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MATHEMATICAL DESCRIPTION OF SYSTEMS FOR SPACE STABILIZATION OF EQUIPMENT ASSIGNED FOR OPERATION ON MOVING VEHICLES

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Abstract**—***The article deals with the development of mathematical description of systems for stabilization of measurement and observation equipment assigned for operation at moving vehicles of the wide class such as land, marine, and air moving vehicles. Mathematical descriptions of one-axis, two-axis, and three-axis stabilization systems are represented including kinematical relations and dynamics models. The general mathematical descriptions and the appropriate models in the space of states are given. The basic approaches to linearization of the generalized models are represented. The sets of turns in the inertial space for two-axial and three-axial stabilizations systems are represented. The obtained mathematical model for one-axis stabilization system has been used for the robust structural synthesis of the system assigned for stabilization of the observation equipment mounted at the land moving vehicles. The obtained results can be spread on moving vehicles of the different type.*

Index Terms—Stabilization systems; multi-axes space stabilization; mathematical description; state-space model; differential equations; matrices.

I. INTRODUCTION AND PROBLEM STATEMENT

Recently, the urgency of creating new promising systems for the spatial stabilization of equipment for moving objects is growing. The creation of such systems requires the use of methods of analysis and synthesis [1] – [3]. This approach ensures the successful development of new systems of the studied class, one of the features of which is operation in conditions of disturbances. In view of this, it is expedient to synthesize these systems on the basis of the theory of robustness. One of the modern approaches to solving the given problem is robust structural synthesis, which requires the creation of a mathematical description of spatial stabilization systems [4].

One of the important features that is decisive for the creation of a mathematical description of systems of the studied type is the number of axes along which spatial stabilization is performed. Single-axis stabilization is appropriate for equipment with significant mass-dimensional characteristics, which are usually installed on land-based mobile objects. The vast majority of equipment for moving objects, for example, laser sights, antennas, requires two-axis stabilization in space. Three-axis stabilization is necessary in situations that require high accuracy. For example, video cameras of unmanned aerial vehicles require three-axis stabilization.

The number of axes of spatial stabilization determines the peculiarities of the kinematics of the stabilization system, which determines the appearance of the mathematical model.

The main goal of the article is to create a mathematical description of stabilization systems depending on the number of axes stabilized in space.

II. GENERAL FEATURES OF MATHEMATICAL DESCRIPTION OF STABILIZATION SYSTEM

The studied stabilization and tracking system provides control by the space orientation of the measurement and observation equipment mounted in the two-axis gimbals. The gimbals consist of the platform representing the internal gimbal and the external gimbal suspended in the ball-bearings. The information-measuring equipment and gyro sensors are mounted on the platform. These sensors measure the absolute angular rate of the information-measuring device lines if sight.

In the general case, the system dynamic may be described by the Euler equations [1]

$$\begin{aligned}
&\dot{\omega}_x J_x + \omega_y \omega_z (J_z - J_y) - (\omega_y^2 - \omega_z^2) J_{yz} \\
&\qquad - (\omega_x \omega_y + \dot{\omega}_z) J_{xz} + (\omega_x \omega_z - \dot{\omega}_y) J_{xy} = M_x, \\
&\dot{\omega}_y J_y + \omega_x \omega_z (J_x - J_z) - (\omega_z^2 - \omega_x^2) J_{xz} \\
&\qquad - (\omega_z \omega_y + \dot{\omega}_x) J_{xy} + (\omega_x \omega_y - \dot{\omega}_z) J_{yz} = M_y, \\
&\dot{\omega}_z J_z + \omega_x \omega_y (J_y - J_x) - (\omega_x^2 - \omega_y^2) J_{xy} \\
&\qquad - (\omega_x \omega_z + \dot{\omega}_y) J_{yz} + (\omega_x \omega_z - \dot{\omega}_x) J_{xz} = M_z,
\end{aligned} \tag{1}$$

where $\omega_x, \omega_y, \omega_z$ are projections of the platform angular rates onto its own axes; J_x, J_y, J_z are the inertia moments of the platform with useful payload mounted on it relative to the gimbals axes; J_{yz}, J_{xz}, J_{xy} are the centrifugal inertia moments relative to the gimbals axes; $\dot{\omega}_x, \dot{\omega}_y, \dot{\omega}_z$ are projections of the platform angular accelerations onto its own axes; M_x, M_y, M_z are the moments acting by the gimbals axes.

