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ATTITUDE DETERMINATION SYSTEM WITH ROTATION OF GYROSCOPES

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Abstract**—***A method of auto-compensation for the influence of constant biases in the output signals of gyroscopes in the system for determining the orientation of an object, based on the rotation of the gyroscopes relative to the object, is considered. The analysis of the obtained models of errors in determining the orientation and the simulation performed show the high efficiency of this method for improving the accuracy of readings. The proposed error model makes it quite easy to evaluate the efficiency of auto-compensation. It is shown that the problem may be reduced to integrating the direction cosine matrix that determine the position of the sensitive elements in the reference coordinate system.*

Index Terms—Auto-compensation; rotation; errors; gyroscope; accuracy; matrix.

I. Introduction

The use of self-compensation of gyroscope errors as an effective means of increasing the accuracy of orientation determination is considered in [1], [2]. In article [3] is proposed a scheme in which two gyroscopes are installed on a common platform in a suspension, which has two degrees of freedom relative to the object. Consider a scheme in which two uniaxial suspensions are used instead of one biaxial suspension.

II. Problem Statement

We will consider a scheme with two uniaxial platforms, on each of which two gyroscopes are installed. The platforms rotate in mutually perpendicular planes (Fig. 1) according to the laws of $\sigma_1 = n_1 t$, $\sigma_2 = n_2 t$.

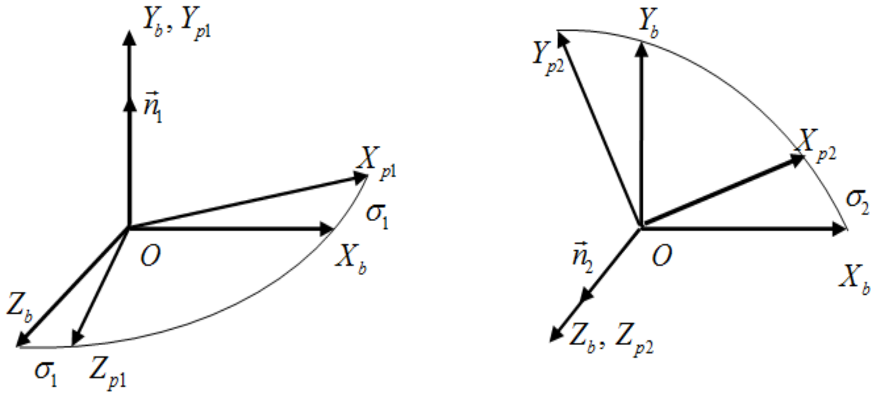

Fig. 1. Used coordinate systems

III. Problem Solution

The transition from the coordinate system $OX_bY_bZ_b$ connected with the object to the coordinate system $OX_{p1}Y_{p1}Z_{p1}$ connected with the first platform (Fig. 1) is defined by the matrix $\mathbf{B}_{p1}$.

$$\mathbf{B}_{p1} = \begin{bmatrix} \cos\sigma_1 & 0 & \sin\sigma_1 \\ 0 & 1 & 0 \\ -\sin\sigma_1 & 0 & \cos\sigma_1 \end{bmatrix}. \quad (1)$$

The transition from the coordinate system $OX_bY_bZ_b$ connected with the object to the coordinate system $OX_{p2}Y_{p2}Z_{p2}$ connected with the second platform is defined by the matrix $\mathbf{B}_{p2}$.

Let us evaluate the effectiveness of this scheme for auto-compensation of gyros errors.

$$\mathbf{B}_{p2} = \begin{bmatrix} \cos\sigma_2 & -\sin\sigma_2 & 0 \\ \sin\sigma_2 & \cos\sigma_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Let's enter vectors of output signals of gyroscopes

$$\boldsymbol{\omega}_{b1} = [\omega_{b1x}\ 0\ \omega_{b1z}]^{\mathrm{T}}, \quad \boldsymbol{\omega}_{b2} = [\omega_{b2x}\ \omega_{b2y}\ 0]^{\mathrm{T}}. \tag{3}$$

The total vector of angular velocities, which is further used to determine the orientation, can be formed in two ways.

