



For this, it is proposed to introduce the additional penalty function in the known combined criterion of the parametric optimization.

## II. PARAMETRIC OPTIMIZATION WITH ADDITIONAL PENALTY FUNCTION

In general, the quality of control systems can be assessed by different groups of criteria.

1) Complex criteria, which represent an assessment of certain averaged properties of the system, for example, on the basis of norms of matrix transfer functions describing the closed-loop system.

2) Criteria that define the indices of the quality of the transient process, for example, the speed of operation of the system. Mostly, the speed of operation can be estimated by the time of damping the transient process.

3) Criteria, which determine the value of the system's stability margin, which can be estimated on the basis of the analysis of the logarithmic amplitude-frequency characteristics of the system.

4) Specific criteria that may be important for a system of the researched type.

The problem of the parametric optimization of control systems for moving vehicles of a wide class in general and stabilization systems in particular requires the use of quality criteria in three aspects. Firstly, solving this problem requires the formation of an objective function. Secondly, optimization problems of this type require the use of a penalty function. Thirdly, a feature of this problem is the need to analyze the obtained results using various quality criteria.

The synthesis of perturbation-resistant systems can be based on the minimization of the norm of the matrix transfer function of the closed-loop system. It is known that an approach to the synthesis of control systems could be based on the minimization of the norm of the matrix transfer function of a closed-loop system, which characterizes accuracy. It should be noted that from the point of view of the organization of computational algorithms,  $H_\infty$ -optimization is much more difficult in comparison with  $H_2$ -optimization due to the need to implement a search procedure.

Methods of the synthesis based on minimization of  $H_2$ -norms ensure high accuracy of the synthesized system, but it remains sensitive both to external disturbances and to parametric disturbances of the control object. The application of the  $H_\infty$ -norm allows us to ensure the stability of the system to external disturbances under the condition of structured and unstructured parametric uncertainty. Optimization according to each of the considered

approaches has advantages. But optimization by a combined criterion allows us to combine these advantages. At the same time, the synthesized system will be characterized by optimal quality, provided that it can function in the presence of disturbances.

It is known that the combined criterion "quality-robustness" is successfully used in the synthesis procedures of robust control systems for aircraft of a wide class. For the studied system, it is advisable to include the quality indicators of the nominal and the system disturbed by parametric structured disturbances for deterministic and stochastic cases in the complex optimization criterion [7]

$$J_{H_2/H_\infty} = \lambda_2^{\text{nom d}} \|\Phi(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{\text{nom d}}\|_2 + \lambda_2^{\text{nom s}} \|\Phi(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{\text{nom s}}\|_2 + \lambda_\infty^{\text{nom}} \|\Phi(\mathbf{K}, \mathbf{x}, \mathbf{u}, j\omega)^{\text{nom}}\|_\infty + PF_1 + PF_2, \quad (1)$$

where  $\|\cdot\|_2^{\text{nom d}}$ ,  $\|\cdot\|_2^{\text{nom s}}$  are  $H_2$ -norms of matrix functions of the sensitivity of the closed-loop nominal system for the deterministic and stochastic cases;  $\|\cdot\|_\infty^{\text{nom}}$  is  $H_\infty$ -norm of the matrix function of the complementary sensitivity function;  $\lambda_2^{\text{nom d}}$ ,  $\lambda_2^{\text{nom s}}$ ,  $\lambda_\infty^{\text{nom}}$  are weighting coefficients of the appropriate norms;  $PF_1$  is the penalty function, which ensures stability of the system during optimization;  $PF_2$  is the penalty function that ensures operating requirements to the initially stabilized platforms;  $\mathbf{K}$  is the vector of parameters to be optimized;  $\mathbf{x}$  is the vector of input parameters;  $\mathbf{u}$  is the vector of observations.

The represented criterion differs from known ones [7], [8] by introducing the new penalty function  $PF_2$ , which takes into consideration operating requirements specific for systems assigned for stabilization of stabilization and observation equipment assigned for operation of moving vehicles.

It is known that requirements for control performance and robustness are mutually contradictory. Therefore, the task of optimal synthesis of the stabilization system consists in finding a compromise between the performance and robustness of the system. This compromise can be achieved by using a combined criterion with variable weighting coefficients (1). Using such a criterion, we can decrease or increase the degree of performance and robustness depending on the analysis of the characteristics of the synthesized system [8].