The moments acting on the platform include the following components: moments of dry friction in the ball-bearing supports of the gimbal; moments developed by stabilization engines, and disturbance moments. When creating a mathematical description of the system in the precise stabilization mode, it is necessary to take into account that each of its control loops includes a stabilization motor, a pulse-width modulator (PWM) and a gyroscopic angular velocity meter [5], [6]. The components of the moments acting along the axes of the gimbals are determined as follows [6]

$$\begin{aligned} &M_{1i} = M_{fr}\operatorname{sign}\omega_i, \quad M_{2i} = c_m U_{ai} / R_a, \\ &M_{3i} = M_{dist\,i}, \qquad i = x_2, y_0, z_1, \end{aligned} \tag{2}$$

where M_{fr} is the nominal friction moment in bearings mounted in axes of gimbals; c_m is the coefficient of loading on the shaft of the motor; U_{ai} are voltages of control windings of the motor armature; R_a is the resistance of circuit in the motor armature; $M_{dist\,i}$ are disturbance moments.

The formation of voltage in the control winding of the motor armature is described by the expression [7]

$$T_a \dot{U}_{ai} + U_{ai} = k_{PWD} U_{PWDi} - n_r c_e \omega_i,\ i = x_2, y_0, z_1, \tag{3}$$

where T_a is the transfer constant of the motor armature; c_e is the coefficient of proportionality between the angular rate of stabilizing motor and electromotive force; k_{PWD} is the transfer coefficient of the linearized PWM; U_{PWDi} is the voltage at the input of PWM. Controlling voltages at the outputs of sensors of the angular rate can be described in the following way $U_{\omega i}$ [7]

$$T_g^2 \ddot{U}_{\omega i} + 2\xi T_g \dot{U}_{\omega i} + U_{\omega i} = k_g \omega_i, \quad i = x, y, z, \tag{4}$$

where T_g is the transfer constant of the gyroscopic sensor of the angular rate; ξ is damping coefficient; k_g is the transfer coefficient of the gyroscopic sensor of the angular rate.

III. Mathematical Description of One-axis Stabilization System

The united model of the motor and stabilization plant can be represented in the following form

$$\begin{aligned} J_{eq}\ddot{\varphi}_{eq} &= -M_{fr}\operatorname{sign}\dot{\varphi}_{eq} - M_{imb}\cos\varphi_{eq} \\ &\quad + \frac{c_r}{n_r}\varphi_{eq} - c_r\varphi_{eq}, \\ J_m\ddot{\varphi}_m &= -M_{frm}\operatorname{sign}\dot{\varphi}_m + \frac{c_m}{R_w}U + \frac{c_r}{n_r^2}\varphi_m - c_r\varphi_m, \\ \dot{U}T_a + U &= U_{\text{PWM}} - c_e\dot{\varphi}_m, \end{aligned} \tag{5}$$

where J_{eq} is the moment of inertia of the measurement or observation equipment; φ_{eq} is an angle of the turn of the equipment; M_{fr} is the nominal friction moment in bearings of the equipment; M_{imb} is the imbalance moment; c_r is the reducer rigidity; n_r is the transfer ratio of the reducer; J_m is the moment of inertia of the motor; φ_m is an angle of the turn of the motor; M_{frm} is the nominal moment of resistance; c_m is the transfer constant of loading at the rotor shaft; R_w is the resistance of the motor armature; U is the voltage of the motor armature; U_{PWM} is the voltage of pulse-width-modulator; c_e is the electromotive constant.

Nonlinear moments of the dry friction can be approximated by linear relationships.

Approximation coefficients are determined as ratio of the first harmonic amplitude to the angular rate amplitude [8]. In this case, expressions for determination of friction moments

$$M_{eq} = M_{fr}\operatorname{sign}\dot{\varphi}_{eq},\ M_m = M_{frm}\operatorname{sign}\dot{\varphi}_m$$

become $M_{eq} = f_{freq}\dot{\varphi}_{eq}$, $M_m = f_{frm}\dot{\varphi}_m$.