In the first case, vectors (3) are used independently, for example

$$\boldsymbol{\omega}_b = \mathbf{B}_{p1}\boldsymbol{\omega}_{b1} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{B}_{p2}\boldsymbol{\omega}_{b2}. \tag{4}$$

You can also accept

$$\boldsymbol{\omega}_b = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\mathbf{B}_{p1}\boldsymbol{\omega}_{b1} + \mathbf{B}_{p2}\boldsymbol{\omega}_{b2}. \tag{5}$$

In the second case, vectors (3) are used together

$$\boldsymbol{\omega}_b = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\mathbf{B}_{p1}\boldsymbol{\omega}_{b1} + \mathbf{B}_{p2}\boldsymbol{\omega}_{b2}\right). \tag{6}$$

The transition from the coordinate system $OX_bY_bZ_b$ to the reference (fixed) coordinate system $OX_OY_OZ_O$ is defined by the matrix $\boldsymbol{B}_b$. This matrix is found as a solution of Poisson's equation

$$\dot{\boldsymbol{B}}_b = \boldsymbol{B}_b\boldsymbol{\Omega}_b, \tag{7}$$

where

$$\boldsymbol{\Omega}_b = \begin{bmatrix} 0 & -\omega_{z_b} & \omega_{y_b} \\ \omega_{z_b} & 0 & -\omega_{x_b} \\ -\omega_{y_b} & \omega_{x_b} & 0 \end{bmatrix},$$

$\boldsymbol{\omega}_b$ is the vector of body angular velocities.

The constant biases of gyroscopes write as

$$\boldsymbol{\omega}_{d1} = [\omega_{d1x}\ 0\ \omega_{d1z}]^{\mathrm{T}}, \quad \boldsymbol{\omega}_{d2} = [\omega_{d2x}\ \omega_{d2y}\ 0]^{\mathrm{T}}. \tag{8}$$

The device value of body direction cosine matrix $\hat{\boldsymbol{B}}_b$ is sought as a solution to the Poisson equation

$$\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b\hat{\boldsymbol{\Omega}}_b, \tag{9}$$

where

$$\hat{\boldsymbol{\Omega}}_b = \begin{bmatrix} 0 & -\hat{\omega}_{z_b} & \hat{\omega}_{y_b} \\ \hat{\omega}_{z_b} & 0 & -\hat{\omega}_{x_b} \\ -\hat{\omega}_{y_b} & \hat{\omega}_{x_b} & 0 \end{bmatrix}.$$

For an analytical evaluation of orientation errors, we will write this expression in the form

$$\hat{\boldsymbol{\Omega}}_b = \boldsymbol{\Omega}_b + \boldsymbol{\Omega}_d, \tag{10}$$

where

$$\Omega_d = \begin{bmatrix} 0 & -\Omega_{dz} & \Omega_{dy} \\ \Omega_{dz} & 0 & -\Omega_{dx} \\ -\Omega_{dy} & \Omega_{dx} & 0 \end{bmatrix}.$$

$$\boldsymbol{\omega}_d = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\left(\mathbf{B}_{p1}\boldsymbol{\omega}_{d1} + \mathbf{B}_{p2}\boldsymbol{\omega}_{d2}\right).$$

Considering the estimation error $\Delta_{\mathbf{B}_b}$ of the "ideal" matrix $\mathbf{B}_b$ to be small, we write

$$\hat{\mathbf{B}}_b = \left(\mathbf{I} + \Delta_{\mathbf{B}_b}\right)\mathbf{B}_b, \tag{11}$$

where

$\Delta_{\boldsymbol{B}_b} = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix}$ is the small skew-symmetric matrix of errors, which is recorded in the coordinate system $OX_OY_OZ_O$.

Then we will write it down

$$\dot{\Delta}_{\mathbf{B}_b}\mathbf{B}_b + (\boldsymbol{I} + \Delta_{\mathbf{B}_b})\dot{\mathbf{B}}_b = (\boldsymbol{I} + \Delta_{\mathbf{B}_b})\mathbf{B}_b\left(\boldsymbol{\Omega}_b + \boldsymbol{\Omega}_d\right).$$

Neglecting the product of errors, we will have the following expression

$$\boldsymbol{\Omega}_\varepsilon = \dot{\Delta}_{\boldsymbol{B}_b} = \mathbf{B}_b\boldsymbol{\Omega}_d\mathbf{B}_b^{\mathrm{T}}. \tag{12}$$

Matrix equation (8) corresponds to a vector equation

$$\dot{\boldsymbol{\varepsilon}} = \mathbf{B}_b \boldsymbol{\omega}_d , \qquad (13)$$

where $\boldsymbol{\varepsilon} = [\varepsilon_x \ \varepsilon_y \ \varepsilon_z]^{\mathrm{T}}$ is the error vector in coordinate system $OX_O Y_O Z_O$.

For constant gyroscopes errors and for a stationary object the task is reduced to matrixes $\mathbf{B}_{p1}$. and $\mathbf{B}_{p2}$. integration.

