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EXPERIMENTAL RESEARCH OF INTEGRATED NAVIGATION SYSTEMS IN PROBLEMS OF QUADROTOR PATH TRACKING

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Abstract**—***The article deals deals with experimental research of the integrated navigation, guidance, and control system of quadrotor. The mathematical grounds of the study of the system are represented. Algorithm of functioning the integrated system is represented. Equations of the optimal estimation are given. Features of tested equipment are described. The quadrotor trajectory in the horizontal plane is simulated. Coordinates of the topocentric reference frame are researched. The results of the experimental test are shown. The components of velocity are represented. The graphical dependences on the yaw, pitch, and roll are shown. The obtained results can be useful for creating perspective navigation systems for quadrotors. They allow to improve the quality of quadrotor navigation.*

Index Terms—Global navigation satellite system; inertial navigation system; guidance and control system; quadrotor; experimental research; flight test.

I. INTRODUCTION AND PROBLEM STATEMENT

Unmanned aerial vehicles (UAVs) or drones are characterized by low cost and ease of operation, which leads to broad prospects for their use for remote sensing, mapping, monitoring and other civilian applications [1] – [4].

However, the complex conditions of their operation and the peculiarities of their application require solving the problems of ensuring high accuracy of trajectory control and stabilization during stochastic and deterministic perturbations. These functions are carried out by the integrated navigation, guidance, and control system.

It is impossible to design such a system without system analysis. System analysis of complex systems represents integration of mathematical, statistical, experimental and other methods [5] – [9].

The aim of this paper is to present some results of testing an integrated navigation, guidance and control system for the quadrotor.

Relatively simple trajectory is selected here for validation, consisting of linear segments, 90-degree turns, climb and descent during straight segment flight.

II. MATHEMATICAL BACKGROUND

This article presents an option of integrated navigation system, where an inertial navigation system (INS) is corrected by Global Navigation Satellite System (GNSS), barometric altimeter and magnetometer. The quaternion $\delta q(t_i)$ representing a small turn of the rigid body at time interval Δt, is approximated by incremental angles vector, and the incremental angles are estimated using the quadratic spline-approximation as was proposed in [10] – [12]:

$$\delta q(t_i)=\begin{bmatrix} 1-\frac{1}{12}\left\|\nabla\theta_i\right\|^2 \\ \frac{1}{2}\nabla\theta_i-\frac{1}{24}\left(\nabla\theta_i\times\nabla\theta_{i-1}\right)\end{bmatrix}, \qquad (1)$$

$$\nabla\theta_i=\frac{\Delta t}{12}\left[5\omega(t_i)+8\omega(t_{i-1})-\omega(t_{i-2})\right].$$

Using (1) and multiplication of the elementary quaternions gives an updated quaternion, from which the Euler angles can be calculated [13]:

$$q(t_i)=q(t_{i-1})\delta q(t_i),$$
$$\delta q(t_i)=\begin{bmatrix}\delta q_0(t_i) & \delta q_1(t_i) & \delta q_2(t_i) & \delta q_1(t_i)\end{bmatrix}^{\mathrm{T}},$$
$$\varphi=\arctan\left(\frac{2(q_2q_3+q_0q_1)}{q_0^2-q_1^2-q_2^2+q_3^2}\right), \qquad (2)$$
$$\theta=\arcsin\left(-2(q_1q_3-q_0q_2)\right),$$
$$\psi=\arctan\left(\frac{2(q_1q_2+q_3q_0)}{q_0^2+q_1^2-q_2^2-q_3^2}\right).$$

The specific forces are transformed in the navigation frame by means of updated quaternion and are corrected to gravitational and centripetal accelerations [13], [14]

$$\tilde{a}=q\otimes a_m^{body}\otimes q^*+g-\Omega\times\Omega\times r, \qquad (3)$$

where q^* is a complex conjugate of q.

Velocity and position are calculated using an approximation similar to the one used for incremental angles calculation [13]:

$$v(t_i)=v(t_{i-1})+\left(5\tilde{a}(t_i)+8\tilde{a}(t_{i-1})-\tilde{a}(t_{i-2})\right)\cdot\frac{\Delta t}{12},$$
$$r(t_i)=r(t_{i-1})+\left(3\tilde{a}(t_i)+10\tilde{a}(t_{i-1})-\tilde{a}(t_{i-2})\right) \qquad (4)$$
$$\cdot\frac{\Delta t^2}{24}+\Delta t\cdot v(t_{i-1}).$$

Such estimation of velocity and position significantly decreases computational workload. Expressions (1) – (4) represent the algorithm of a functioning integrated system.