Coefficients f_{freq}, f_{frm} will be determined by expressions

$$f_{freq} = 4M_{fr}/(\pi\Omega_{eq}),\ f_{frm} = 4M_{frm}/(\pi\Omega_m).$$

It should be noted that the expression for unbalanced moment determination $M_{imb}\cos\dot{\varphi}_{eq}$ can be linearized for platform angles changing in the range 5–10 degrees.

The developed mathematical description (5) can be reduced to the linearized form

$$
\begin{aligned}
&J_{eq}\ddot{\varphi}_{eq} = -f_{eq}\,\mathrm{sign}\dot{\varphi}_{eq} - M_{imb}\cos\varphi_{eq} \\
&\qquad + \frac{c_r}{n_r}\varphi_{eq} - c_r\varphi_{eq}, \\
&J_m\ddot{\varphi}_m = -f_m\,\mathrm{sign}\dot{\varphi}_m + \frac{c_m}{R_w}U + \frac{c_r}{n_r^2}\varphi_m - c_r\varphi_m, \\
&\dot{U}T_a + U = U_{\mathrm{PWM}} - c_e\dot{\varphi}_m,
\end{aligned} \tag{6}
$$

where f_{eq}, f_m are coefficients of friction moments of the measurement or observation equipment and motor respectively. In model (6), the most significant thing is the replacement of nonlinear moments of friction with linear dependencies.

The presented model can be transformed to a model in the state space in the generalized form

$$
\begin{aligned}
\dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu}, \\
\mathbf{y} &= \mathbf{Cx} + \mathbf{Du},
\end{aligned} \tag{7}
$$

where $\mathbf{x}$ is the state space vector; u is the vector of controls; **A, B, C, D** are matrices that characterize features of the system and control; $\mathbf{y}$ is the vector of observations.

For model (3), vectors of the state and controls and appropriate matrices look like

$$
\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \dot{\varphi}_{eq} \\ \dot{\varphi}_m \\ \varphi_{eq} \\ \varphi_m \\ U \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 0 & 0 \\ \frac{-M_{imb}}{J_{eq}} & 0 \\ 0 & 0 \\ 0 & \frac{U_{\mathrm{PWM}}}{T_a} \\ 0 & 0 \end{bmatrix},
$$

$$
\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{-c_r}{J_{eq}} & \frac{-f}{J_{eq}} & \frac{c_r}{n_r J_{eq}} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \frac{-c_r}{n_r J_m} & 0 & \frac{-c_r}{n_r^2 J_m} & \frac{-f_m}{J_m} & \frac{c_m}{R_w J_m} \\ 0 & 0 & 0 & \frac{-c_e}{T_a} & \frac{-1}{T_a} \end{bmatrix},
$$

$$
\mathbf{C} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},
$$

$$
\mathbf{B}^{\mathrm{T}} = \begin{bmatrix} 0 & \frac{-M_{imb}}{J_{eq}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{U_{pwm}}{T_a} \end{bmatrix}. \tag{8}
$$

IV. Mathematical Description of Two-axis Stabilization System

It should be noted that use of the two-axis gimbals is the most acceptable for many applications, as two mutually-orthogonal axes represent the minimum quantity necessary for determination of the direction in the three-dimensional space [8].

The mutual position of the coordinate axes necessary for representation of kinematic of the stabilization and tracking system is represented in Fig. 1.

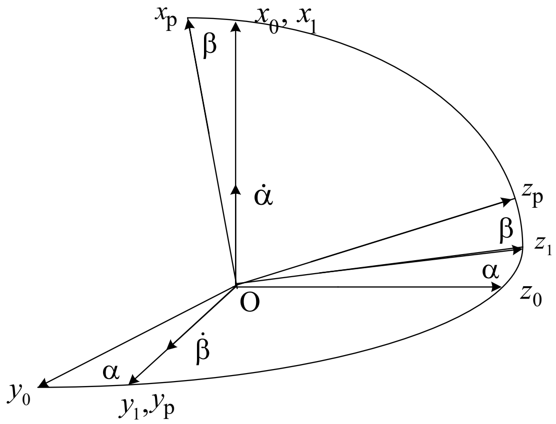

Fig. 1. Mutual position of the coordinate system connected with the vehicle and the platform

The represented coordinate systems may be characterized in the following way. The coordinate system $Ox_0y_0z_0$ is connected with the moving vehicle. The coordinate system $Ox_1y_1z_1$ is connected with the gimbal. The coordinate system $Ox_\mathrm{P}y_\mathrm{P}z_\mathrm{P}$ is connected with the platform.

