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INCREASING THE ACCURACY OF ORIENTATION DETERMINATION BASED ON AUTO-COMPENSATION OF ERRORS

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Abstract**—***An increasing the accuracy of orientation determination of strapdown system based on the spatial rotation of gyroscopes relative to an object is considered. The subject of analysis is a mathematical model of the measurement errors of orientation angles, taking into account the constant biases of gyroscopes. The proposed error model makes it quite easy to evaluate the efficiency of auto-compensation. It is shown that the problem may be reduced to integrating the direction cosine matrix that determine the position of the sensitive elements in the reference coordinate system. The results of theoretical analysis and simulation confirm the high efficiency of this method for increasing the accuracy of measuring kinematic parameters.*

Index Terms—Auto-compensation; rotation; errors.

I. INTRODUCTION

One of the methods for creating autonomous high-precision strapdown inertial systems is the auto-compensation of sensor errors. A promising scheme for implementing this approach is the forced rotation of the block of sensitive elements [1] – [3]. An integral part of the general task is the task of increasing the accuracy of determining the angular orientation of an object, which is also of great independent importance. Of paramount importance is the choice of the laws of spatial rotation of sensitive elements, which, in turn, necessitates the construction of a mathematical model of errors.

II. PROBLEM STATEMENT

We consider that the sensitive elements (three gyroxcopes) are installed on a platform that rotates in the suspension (Fig. 1) relative to the object according to the laws $\sigma_1 = n_1 t$, $\sigma_2 = n_2 t$.

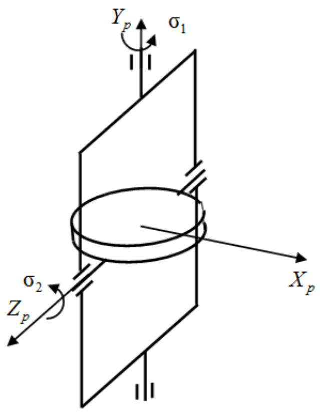

Fig. 1. Suspension

Let us evaluate the effectiveness of this scheme for auto-compensation of gyros errors.

III. PROBLEM SOLUTION

The transition from the coordinate system $OX_bY_bZ_b$ connected with the object to the coordinate system $OX_pY_pZ_p$ connected with the platform (Fig. 2) is defined by the matrix $\boldsymbol{B}_p$.

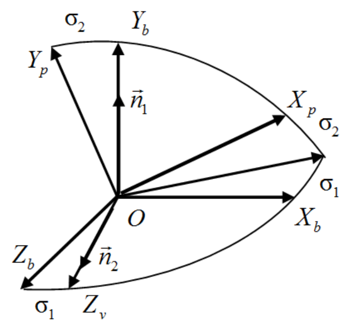

Fig. 2. Used coordinate systems

$$\boldsymbol{B}_p = \begin{bmatrix} \cos\sigma_1\cos\sigma_2 & -\cos\sigma_1\sin\sigma_2 & \sin\sigma_1 \\ \sin\sigma_2 & \cos\sigma_2 & 0 \\ -\sin\sigma_1\cos\sigma_2 & \sin\sigma_1\sin\sigma_2 & \cos\sigma_1 \end{bmatrix} \quad (1)$$

We see that the matrix $\boldsymbol{B}_p$ does not contain constant components if the angular velocities n_1 and n_2 are different.

The transition from the coordinate system $OX_bY_bZ_b$ to the reference (fixed) coordinate system $OX_oY_oZ_o$ is defined by the matrix $\boldsymbol{B}_b$. This matrix is found as a solution of Poisson's equation

$$\dot{\boldsymbol{B}}_b = \boldsymbol{B}_b \boldsymbol{\Omega}_b, \tag{2}$$

where

$$\boldsymbol{\Omega}_b = \begin{bmatrix} 0 & -\omega_{z_b} & \omega_{y_b} \\ \omega_{z_b} & 0 & -\omega_{x_b} \\ -\omega_{y_b} & \omega_{x_b} & 0 \end{bmatrix}.$$

$\boldsymbol{\omega} = \begin{bmatrix} \omega_{x_b} & \omega_{y_b} & \omega_{z_b} \end{bmatrix}^{\mathrm{T}}$ is the vector of body angular velocities.

In addition to the useful angular velocities, the vector of the output signals of the gyroscopes $\begin{bmatrix} \omega_{x_p} & \omega_{y_p} & \omega_{z_p} \end{bmatrix}^{\mathrm{T}}$ also contains the constant biases of gyroscopes $\boldsymbol{\omega}_d = [\omega_{dx}\ \omega_{dy}\ \omega_{dz}]^{\mathrm{T}}$ and the projections of the angular velocities of the additional rotations $[n_1 \sin\sigma_2\ n_1\cos\sigma_2\ n_2]^{\mathrm{T}}$. Therefore to find the angular position of the object using the output signals of the gyroscopes, it is necessary to subtract the corresponding angular velocities of rotation of the platform relative to the object and difference project into the coordinate system $OX_bY_bZ_b$:

$$\begin{bmatrix} \hat{\omega}_{x_b} \\ \hat{\omega}_{y_b} \\ \hat{\omega}_{z_b} \end{bmatrix} = \boldsymbol{B}_p \left(\begin{bmatrix} \omega_{x_p} \\ \omega_{y_p} \\ \omega_{z_p} \end{bmatrix} - \begin{bmatrix} n_1 \sin\sigma_2 \\ n_1\cos\sigma_2 \\ n_2 \end{bmatrix} \right). \tag{3}$$