The inertial navigation system is corrected from GNSS receiver, magnetometer, and barometric altimeter using Kalman filtering equations. Vector of INS errors is defined as: μ is an orientation error, $\delta\mathbf{v},\delta\mathbf{r}$ are the vectors of velocity and position calculation errors. To save the place, the full derivation of the used equations is not given here. More details can be found in [13], [15], [16]. The correction task is solved as:

$$\hat{\mathbf{x}}_k=\bar{\mathbf{x}}_k+\mathbf{K}_k\left(\mathbf{z}_k-\mathbf{H}\bar{\mathbf{x}}_k\right),\quad \bar{\mathbf{x}}_{k+1}=\Phi_k\hat{\mathbf{x}}_k, \qquad (5)$$

In addition to (5), Kalman filtering equations to provide the vector of the optimal estimate $\hat{\mathbf{x}}_k$ are:

$$K_k=P_k(-)H^{\mathrm{T}}\left(HP_k(-)H^{\mathrm{T}}+R_k\right)^{-1},$$
$$P_{k+1}(-)=\Phi_kP_k(+)\Phi_k'+Q_k^{\mathrm{T}}, \qquad (6)$$
$$P_k(+)=P_k(-)-K_k\left(HP_k(-)H^T+R_k\right)K_k^{\mathrm{T}},$$

matrices Q_k, R_k in (6) are the covariance matrices of errors in INS error propagation equation and measurement equation. Features of designing guidance and control systems are represented in [17] – [20].

III. EXPERIMENTAL VALIDATION

A. Description of experiment set up and equipment used

A flight test was performed in October in Kyiv. Weather conditions were the following: cloudy, air temperature about 8 C, humidity about 80 %, wind-south-west, about 2 m/s.

In the presented experiment the task was to fly along the piecewise linear trajectory specified by 8 waypoints (Fig. 1). Sections WP1-WP2, WP5-WP6, WP6-WP7, WP7-WP8, WP8-WP1 are straight-line segments that has to be flown with a constant speed 5 m/s and a relative height 50 m. Section WP2-WP3 has to be performed with smooth descent from 50 to 30 m, WP3-WP4 – is a straight-line segment that has to be flown at a relative height 30 m with a constant speed 5 m/s, WP4-WP5 – a smooth climb from 30 to 50 m. The task was to fly along the given spatial trajectory 3 times, which was successfully performed by the control system.

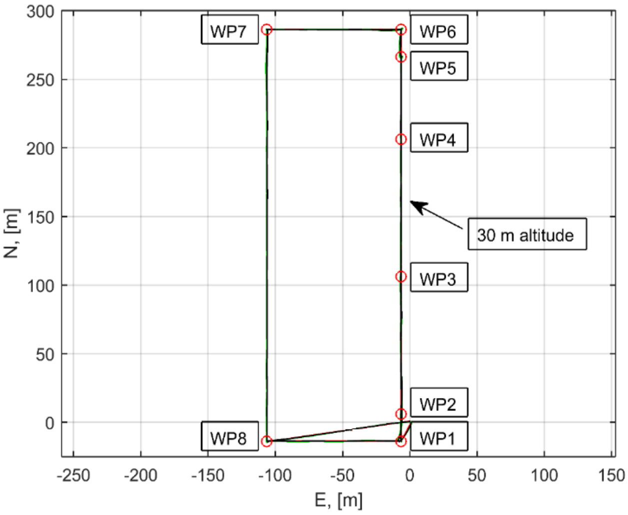


Fig. 1. Experimental trajectory of a quadrotor in the horizontal plane

The equipment used for experimental research (Figs. 2 and 3): quadrotor, designed experimental sample of integrated navigation, guidance and control system, double-frequency GNSS receivers "rover" and "base", GNSS antennas