In accordance with Fig. 1, the angular rate projections of the platform with mounted on it useful payload onto its own axes may be represented in the following form

$$
\begin{aligned}
\omega_x &= \dot{\alpha}\cos\beta, \\
\omega_y &= \dot{\beta}, \\
\omega_z &= \dot{\alpha}\sin\beta.
\end{aligned} \tag{9}
$$

Based on the relationships (9), the Euler kinematic equations, which correspond to the sequence of angles represented in Fig. 1, become

$$
\begin{aligned}
\dot{\alpha} &= \omega_x\cos\beta + \omega_z\sin\beta, \\
\dot{\beta} &= \omega_y.
\end{aligned} \tag{10}
$$

The mathematical model of the control object of the system for stabilization and tracking of the information-measuring devices operated on vehicles depends essentially on the type of the vehicle. In the represented paper the mathematical model assigned for operation on the ground vehicles is considered.

Based on the expressions (2) – (4) and (9), (10), the mathematical model of the two-axis system for the space stabilization of measurement and observation equipment operated on moving vehicle can be obtained. It should be noted that for creation of the system control object model it is expedient to accept some simplifications for the set of equations (1), namely, to neglect by the centrifugal inertia moments. Taking into account this simplification the mathematical model of the system providing stabilization and control by the lines of sight of the information-measuring equipment operated on the land vehicles in the horizontal and vertical planes becomes

$$\dot\alpha = \omega_x \cos\beta + \omega_z \sin\beta ,$$
$$\dot\beta = \omega_y ,$$
$$\dot\alpha_e = \omega_{e\alpha} ,$$
$$\dot\beta_e = \omega_{e\beta} ,$$
$$\dot U_{\omega\alpha} = U_{\omega d\alpha} ,$$
$$\dot U_{\omega\beta} = U_{\omega d\beta} ,$$
$$\dot\omega_x = [-(J_z - J_y)\omega_y\omega_z - M_{frx}\mathrm{sign}\omega_x - M_{unbx}\cos\alpha + c_r(\alpha_g - \alpha)/n_r]/J_x ,$$
$$\dot\omega_y = [-(J_y - J_x)\omega_x\omega_z - M_{fry}\mathrm{sign}\omega_y - M_{unby}\cos\beta + k_{spr}(A-\beta) + \frac{c_r(\beta_g - \beta)}{n_r}]/J_y ,$$
$$\dot\omega_{e\alpha} = \left[-M_{fre}\mathrm{sign}\omega_{e\alpha} + \frac{c_m}{R_w}U_\alpha + \frac{c_r(\alpha_g-\alpha)}{n_r}\right]/J_e ,$$
$$\dot\omega_{e\beta} = \left[-M_{fre}\mathrm{sign}\omega_{e\beta} + \frac{c_m}{R_w}U_\beta + \frac{c_r(\beta_g-\beta)}{n_r}\right]/J_e ,$$
$$\dot U_\alpha = [-U_\alpha + k_{\mathrm{PWM}}U_{\mathrm{PWM}\alpha} - c_{ed}\omega_{e\alpha}]/T_{arm} ,$$
$$\dot U_\beta = [-U_\beta + k_{\mathrm{PWM}}U_{\mathrm{PWM}\beta} - c_{ed}\omega_{e\beta}]/T_{arm} ,$$
$$\dot U_{\omega d\alpha} = [-2\nu T_0 U_{\omega e\alpha} - U_{\omega\alpha} + k_{ars}\omega_x]/T_0^2 ,$$
$$\dot U_{\omega d\beta} = [-2\nu T_0 U_{\omega e\beta} - U_{\omega\beta} + k_{ars}\omega_y]/T_0^2 , \quad (11)$$