It should be noted that during the parametric optimization procedure, it is necessary to ensure that the closed system remains stable in the process of variations in the characteristics of the control object

and the set parameters of the controller. For this purpose, the penalty function  $PF_1$  is added to the quality indicator, which ensures the finding of the poles of the closed system in the left half-plane of the plane of the complex variable. To determine the penalty function during the synthesis procedure, a check is made to find the poles of the system in the section, on the half-plane of the complex variable, which meets the criteria of system stability.

For the system of the studied class, it is reasonable to use performance indices as restrictions, which are subjected to unconditional fulfilment by entering the additional penalty function  $PF_2$ . This is the novelty of the proposed approach.

Let us consider the most important operational requirements. The accuracy of the system is defined by three basic types of errors.

The first component characterized tracking error. It is defined by the expression [10]

$$a_1 = x_{tr} = \frac{U_{cc}}{[1+W(p)]}, \quad (2)$$

where  $U_{cc}$  is the signal of control console;  $W(p)$  is the transfer function of the open stabilization system.

The second component characterizes the error caused by influence of external disturbance moments. Usually, this error is defined relative to the stabilization object's location. It can be described by the equation [10]

$$a_2 = x_m = \frac{W_{m1}(p)M}{[1+W(p)]p}, \quad (3)$$

where  $W_{m1}(p)$  is the transfer function of disturbing moment, for example, unbalanced moment,  $M$  is disturbing moment.

The third component is the stabilization error and takes into account the influence of the angular rate of the moving object. It is also determined relative to the angular position of the control object. As a source of the stabilization error we consider also the moment applied to the drive of moving object. The total error will be determined by the expression [10]

$$a_3 = x_{st} = \frac{[1 - W_{m2}(p)W_r(p)Jp]\omega_{mo}}{[1+W(p)]p}, \quad (4)$$

where  $W_{m2}(p)$  is the transfer function by the moment caused by rotation of moving object;  $W_r(p)$  is the transfer function of reduction of disturbing moment to the input of the stabilization object.

For inertially stabilized platforms, one of the main requirements include stiffness by moment of disturbance. In order to estimate the stiffness based on the moment of disturbance, it is necessary to set the law of changing the load moment of the control object, for example in the form of a jump, and analyze the corresponding change in the absolute angle of the position of the control object.

The angular stiffness due to unbalanced moment is determined by the ratio

$$a_4 = \frac{M_1(t_0) - M_2(t_0 + \Delta t)}{\frac{W_{m1}(p)M_1(t_0)}{[1+W(p)]p} - \frac{W_{m1}(p)M_2(t_0 + \Delta t)}{[1+W(p)]p}}, \quad (5)$$

where  $M_1(t_0), M_2(t_0 + \Delta t)$  are moments at instants of time  $t_0, t_0 + \Delta t$ ,  $M_2(t_0) = M_1(t_0 + \Delta t) + M$ ; and  $M_2(t_0) \gg M_1(t_0 + \Delta t)$ .

For systems of the considered type, it is important to estimate the dynamic properties of the system under investigation. During movement according to the harmonic law, the direction of movement of the ground object and, accordingly, of the stabilization objects changes continuously, while simultaneously changing the direction of action of the dry friction forces, which allows the most complete assessment of the dynamic properties of the stabilizer. Given the task of such movement, the error of the angular position in the steady state will change according to the harmonic law. The accuracy of the stabilization system can be estimated by the maximum amplitude

In conditions of such a motion, the error of the angular position will be changed in accordance with the harmonic law with the frequency  $\omega_k$  and shift of phase  $\psi$ :  $x(t) = x_{\max} \sin(\omega_k t + \psi)$ . The accuracy of the stabilization system can be estimated by the maximum amplitude of the error  $x_{\max}$ . The magnitude of this amplitude can be estimated by substitution  $p = j\omega_k$  [10]

$$x_{\max} = \frac{\omega_{mo} - |W(p)_{m2}(j\omega)W_r(j\omega)Jj\omega| \omega_{mo}}{|1+W(j\omega)| j\omega}. \quad (6)$$

Since the amplitude of the error is much smaller than the amplitude of the input, the expression (6) can be replaced by an approximate expression

$$a_5 = x_{\max} = \frac{\omega_{mo} - |W(p)_{m2}(j\omega)W_r(j\omega)Jj\omega| \omega_{mo}}{|W(j\omega)| j\omega}, \quad (7)$$

where  $|W(j\omega)|$  is the module of the frequency transfer function in condition  $\omega = \omega_k$ .