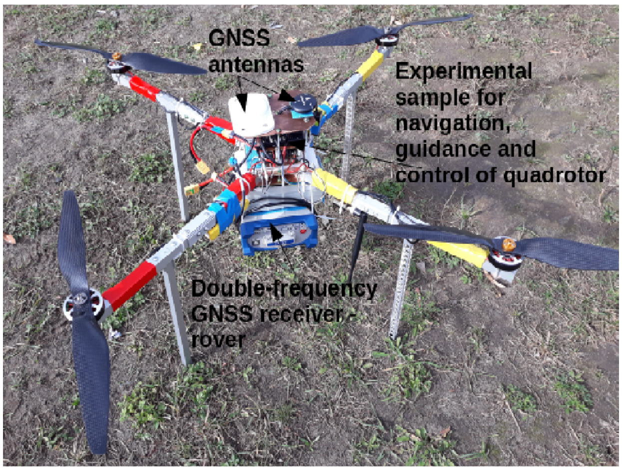


Fig. 2. Photo of drone and used equipment

The represented aids have been allowed to carry out the experimental research of the integrated navigation, guidance and control system.

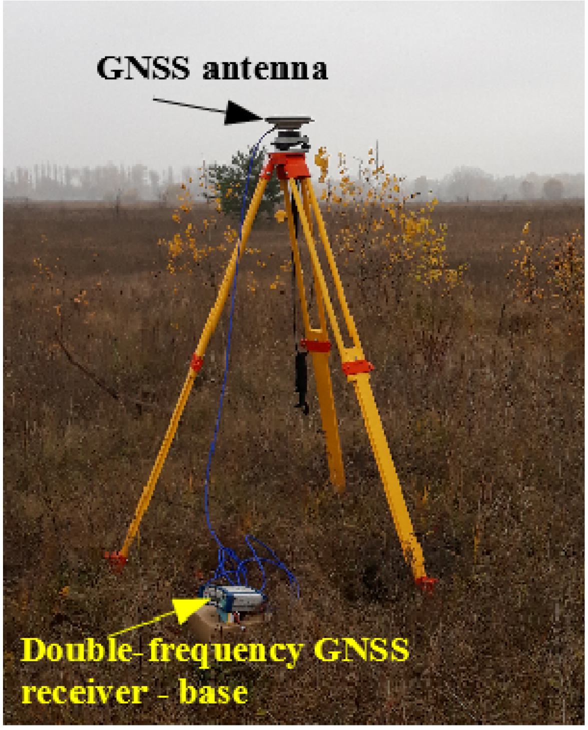


Fig. 3. Photo of GNSS equipment used for construction in post-processing of the reference trajectory: antenna and double-frequency receiver

B. Results of Experiment

In Figure 4, the enlarged fragments of the experimental trajectory of the quadrotor in the horizontal plane around waypoints WP4 and WP6 is shown. It appears that while moving along a straight line, the deviation of the quadrotor from a given trajectory was not more than 30 cm (WP4). However, we may observe so-called "loop" around WP6, which was caused by the fact that the quadcopter flew over point 4, and to return back to the trajectory, made a maneuver "loop". The same thing happened when turning at point WP6. Deviations from the specified trajectory were not more than 1.5 m, which is a pretty good result.

In Figures 5 and 6, the values of coordinates and velocity components in the local topocentric coordinate frame during the experiment are presented. The following color designations are accepted here: red – specified reference values, green – actually performed obtained from single GNSS, black – actually performed obtained from the integrated navigation system.

Figure 7 presents the values of relative altitude during the experiment from three sources: GNSS only (green line), barometric altimeter (blue line) and, the integrated navigation system (black line). It can also be seen that the relative altitude obtained from the integrated navigation system is more accurate and smoother compared to the ones obtained separately from GNSS and barometric altimeter.