The device value of body direction cosine matrix $\hat{\boldsymbol{B}}_b$ is sought as a solution to the Poisson equation

$$\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b \hat{\boldsymbol{\Omega}}_b, \tag{4}$$

where

$$\hat{\boldsymbol{\Omega}}_b = \begin{bmatrix} 0 & -\hat{\omega}_{z_b} & \hat{\omega}_{y_b} \\ \hat{\omega}_{z_b} & 0 & -\hat{\omega}_{x_b} \\ -\hat{\omega}_{y_b} & \hat{\omega}_{x_b} & 0 \end{bmatrix}.$$

For an analytical evaluation of orientation errors, we will write this expression in the form

$$\dot{\hat{\boldsymbol{B}}}_b = \hat{\boldsymbol{B}}_b \left(\boldsymbol{\Omega}_b + \boldsymbol{B}_p \boldsymbol{\Omega}_d \boldsymbol{B}_p^{-1} \right), \tag{5}$$

where

$$\boldsymbol{\Omega}_d = \begin{bmatrix} 0 & -\Omega_{dz} & \Omega_{dy} \\ \Omega_{dz} & 0 & -\Omega_{dx} \\ -\Omega_{dy} & \Omega_{dx} & 0 \end{bmatrix}.$$

Considering the estimation error $\Delta_{\boldsymbol{B}_b}$ of the "ideal" matrix $\boldsymbol{B}_b$ to be small, we write

$$\hat{\boldsymbol{B}}_b = (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \boldsymbol{B}_b, \tag{6}$$

where

$\Delta_{\boldsymbol{B}_b} = \begin{bmatrix} 0 & -\varepsilon_z & \varepsilon_y \\ \varepsilon_z & 0 & -\varepsilon_x \\ -\varepsilon_y & \varepsilon_x & 0 \end{bmatrix}$ is the small skew-symmetric matrix of errors, which is recorded in the coordinate system $OX_oY_oZ_o$.

Then we will write it down

$$\begin{aligned} &\dot{\Delta}_{\boldsymbol{B}_b} \boldsymbol{B}_b + (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \dot{\boldsymbol{B}}_b \\ &\quad = (\boldsymbol{I} + \Delta_{\boldsymbol{B}_b}) \boldsymbol{B}_b \left(\boldsymbol{\Omega}_b + \boldsymbol{B}_p \boldsymbol{\Omega}_d \boldsymbol{B}_p^{-1} \right) \end{aligned}. \tag{7}$$

Neglecting the product of errors, we will have the following expression

$$\boldsymbol{\Omega}_\varepsilon = \boldsymbol{B} \boldsymbol{\Omega}_d \boldsymbol{B}^{\mathrm{T}}. \tag{8}$$

where $\boldsymbol{\Omega}_\varepsilon = \dot{\Delta}_{\boldsymbol{B}_b}$, $\boldsymbol{B} = \boldsymbol{B}_b \boldsymbol{B}_p$ ~ the transition matrix from the coordinate system $OX_pY_pZ_p$ to the coordinate system $OX_oY_oZ_o$.

This expression can be thought of as changing the transformation when the basis changes

Matrix equation (8) corresponds to a vector equation

$$\dot{\boldsymbol{\varepsilon}} = \boldsymbol{B} \boldsymbol{\omega}_d, \tag{9}$$

where $\boldsymbol{\varepsilon} = [\varepsilon_x\ \varepsilon_y\ \varepsilon_z]^{\mathrm{T}}$ is the error vector in the coordinate system. $OX_oY_oZ_o$.

Using an expression (9) simplifies the solution of the problem, since in this case the analysis of the movement of the object is performed before the analysis of errors, while when using an equation (4), these tasks are studied together.

That is, for constant gyroscope errors, the task is reduced to matrix $\boldsymbol{B}$ integration, and for a stationary object – to matrix $\boldsymbol{B}_p$ integration.

