^{m_x-b} f_2(x,y)dy + \int_{m_x+b}^{\infty} f_2(x,y)dy\right]dx, \tag{1}$$

$$P_{if} = \int_{-\infty}^{m_x-a} f_1(x)\left[\int_{m_x-b}^{m_x+b} f_2(x,y)dy + \int_{m_x+a}^{\infty} f_1(x)\left[\int_{m_x-b}^{m_x+b} f_2(x,y)dy\right]\right]dx. \tag{2}$$

In formulas (1), (2) functions $f_1(x)$, $f_2(x)$ are determined in the following way

$$f_1(x) = \frac{1}{\sigma_x\sqrt{2\pi}} e^{\frac{-(x-m_x)^2}{2\sigma_x^2}}, \quad f_2(x,y) = \frac{1}{\sigma_y\sqrt{2\pi}} e^{\frac{-(y-x)^2}{2\sigma_y^2}},$$

where m_x is the mathematical expectation of the checked parameter; a is the bounded deviation of the checked parameter in operating conditions; $f_1(x)$ is the density of the probabilistic distribution of the checked parameter x; b is the bounded deviation of the checked parameter in conditions of checks; $f_2(x,y)$ is the density of the probabilistic distribution of the checked parameter y; σ_x is the mean-root deviation of the checked parameter; σ_y is the mean-root deviation of measuring error.

In the case of symmetrical bounded deviations, expressions (1), (2) can be represented in the form

$$P_{ff} = 4\int_{0}^{m_x+a} f_1(x)\left[\int_{m_x+b}^{\infty} f_2(x,y)dy\right]dx\,; \quad (3)$$

$$P_{if} = 4\int_{m_x+a}^{\infty} f_1(x)\left[\int_{0}^{m_x+b} f_2(x,y)dy\right]dx\,. \quad (4)$$

In practical situations, checked parameters are characterized by the finite distribution. Therefore, it is necessary to determine bounds of the integration in expressions (3), (4).

It worth be noticing that the following condition always takes place [5]

$$P_{ff} = \frac{2}{\sigma_x\sqrt{2\pi}}\int_{m_x+a}^{\infty} e^{\frac{-(x-m_x)^2}{2\sigma_x^2}}\left[\frac{2}{\sigma_y\sqrt{2\pi}}\int_{0}^{m_x+b} e^{\frac{-(y-x)^2}{2\sigma_y^2}}dy\right]dx < \frac{2}{\sigma_x\sqrt{2\pi}}\int_{m_x+a}^{\infty} e^{\frac{-(x-m_x)^2}{2\sigma_x^2}}dx, \quad (5)$$

To determine finite values of integration bounds, it is sufficient to consider the right part of inequality (5), which after change of variables

$$\frac{x-m_x}{\sigma_x} = u,\; dx = \sigma_x du$$

becomes

$$J = \frac{2}{\sqrt{2\pi}}\int_{\frac{a}{\sigma_x}}^{\infty} e^{\frac{-u^2}{2}}du\,. \quad (6)$$

Based on the theorem about segmentation of an integral, relation (6) can be represented in the following form

$$\frac{2}{\sqrt{2\pi}}\int_{\frac{a}{\sigma_x}}^{\infty} e^{\frac{-u^2}{2}}du = \frac{2}{\sqrt{2\pi}}\int_{\frac{a}{\sigma_x}}^{\frac{ka}{\sigma_x}} e^{\frac{-u^2}{2}}du + \frac{2}{\sqrt{2\pi}}\int_{\frac{ka}{\sigma_x}}^{\infty} e^{\frac{-u^2}{2}}du\,, \quad (7)$$

where k is an integer positive value.

As follows from (7), to determine the integration bounds, which will ensure calculation of the integral with the given accuracy, it is necessary to satisfy the following condition

$$\frac{2}{\sqrt{2\pi}}\int_{\frac{ka}{\sigma_x}}^{\infty} e^{\frac{-u^2}{2}}du = \varepsilon\,. \quad (8)$$

Values of integration bounds (8) on the basis of the given accuracy ε can be determined by means of appropriate table values.

For n parameters, the probability of arising at least one failure (false or indefinite) can be determined by relationships

$$P_{ff} = 1-\prod_{i=1}^{n}(1-P_{ff_i})\,. \quad (9)$$

In equation (9), P_{ff}, P_{if} are probabilities of the false and indefinite failures for ith parameter.

III. Correlation of Reliability of Checking and Accuracy Characteristics of Gyroscopic Devices

The research of correlations between indices of checking reliability and mass-dimensions and accuracy characteristics of measuring instruments of navigation parameters has a great significance for solving problems of designing control systems generally. This is explained in the following way. Errors of control system loops depend largely on accuracy characteristics of measuring instruments of navigation parameters. Moreover, the a change of accuracy characteristics of a control system influences sufficiently on the system's mass-dimension characteristics.