The disadvantage of this method is using of a specific value of the test signal. This disadvantage can be avoided by using the relative amplitude error. For this, it is advisable to introduce two control measurements under the condition of angular rates  $\omega_{\max_1}, \omega_{\max_2}$  with amplitudes  $x_{\max_1}, x_{\max_2}$  and consider the relative amplitude error

$$a_6 = \Delta x_{\max} = \left( \frac{x_{\max_1}}{\varphi_{\max_1}} - \frac{x_{\max_2}}{\varphi_{\max_2}} \right) \cdot 100\%, \quad (8)$$

where  $\varphi_1, \varphi_2$  are angles of the platform position, which correspond to given angular rates.

The additional penalty function is formed as follows

$$PF_2 = N \text{ if } \forall [i] a_i < \Delta_i, i = 1 \dots 4, \quad (9)$$

here  $N$  is some great number;  $a_i$  are determined by formulas (2) – (9),  $\Delta_i$  are increased tolerances.

Next, a decision is made to complete the parametric synthesis or to repeat the optimization procedures. A new optimization procedure is performed after changing the initial conditions or after introducing new weighting factors into the system optimization criteria.

### III. APPLICATION OF GENETIC ALGORITHM

The parameters optimization can be implemented by means of the genetic algorithm. In MatLab Optimization toolbox one can find genetic algorithm optimization section. Unlike the Nelder–Mead method, where we should set the starting point, in genetic algorithm we set number of variables and also the initial and final values of variable. The genetic algorithm also has other parameters, which are intended to modify it for the certain problem [11], [12].

The procedure of genetic algorithm consists of next steps [12]:

1) The initial individual's population of size  $\mu$  ( $\mu < N$ ), where  $N$  is dimension, in the search space  $E^N$ . The initial population is usually created in a random way in a symbol form.

2) Translation of each vector  $X_i = [x_{1i} \ x_{2i} \ \dots \ x_{ii}]$ ,  $i \in \overline{0, \mu}$  from the symbol form into decimal one and calculation of the fitness function for each coordinate point  $f_i(X_i)$ ,  $i \in \overline{0, \mu}$ .

3) Estimation of the population on degeneracy. The population degeneracy is valued from difference of maximal  $f^{\max}$  and minimal  $f^{\min}$  values of fitness function. If the condition  $|f^{\max} - f^{\min}| \leq \varepsilon$ , is

satisfied, where  $\varepsilon$  is a sufficiently small number, the population degenerates into the point corresponding to problem solution. Otherwise, the next step is performed.

4) Deletion of the least adapted individuals  $\rho \cdot \mu$  taking into account the fitness function value, where  $\rho$  is elimination coefficient (usually it equals 0.1 [12]). The rest  $(1 - \rho) \cdot \mu$  individuals compose the new parental group that is used for descendant generation (new coordinate points).

5) Selection of equiprobable  $\rho \cdot \mu$  times individuals from the parental group for parental couples, to which correspondingly the genetic operators are applied. As a result of genetic operations we obtain  $\rho \cdot \mu$  descendants (new coordinate points). Obtained descendants are set in initial population and they are valued at fitness function.

6) The algorithm goes to 3rd step beginning a new evolution stage.

The genetic algorithms when searching the global extreme use the probabilistic approach. In view of this it is expedient not to talk about a global extreme but about the best achieved solution in accepted search range. The success in genetic algorithm procedure is provided first of all with the collective search idea or the search provided with the help of population of searching points and genetic operators taken from the nature. The genetic operators affecting with some probability on parental chromosomes provide from the one side the information transfer to descendants about population state and from the other side – support the sufficient level of changeability, this factor retains the algorithm's searching ability.

The genetic algorithms searching ability to a considerable extent depends on the population size. It is obvious that the bigger population size, the higher approximation probability to searched global extremes. However, in practice, the population size is bounded by computer technology opportunities and keeps in range 10 ... 500 individuals [12].

The one of genetic algorithms important peculiarities is that no one of genetic operators (crossover, mutation, inversion) during generation process relies on information about local relief of fitness function surface [12]. The descendant formation happens in a random manner and there is no guarantee that the found solutions will be better than the parental ones. Therefore, during the evolution process, one can meet the 'unsuccessful' descendants which extent the fitness function call number and thereby the global extreme search time.