where α, β are the turn angles of the platform with the useful payload installed on it; ω_x, ω_y are the platform angular rates in the horizontal and vertical planes correspondingly; $\omega_{e\alpha}$, $\omega_{e\beta}$ are the rates of the engines mounted by the axes x, y correspondingly; α_e, β_e are the turn angles of the engines mounted by the axes x, y; $U_{\omega\alpha}$, $U_{\omega\beta}$ are the output signals of the angular rate signals measuring the absolute angular rates of the platform with mounted on it useful payload by the axes x, y; $U_{\omega d\alpha}$, $U_{\omega d\beta}$ are the derivatives of the sensor output signals; J_x, J_y, J_z are the inertia moments of the platform with mounted on it useful payload relative its own axes x, y, z; M_{frx}, M_{fry} are the nominal dry friction moments acting by the gimbals axes x, y; M_{unbx}, M_{unby} are the unbalanced moments by the axes x, y; k_{spr} is the rigidity coefficient of the spring compensator; A is the initial angle of spring resetting; c_r is the reducer rigidity; α_g, β_g are the turn angles of the platform taking into account presence of the drive gap; M_{frex}, M_{frey} are the nominal dry friction moments of engines installed at the gimbals axes x, y; c_m is the constant of the load moment at the engine shaft; R_w is the resistance of the engine armature winding; U_α, U_β are the armature voltages of engines mounted by the gimbals axes; n_r is the reducer gear ratio; T_{arm} is the time constant of the engine armature circuit; k_{PWM} is the transfer constant of the linearized pulse width modulator; U_{PWM} is the voltage at the pulse width modulator input; c_{ed} is the coefficient of proportionality between the engine angular rate and the electromotive force; ν is the relative damping coefficient; T_0 is the time constant of the angular rate sensor, k_{ars} is the transfer constant of the angular rate sensor. In the represented non-linear equations (4) the angles α_g, β_g may be defined in accordance with the expressions

$$\alpha_g = \alpha_e/n_p , \quad \text{if } |\alpha_e/n_p - \alpha| \ge 0.5\Delta ,$$
$$\alpha_g = \alpha , \quad \text{if } |\alpha_e/n_p - \alpha| < 0.5\Delta ,$$
$$\beta_g = \beta_e/n_p , \quad \text{if } |\beta_e/n_p - \beta| \ge 0.5\Delta ,$$
$$\beta_g = \beta , \quad \text{if } |\beta_e/n_p - \beta| < 0.5\Delta , \quad (12)$$

where Δ is the value of the experimentally determined system drive gap.

For further researches it is necessary to implement linearization of the equations (11), (12) relative to the nominal values of the phase coordinates. Such linearization must include the following stages:

1) linearization of the expressions for the friction and unbalanced moments of the engine and stabilization object;

2) neglect by the drive gap and the friction moments at the bearings of the gimbals and at the engine shaft;

3) neglect by the error of the angular rate sensor;

4) assumption of smallness of the platform turn angles for linearization of the trigonometric functions. After these actions the set of equations (11) can may be transformed to the linearized form and represented in the space of states.

V. Mathematical Description of Three-axis Stabilization System

The position of the coordinate system O*XYZ* associated with the platform in inertial space relative to the original coordinate system O is determined by a sequence of three rotations through angles ψ, ϑ, γ as it is shown in Fig. 2.

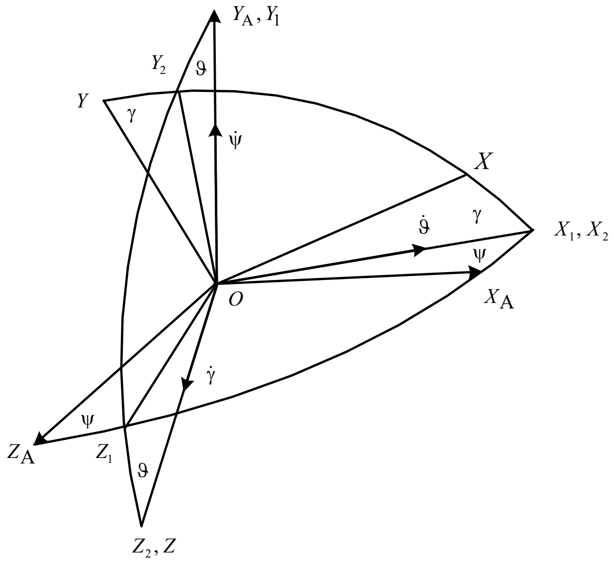

Fig. 2. Sequence of turns that determines the position of the platform with the equipment