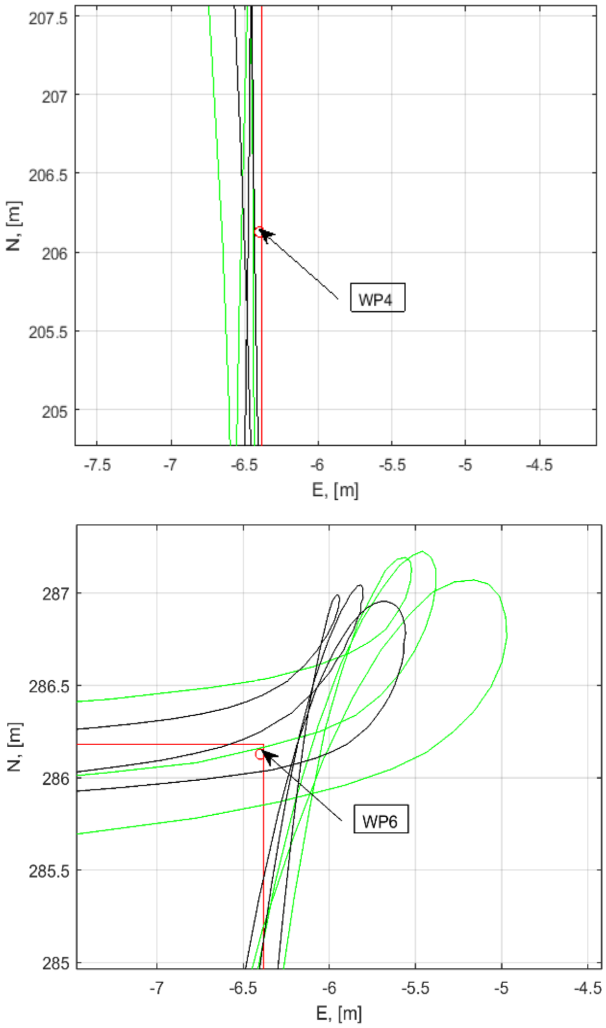


Fig. 4. Enlarged fragments of the experimental trajectory of drone in the horizontal plane: waypoints WP4 and WP6

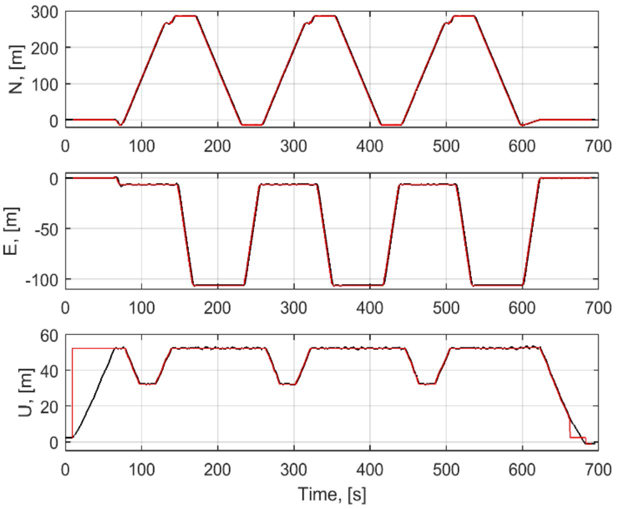


Fig. 5. Coordinates in the local topocentric coordinate frame: North, East, Up.