In presence, the genetic algorithms have mainly the particularized application in neuronets technologies for multi-parametric problems solution (object recognition and forecasting). However, having the simple superficial conception genetic algorithms require considerable efforts in order to adapt them to a certain problem. First of all, adaptation is required in genetic operator application probability.

Taking into account the mentioned above one proposes the genetic algorithm modification for universal application to problems having comparatively small dimensions. The modified genetic algorithm retains genetic qualities of static searching points population selection. In order to exclude unsuccessful descendants there realized the local extremes regular search procedure with usage of deformable polyhedron operators.

Consider the parameters optimization by means of genetic algorithm. In MatLab optimization toolbox one can find genetic algorithm optimization section. The fitness function remains the same and is described above. Unlike the Nelder–Mead method, where we should set the starting point, in genetic algorithm one should set number of variables and the initial and final values of variable. The genetic algorithm also has other parameters, which are intended to modify it for the certain problem.

Doing the optimization by the genetic algorithm, it is expedient to mention that this algorithm is universal, as it does not impose constraints for fitness function type. In addition, it gives us an opportunity to perform the multi-sequencing.

Otherwise, there exist such situations, when one should terminate the algorithm because of such reasons [12]:

- the achievement of certain number of populations;
- the evolution time expiration;
- the population convergence.

First two criteria depend on the problem type, and sometimes there occurs a situation, when the algorithm cannot find the function extreme or when the obtained after some number of populations result satisfies the requirement. Under the population convergence one means that neither crossover nor mutation operations make the change into algorithm result during a few populations creation. Such a situation takes place either when reaching the ‘plato’, at which the fitness function does not changes is value through the whole ‘plato’ surface, or when the population falls into local extreme zone.

The optimization can be implemented such a code:

```
clear all
clc
global ps
ps=ga(@fh_16,[3],[],[],[],[0.28 0.078
0.18],[0.3 0.08 0.33])
```

After running the code, we obtain such a set of adjustable coefficients:

$$k1 = 0.2763; k2 = 0.0779; k3 = 0.2731.$$

Results of parametrization are represented in Table I.

TABLE I. COMPARATIVE ANALYSIS

Parameters	Genetic algorithm	Nelder–Mead method
$H_2$ -norm	0.207	0.399
$H_\infty$ -norm	0.632	0.793
Setting time,s	0.59	0.727
Oscillation factor	3.5	2.91
Number of oscillations	3	3
Delay time	0.0542	0.0543
Rise time, s	0.0315	0.0314

The difference, between application of the Nelder–Mead method and the genetic algorithm is shown in Figs 2 and 3.

As we can see, the result obtained by the genetic algorithm is more optimal, as the transient process goes to its equilibrium point more quickly and smoothly [10].

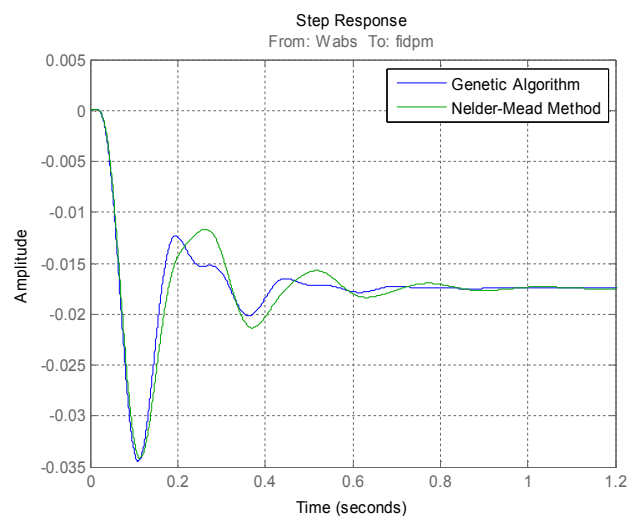


Fig. 2. Step response obtained by both the Nelder–Mead method and genetic algorithm

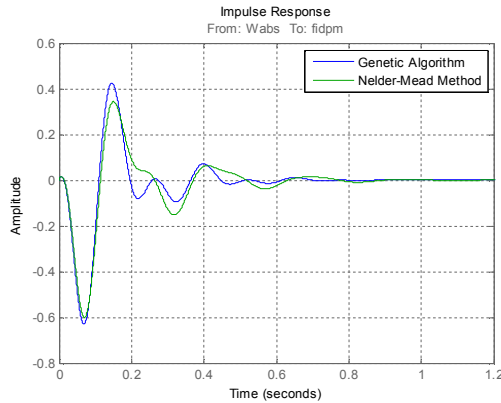


Fig. 3. Impulse responses obtained by both Nelder–Mead method and Genetic Algorithm