When creating a mathematical description of the stabilization system, it is necessary to take into account that the control moments generated in the controller control the movement of the gimbal frames, and the system dynamics model (1) is determined in projections onto its own axes [9], [10]. Expressions for determining the components of control moments can be determined based on the following transformations

$$\begin{bmatrix} M_{X_1} \\ M_{Y_1} \\ M_{Z_1} \end{bmatrix} = A_3^{\mathrm{T}} \begin{bmatrix} 0 \\ 0 \\ M_{Z_2} \end{bmatrix}, \quad (13)$$

$$\begin{bmatrix} M_{X_2} \\ M_{Y_2} \\ M_{Z_2} \end{bmatrix} = A_3^{\mathrm{T}} A_2^{\mathrm{T}} \begin{bmatrix} M_{X_1} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\varphi M_{X_1} \\ -\sin\varphi M_{X_1} \\ 0 \end{bmatrix}, \quad (14)$$

$$\begin{bmatrix} M_{X_3} \\ M_{Y_3} \\ M_{Z_3} \end{bmatrix} = A_3^{\mathrm{T}} A_2^{\mathrm{T}} A_1^{\mathrm{T}} \begin{bmatrix} 0 \\ M_{Y_A} \\ 0 \end{bmatrix} = \begin{bmatrix} \cos\beta\sin\varphi M_{Y_A} \\ \cos\beta\cos\varphi M_{Y_A} \\ -\sin\beta M_{Y_A} \end{bmatrix}. \quad (15)$$

Based on expressions (13) – (15), the projections of the platform control moments onto its axes take the form

$$\begin{aligned} M_X &= \cos\varphi M_{X_1} + \cos\beta\sin\varphi M_{Y_A}, \\ M_Y &= -\sin\varphi M_{X_1} + \cos\beta\cos\varphi M_{Y_A}, \quad (16) \\ M_Z &= M_{Z_2} - \sin\beta M_{Y_A}. \end{aligned}$$

Relations (2) – (4) and (13) – (16) represent a mathematical description of the three-axis stabilization system for surveillance equipment operated on a UAV. In modern control theory, the design of robust systems is usually carried out in two stages. At the first stage, robust synthesis is carried out, based on the use of a linearized model represented in state space. At the second stage, the synthesized system is tested using simulation.

Depending on the results obtained, the first stage can be repeated after making changes to the initial data and optimization criteria.

The implementation of robust synthesis requires the use of a linearized state-space model. Such a model can be obtained based on the above-mentioned relations using the following assumptions:

1) neglect of the centrifugal moments of the platform and the difference in axial moments, which makes it possible to significantly simplify expressions (1);

2) taking into account only small angles of rotation of the platform, which makes it possible to simplify expressions (3), (9), (10);

3) replacement of nonlinear friction moments with linearized moments;

4) use of a linearized PWM model;

5) neglect of some moments of disturbance acting on the platform.

VI. Simulation Results

The results of the robust synthesis by the method of the mixed sensitivity of the stabilization system using the developed mathematical description are represented in Fig. 3. The represented results of the synthesis of the one-axis stabilization system have been obtained for the equipment assigned for operation at the land moving vehicle (horizontal and vertical channels).