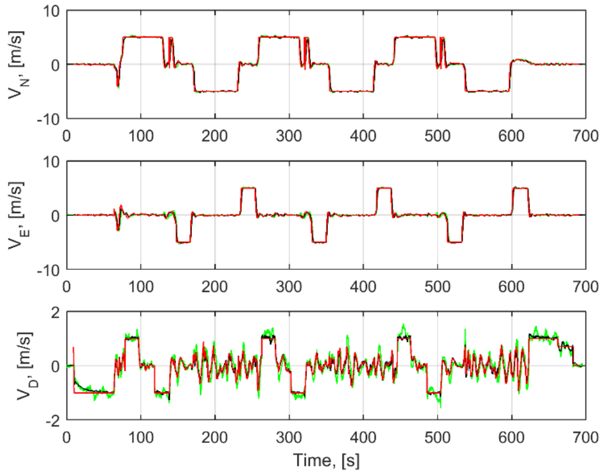


Fig. 6. Components of velocity: North, East, Down

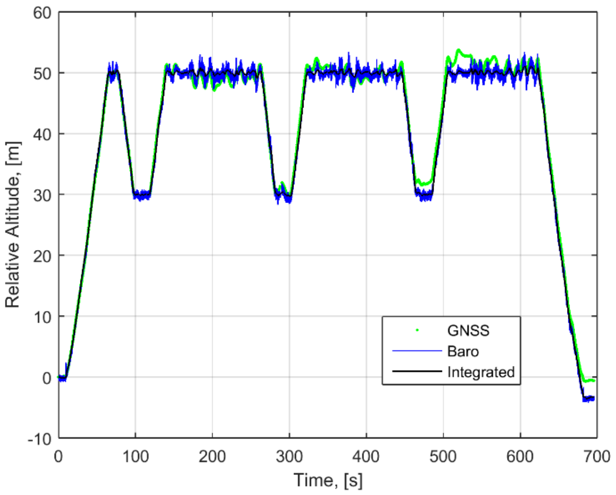


Fig. 7. Components of velocity: North, East, Down

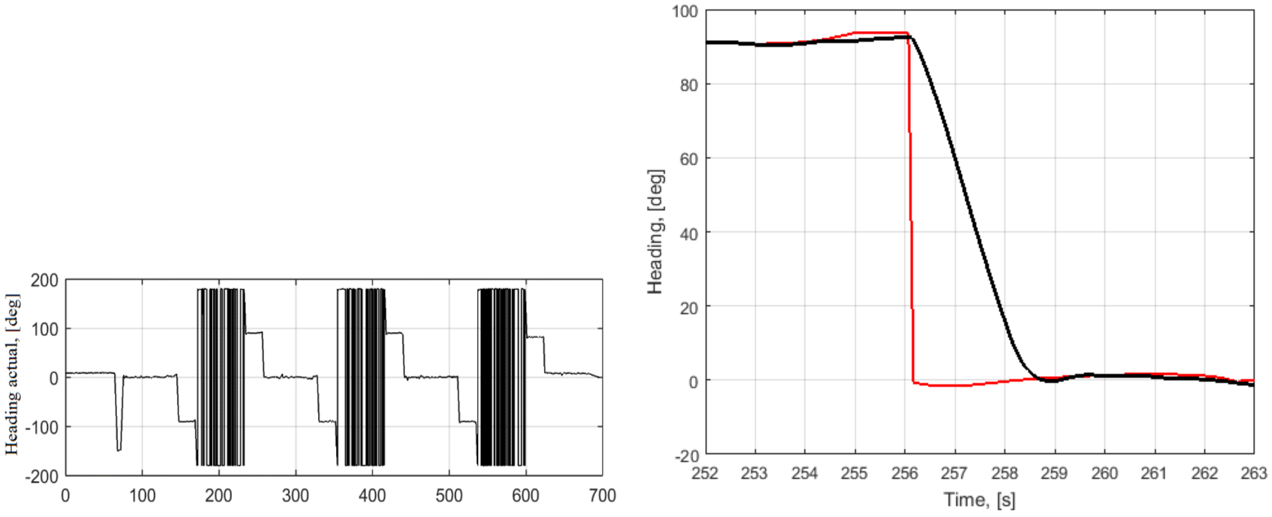
In Figures 8 – 10, the specified and actual values of heading, roll and pitch angles are presented.

The upper parts of the figures correspond to the actual obtained values during whole experiment, and the lower parts correspond to scaled angle’s values for the timeframe [252–263 s] timeframe of experiment.

During this time interval there was a change of heading from 90° to 0°. It appears that changing the heading by 90° requires about 4–5 s.

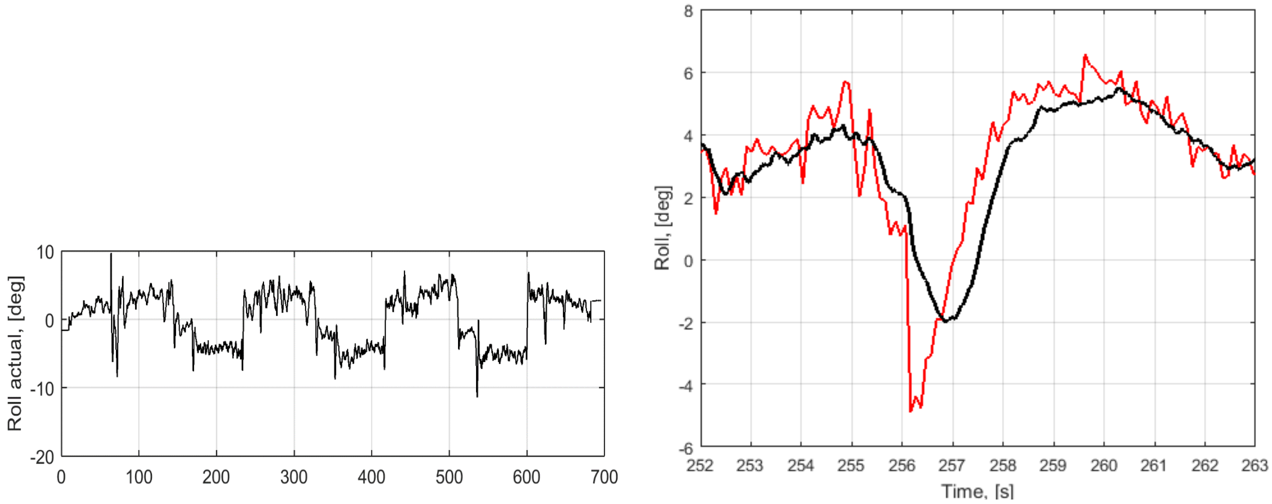
Also, heading change of the quadrotor requires to perform appropriate oscillating maneuvers for the roll and pitch angles.