Interconnection of accuracy performances and mass-dimension characteristics of gyroscopic devices, which are one of the most widespread measuring instruments of navigation parameters, can be described by the relationship [8], [9] and statistical information about design feature of gyroscopic devices of the different type

$$\sigma_x = \frac{1}{3}\sqrt{\frac{k_t}{\left(\frac{V}{\pi k k_{kn} N^3}\right)^{4/3}\Omega^2 + k_f}}\,, \quad (10)$$

where V is the volume of the gyroscopic device; k is the mathematical expectation between the height d and radius of the rotor of the gyroscopic motor R, $k = m[d/R]$; k_{kn} is the mathematical expectation of the criterion of the design similarity, $k_{kn} = m[V/V_p]$; V_p is the volume of the rotor; N, k_f, k_t are coefficients, which take into consideration design features of gyroscopic devices of different types; Ω is the angular rate of the proper rotation of the rotor. In formula (10), coefficients k_f, k_t are determined in the following way [10]

$$k_f = \begin{cases} 1 \text{ in the case of sensor of pitch and roll,} \\ 0 \text{ in the case of angular rate,} \end{cases}$$

$$k_t = \begin{cases} S \text{ in the case of sensor of pitch and roll,} \\ 0 \text{ in the case of angular rate,} \end{cases}$$

where S is a coefficient taking into consideration construction features of sensors of pitch and roll.

Interconnection of the instrumental component of the reliability of checks with accuracy and mass-dimensions characteristics of gyroscopic measuring instruments of navigation parameters is illustrated by dependences given in Fig. 1.

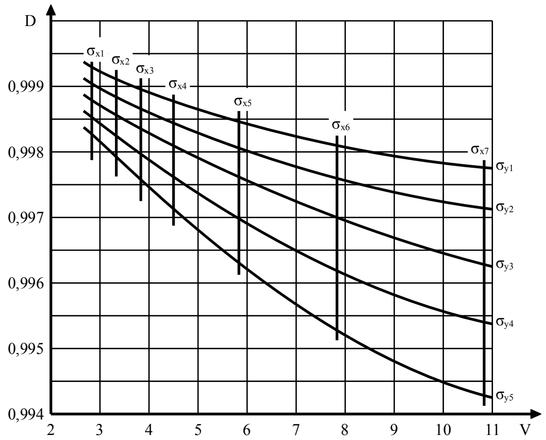

Fig. 1. Interconnection of instrumental reliability of checks with characteristics of navigation measuring instruments

IV. PROBABILISTIC REPRESENTATION OF METHODICAL COMPONENTS OF RELIABILITY OF CHECKS

False and indefinite failures can be caused by methodical errors of checks. The probability of arising indefinite failures can be analysed taking into consideration the instrumental and methodical components. The methodical component can be described by the expression

$$P_{if}^{'} = 1 - (1 - P_{if})(1 - P_{ifi_1})(1 - P_{ifi_2}), \qquad (11)$$

In equation (11), $P_{if}^{'}$ is the probability of arising an indefinite failure taking into consideration the methodical component of the reliability of checks; P_{ifi1} is a component caused by insufficient information about states of separate devices; P_{ifi2} is a component caused by the difference of checking and operating conditions.

The methodical component caused by insufficient information about states of separate devices can be determined by the relationship [5]

$$P_{ifm_1} = 1 - e^{-\sum_{i=1}^{l} \lambda_i t_{st}}, \qquad (12)$$

where l is the quantity of elements, which are not checked; λ_i is the intensity of the failure of ith element in conditions of storage; t_{st} is the time of storage of the gyroscopic device.

A value of the methodical component (12) caused by insufficient information about the state of separate device is determined by structural features of measuring instruments of navigation parameters. This component can be decreased due to increasing the completeness of checking. This means increasing the quantity of checked elements.

We will analyse the methodic component of the reliability of checking, which is caused by difference in checking and operating conditions. This methodical component for x_i parameter is defined by the expression

$$P_{ifm_{2i}} = 2\Phi\left(\frac{b_i}{\sigma_{x0_i}}\right)\left[1 - 2\Phi\left(\frac{a_i}{\sigma_{x_i}}\right)\right], \qquad (13)$$

where 2Φ is the double Laplace integral, which determines the probability of determining errors of a parameter with root mean square deviations $\sigma_{x0_i}(\sigma_{x_i})$ in the interval $b_i(a_i)$; b_i is the bounded deviation of the parameter x_i in checking conditions; σ_{x0_i} is the root mean square deviation in checking conditions; a_i is the bounded deviation of the parameter x_i in operating conditions; σ_{x0_i} is the root mean square deviation in operating conditions.

The methodical component of the checking reliability (13) takes into consideration the possibility to determine the parameter x_i in the range of limited values b_i during checks and in the process of the parameter x_i going beyond the limits a_i in operating conditions.

For r kinds of checks of the technical state, expression (13) becomes

$$P_{ifm_{2i}} = 2_r \prod_{j=1}^{r} \Phi\left(\frac{b_{ij}}{\sigma_{x0_{ij}}}\right)\left[1 - 2\Phi\left(\frac{a_i}{\sigma_{x_i}}\right)\right], \qquad (14)$$

where b_{ij} is the limited value of ith parameter for jth kind of a check.