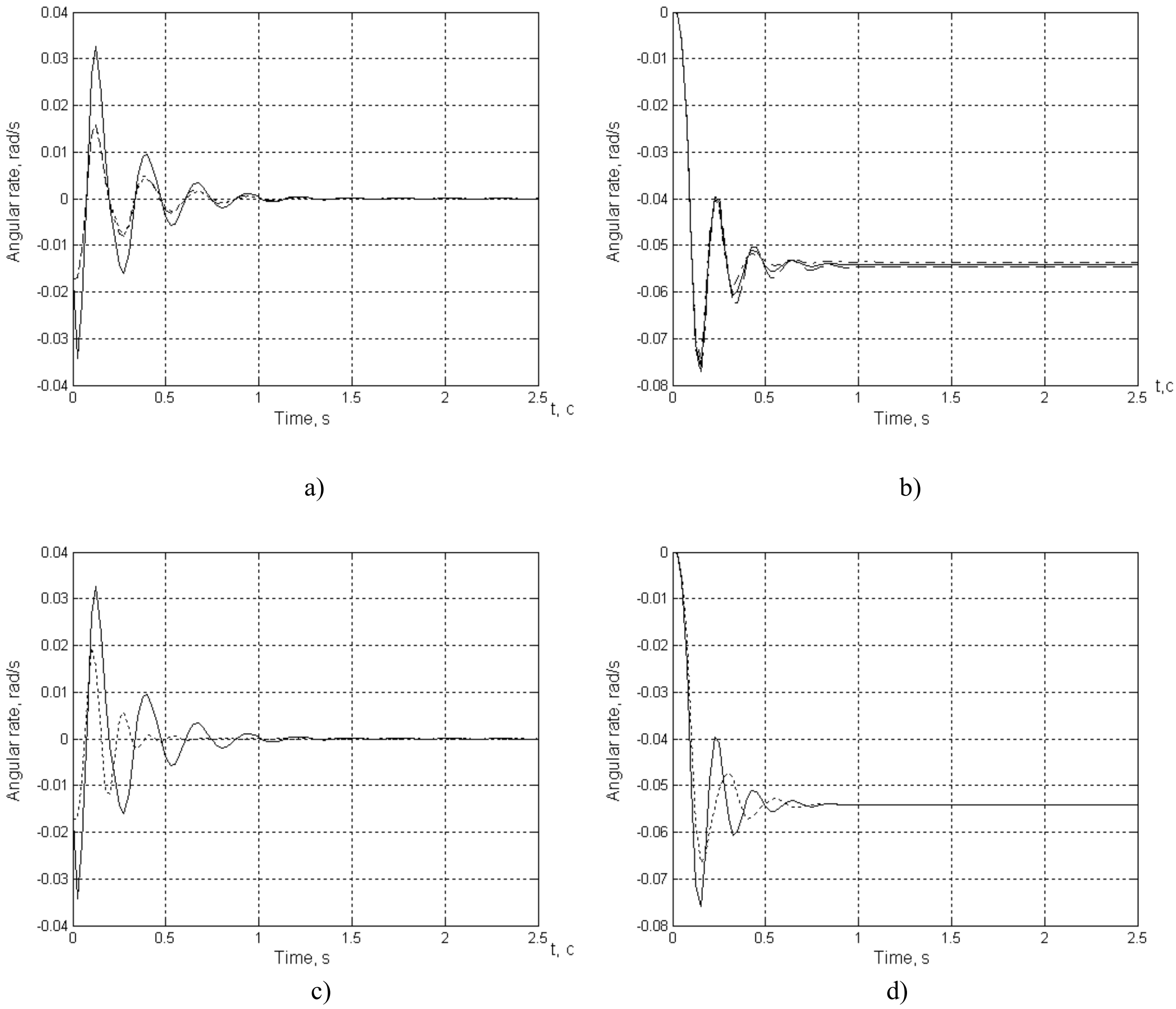

Fig. 3. Simulation results: transition processes of the nominal (solid) and parametrically disturbed (dotted line) systems in the stabilization mode (a) and the tracking mode (b) for the horizontal channel; transition process of the nominal (solid) and parametrically disturbed (dotted line) systems in the stabilization mode (a) and the tracking mode (b) for the vertical channel

VII. CONCLUSIONS

The generalized approaches to development of mathematical description of stabilization systems assigned for stabilization of measurement and observation equipment mounted at moving vehicles of the different classes.

Mathematical descriptions for one-axis, two-axis, and three-axis systems are represented. The appropriate kinematical relations are represented.

Simulations results of synthesis for the land moving vehicle based on the mathematical description of one-axis stabilization system are represented.

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О. О. Салюк. Математичний опис систем просторової стабілізації обладнання призначеного для експлуатації на рухомому транспортному засобі

Статтю присвячено розробці математичного опису систем стабілізації вимірювальної та спостережувальної апаратури, призначеної для роботи на рухомих об'єктах широкого класу – наземних, морських та повітряних. Представлені математичні описи одноосьових, двовісних і тривісних систем стабілізації, включаючи кінематичні співвідношення та моделі динаміки. Наведено загальні математичні описи та відповідні моделі в просторі станів. Представлено основні підходи до лінеаризації узагальнених моделей. Наведено сукупності поворотів в інерційному просторі для двовісної та тривісної систем стабілізації. Отриману математичну модель одновісної системи стабілізації використано для робастного структурного синтезу системи, призначеної для стабілізації обладнання спостереження, встановленого на наземних рухомих об'єктах. Отримані результати можна поширити на рухомі об'єкти різного типу.

Ключові слова: системи стабілізації; багатовісна просторова стабілізація; математичний опис; модель у просторі станів; диференціальні рівняння; матриці.

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