We may see that in general the autopilot system worked out the set values quite accurately. The actual values of roll and pitch angles were mostly within ± 10° and ± 20° correspondently.



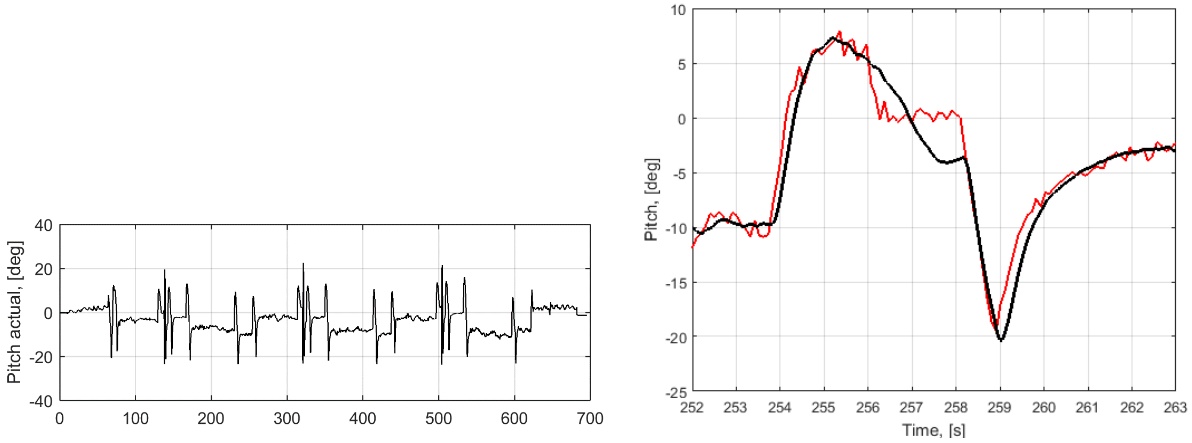
a) b)

Fig. 8. Heading: (a) during whole experiment; (b) scaled for timeframe (252–263 s)



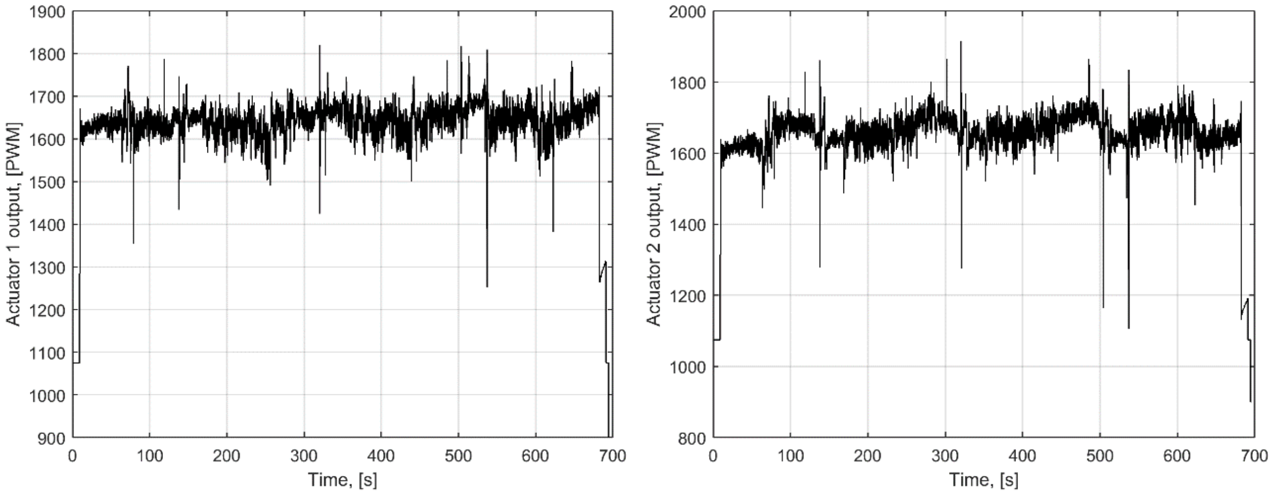
a) b)