For $n_1 = n_2 = n$, $\sigma_1 = \sigma_2 = \sigma$ we have

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{bmatrix} = \frac{1}{n} \begin{bmatrix} \frac{1}{2}\begin{pmatrix} \omega_{1dx}\sin\sigma + \omega_{1dz}(1-\cos\sigma) \\ + \omega_{2dx}\sin\sigma - \omega_{2dy}(1-\cos\sigma) \end{pmatrix} \\ \omega_{2dy}\sin\sigma + \omega_{2dx}(1-\cos\sigma) \\ \omega_{1dz}\sin\sigma - \omega_{1dx}(1-\cos\sigma) \end{bmatrix}.$$

To assess the accuracy of this method will accept

$$\omega_{dx} = \omega_{dy} = \omega_{dz} = 5,^0/h, \quad n_1 = n_2 = 0.5\,s^{-1},$$
$$\omega_{x_b} = \omega_{y_b} = \omega_{z_b} = 1\cos 0.1t,^0/s.$$

Errors in the absence of rotations according to the expression (6) are shown in Fig. 2.

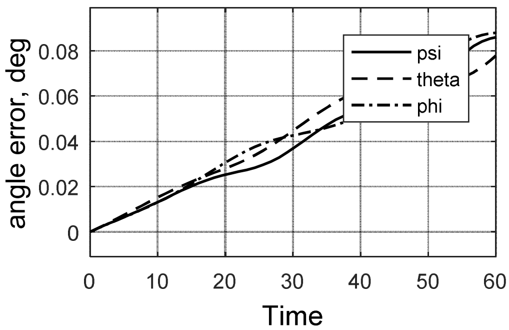

Fig. 2. Errors of angles

Errors are defined as the difference between the values of the angles in the presence of gyros errors and the values in their absence.

Errors in the presence of rotations are shown in Fig. 3.

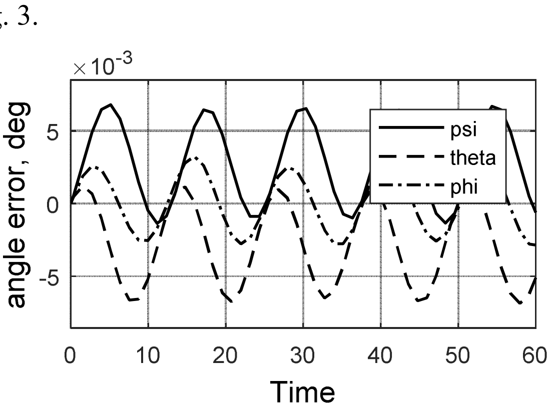

Fig. 3. Errors of angles

The errors presented in Fig. 4 are determined in the presence of rotations by the formula (13)

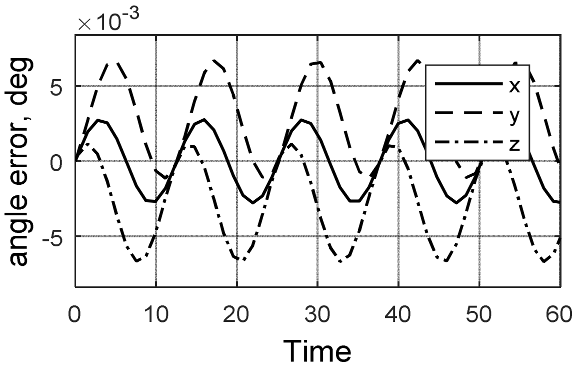

Fig. 4. Errors of angles

We see that during rotation there is no unlimited growth of errors over time.

Also we see expediency of using an approximate formula (13) for performing analysis.

IV. Conclusion

The high efficiency of using the rotation of sensitive INS elements to reduce the influence of their constant errors on the accuracy of determining navigation parameters is shown.

A further improving the accuracy can be ensured by increasing the angular velocities of rotation and using of rotation reversal.

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Л. М. Рижков, М. Г. Черняк. Система визначення положення з обертанням гіроскопів

Розглянуто метод автокомпенсації впливу постійних зміщень у вихідних сигналах гіроскопів у системі визначення орієнтації об’єкта на основі обертання гіроскопів відносно об’єкта. Аналіз отриманих моделей похибок визначення орієнтації та проведене моделювання показують високу ефективність даного методу для підвищення точності вимірювань. Запропонована модель похибок дозволяє досить легко оцінити ефективність автокомпенсації. Показано, що задачу можна звести до інтегрування матриці напрямних косинусів, яка визначає положення чутливих елементів у опорній системі координат.

Ключові слова: автокомпенсація; обертання; помилки; гіроскоп; точність; матриця.

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