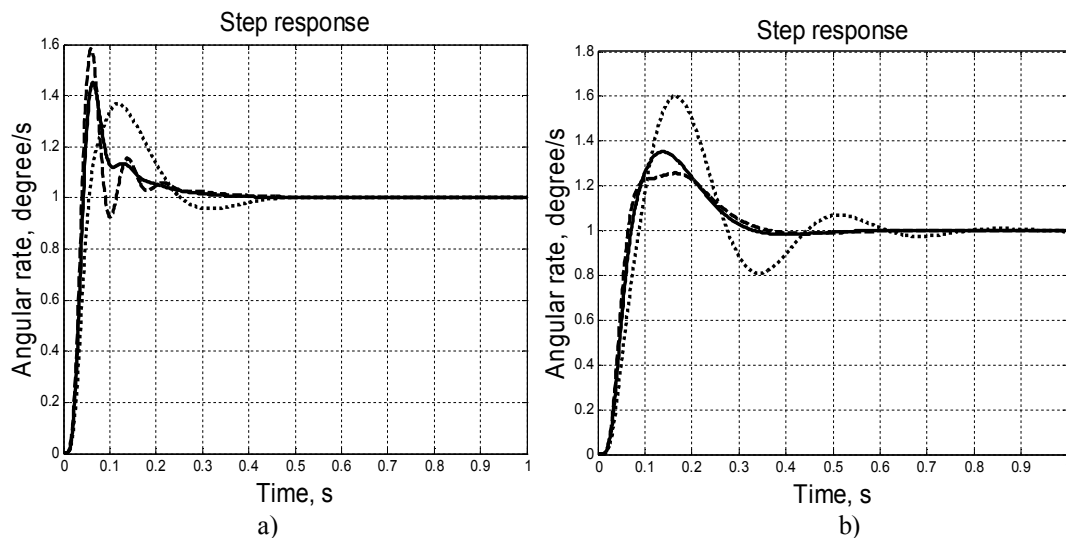


Fig. 4. Results of simulation of the stabilization system: (a), (b) are step responses for horizontal and vertical channels

## V. CONCLUSIONS

Introducing the additional penalty function based on requirements to operating characteristics has been grounded and proposed. Expressions for the penalty function are represented. The efficiency of the proposed approach lies in increasing time of the optimization process.

The comparative analysis of the Nelder–Mead and genetic algorithm for realization of the parametrical optimization has been carried out.

The obtained results in the form of graphical dependences prove the efficiency of the proposed approach.

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## IV. SIMULATION RESULTS

The results of simulation taking into account parametric perturbations of a system [14] are shown in Fig. 4.

To test the proposed approaches to designing a robust system, changes in the moment of inertia of the plant, were considered as parametric disturbances, since for the example under consideration, changes in this parameter during operation can reach 50%. [12], [13].

With a zero reference signal, an increased constant value of a moment on the system's input by perturbation is applied.

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**О. А. Сущенко, О. О. Салюк. H2/Hinf оптимізація системи стабілізації обладнання рухомого об'єктів з використанням двох типів штрафних функцій**

У статті розглядаються особливості H2/Hinf оптимізації системи стабілізації з використанням двох типів штрафних функцій, спрямованих на забезпечення як стійкості, так і експлуатаційних характеристик системи. Досліджувані системи призначені для стабілізації обладнання, що експлуатується на рухомих об'єктах. Новизна дослідження полягає у введенні нового типу штрафної функції. Наведено вирази для основних експлуатаційних вимог. Представлено вибір алгоритмів оптимізації, включаючи метод Нелдера–Міда та генетичний алгоритм. Описано особливості генетичного алгоритму. Проведено порівняльний аналіз оптимізації обома методами. Представлено результати оптимізації у вигляді перехідних процесів. Отримані результати можуть бути корисними для систем, призначених для стабілізації обладнання, що експлуатується на транспортних засобах широкого класу.

**Ключові слова:** система стабілізації; оптимізація; експлуатаційні вимоги; штрафна функція; похибка; моментна жорсткість.

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