The nature of a change of the methodical component of the check's reliability (14), which is caused by the difference in checking and operating conditions, is illustrated in Figs 2, 3. Figures 2, 3 represent graphical dependences for gyroscopic devices of different types.

For n parameters, the methodical component of the check's reliability P_{ifm2} is determined by the relationship

$$P_{ifm_2} = 1 - \prod_{i=1}^{n} (1 - P_{ifm_{2i}}) . \qquad (15)$$

Appropriate relationships for the analysis of the methodical component of the check's reliability, caused by arising false failures, look like

$$P_{ffm} = 2\Phi\left(\frac{a_i}{\sigma_{x_i}}\right)\left[1 - 2\Phi\left(\frac{b_i}{\sigma_{x_{0i}}}\right)\right]. \qquad (16)$$

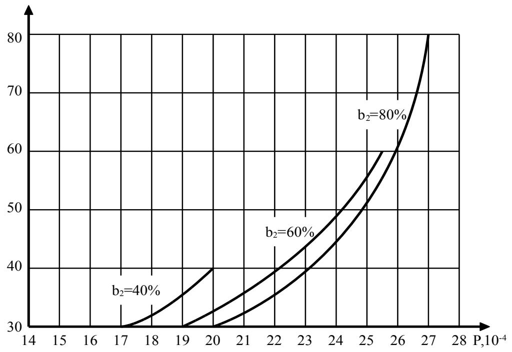

Fig. 2. The methodical component of checking reliability for the sensor of heading

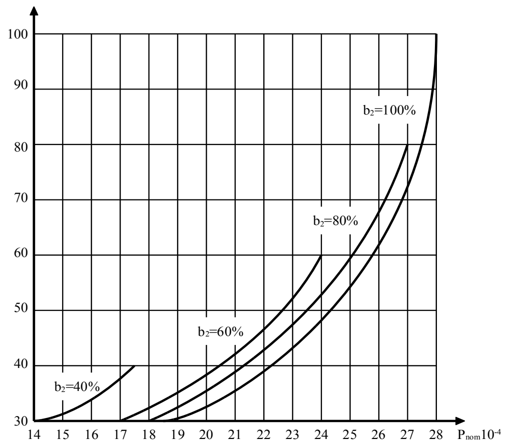

Fig. 3. The methodical component of checking reliability for the gyroscopic vertical

As follows from (16), harrowing of a tolerance leads to increase of the probability of false failures.

Omitting intermediate conversions, relationships for estimating the methodical component of the check's reliability for n parameters can be represented in the following form

$$P_{ffmn} = 2_r \prod_{j=1}^{r} \Phi\left(\frac{a_{ij}}{\sigma_{x_{ij}}}\right)\left[1 - 2\Phi\left(\frac{b_i}{\sigma_{x_{0i}}}\right)\right]. \qquad (17)$$

Based on relationships (14) and (15), the methodical component of the check's reliability looks like

$$D_m = 1 - P_{ffm} - P_{ifm}, \qquad (18)$$

$$D_m = 1 - P_{ffi} - P_{ifi}. \qquad (19)$$

The full expression for determination of reliability of checks including both instrumental and methodical component (18), (19) can be represented in the following form

$$D = D_i D_m . \qquad (20)$$

In equation (20), components D_i and D_m are believed to be independent.

V. CONCLUSIONS

To improve the quality of the earlier designing phases, the analysis of correlations between indices of the check's reliability and operating characteristics of measuring instruments of navigation parameters has been carried out.

To take into considerations a difference in checking and operating conditions, the way to estimate the methodical component of the check's reliability of instruments for measuring navigation parameters is proposed.

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О. А. Сущенко, О. О. Салюк, С. Г. Єгоров. Дослідження показників вірогідності контролю вимірювачів навігаційних параметрів

У статті розглянуто процес вимірювання навігаційних параметрів за допомогою гіроскопічних вимірювальних приладів. Тема дослідження пов'язана з дослідженням вірогідності контролю гіроскопічних вимірювальних приладів. Визначено інструментальну та методичну складові вірогідності контролю. Отримано ймовірності виникнення як хибних, так і невизначених відмов. Представлено зв'язок, що зв'язує показники точності та масогабаритні характеристики. Представлено графічні залежності, що характеризують взаємозв’язок інструментальної складової вірогідності контролю з характеристиками точності навігаційних вимірювальних приладів. Наведено графічні залежності, які ілюструють зміну методичної складової для різних видів контролю. Проведено аналіз інструментальної та методичної складових вірогідності контролю. Отримані результати можуть бути корисними для обґрунтованого призначення допусків на вимірювані навігаційні параметри.

Ключові слова: вірогідність контролю; інструментальна складова; методична складова; ймовірність хибної відмови; ймовірність невизначеної відмови; точність і масогабаритні характеристики.

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