The integral of the matrix $\boldsymbol{B}_p$ is equal to

$$J_1 = \begin{bmatrix} \frac{1}{2}\left(\frac{\sin(n_1-n_2)t}{n_1-n_2}+\frac{\sin(n_1+n_2)t}{n_1+n_2}\right) & \frac{1}{2}\left(\frac{\cos(n_1+n_2)t-1}{n_1+n_2}-\frac{\cos(n_1-n_2)t-1}{n_1-n_2}\right) & -\frac{\cos n_1 t-1}{n_1} \\ -\frac{\cos n_2 t-1}{n_2} & -\frac{\sin n_2 t}{n_2} & 0 \\ \frac{1}{2}\left(\frac{\cos(n_1+n_2)t-1}{n_1+n_2}+\frac{\cos(n_1-n_2)t-1}{n_1-n_2}\right) & \frac{1}{2}\left(\frac{\sin(n_1-n_2)t}{n_1-n_2}-\frac{\sin(n_1+n_2)t}{n_1+n_2}\right) & \frac{\sin n_1 t}{n_1} \end{bmatrix}.$$

The matrix of time-independent components is

$$J_2 = \begin{bmatrix} 0 & \frac{n_2}{n_1^2-n_2^2} & \frac{1}{n_1} \\ \frac{1}{n_2} & 0 & 0 \\ -\frac{n_1}{n_1^2-n_2^2} & 0 & 0 \end{bmatrix}. \qquad (10)$$

From these matrices, we can see that the angular velocities of rotation n_1 and n_2 must be different.

The presence of odd powers of rotation frequencies indicates the feasibility of using reverse rotation to increase the effectiveness of this method of self-compensation of errors

Note that the matrix with constant components is of great importance when calculating velocities and coordinates in INS due to the projection of gravitational acceleration on the sensitivity axes of accelerometers.

To assess the accuracy of this method will accept

$$\omega_{dx} = \omega_{dy} = \omega_{dz} = 10,^0/h, \quad n_1 = 1c^{-1}, \quad n_2 = 2c^{-1},$$
$$n_2 = 2c^{-1}, \quad \omega_{x_b} = 1\cos 0.1t,^0/s, \ \omega_{y_b} = 2\cos 0.2t.^0/s,$$
$$\omega_{z_b} = 3\cos 0.3t,^0/s.$$

Errors in the absence of rotations according to the equation (4) are shown in Fig. 3.

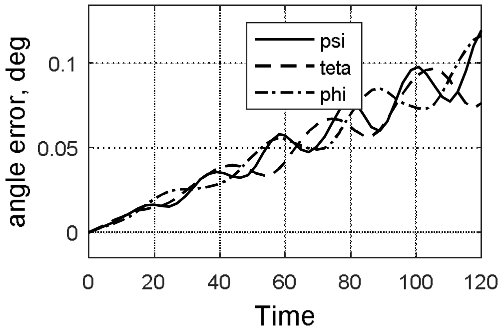

Fig. 3. Errors of angles

Errors are defined as the difference between the values of the angles in the presence of gyros errors and the values in their absence.

Errors in the presence of rotations are shown in Fig. 4.

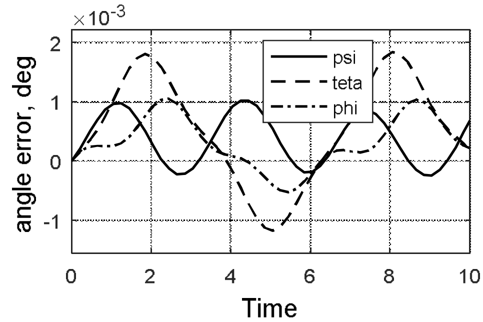

Fig. 4. Errors of angles

The errors presented in Fig. 5 are determined in the presence of rotations by the formula (9)

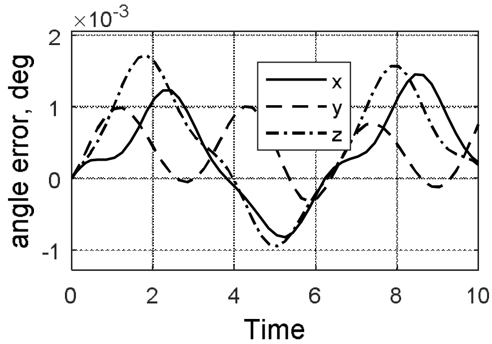

Fig. 5. Errors of angles

We see that during rotation there is no unlimited growth of errors over time.

Also we see expediency of using an approximate formula (9) for performing analysis.

IV. Conclusion

The high efficiency of using the rotation of sensitive INS elements to reduce the influence of their constant errors on the accuracy of determining navigation parameters is shown.

A further improving the accuracy can be ensured by increasing the angular velocities of rotation and using of rotation reversal.

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Л. М. Рижков, М. Г. Черняк. Підвищення точності визначення орієнтації на основі автокомпенсації похибок

Розглянуто підвищення точності визначення орієнтації безплатформною системою на основі просторового обертання гіроскопів відносно об’єкта. Предметом аналізу є математична модель похибки вимірювання кутів орієнтації з урахуванням постійних зсувів гіроскопів. Запропонована модель похибок дозволяє досить легко оцінити ефективність автокомпенсації. Показано, що задачу можна звести до інтегрування матриці напрямних косинусів, які визначають положення чутливих елементів у системі відліку координат. Результати теоретичного аналізу та моделювання підтверджують високу ефективність даного методу для підвищення точності вимірювання кінематичних параметрів.

Ключові слова: автокомпенсація; обертання; помилки.

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