Fig. 9.Roll: (a) during whole experiment; (b) scaled for timeframe (252–263 s)



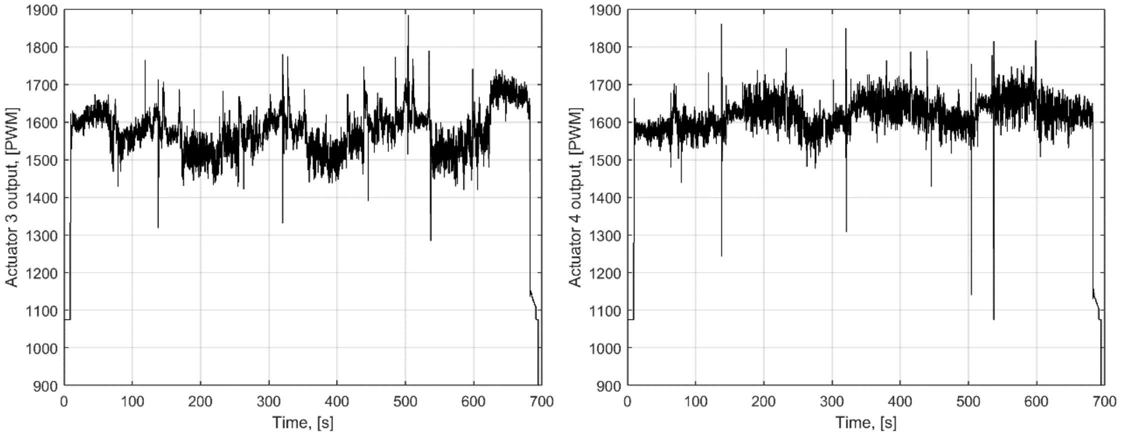
a) b)

Fig. 10. Pitch: (a) during whole experiment; (b) scaled for timeframe (252–263 s)



a) b)

Fig. 11. Pulse width modulation signals of actuators: (a) – (d) – for 1st, 2nd, 3rd, 4th actuators respectively



c) d)

Fig. 11. Ending. (See also p. 84)

Figure 11 presents the values of pulse width modulation (PWM) signals supplied separately for each actuator. We see that the PWM values were in the range of 900–1900. These limits may vary slightly depending on the settings of the control panel.

IV. CONCLUSIONS

The actual values of roll and pitch angles were mostly within ± 10° and ± 20° correspondently, which corresponds to quite safe and reliable mode.

We can conclude that the autopilot worked quite well, despite the noise in the sensors.

The represented results can be considered as components of the system analysis directed to improving quality of designing navigation and control system of the quadrotor.

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REFERENCES

[1] V. Kharchenko, V. Kondratyuk, S. Ilnytska, O. Kutsenko, and V. Larin, "Urgent problems of UAV navigation system development and practical implementation," *2013 IEEE 2nd International Conference Actual Problems of Unmanned Air Vehicles Developments Proceedings (APUAVD)*. Proceedings, 2013, pp. 157–160. https://doi.org/10.1109/APUAVD.2013.6705313

[2] S. Ilnytska, V. Kondratiuk, O. Kutsenko, and V. Konin, "Potential Possibilities of Highly Accurate Satellite Navigation Use for Landing Operations of Unmanned Aerial Systems," *2019 IEEE 5th International Conference Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)*. Proceedings, 2019, pp. 174–177, https://doi.org/10.1109/APUAVD47061.2019.8943873.

[3] I. V. Ostroumov, K. Marais, and N. Kuzmenko, “Aircraft positioning using multiple distance measurements and spline prediction,” *Aviation*, vol. 26, no. 1, pp. 1–10, 2022. https://doi.org/10.3846/aviation.2022.16589.

[4] Y. N. Bezkorovainyi and O. A. Sushchenko, "Improvement of UAV positioning by information of inertial sensors," *2018 IEEE 5th International Conference on Methods and Systems of Navigation and Motion Control (MSNMC)*. Proceedings, October 16-19, Kyiv, Ukraine, 2018, pp. 123–126. https://doi.org/10.1109/MSNMC.2018.8576307

[5] A. Dennis, B. H. Wixom, and R. M. Roth, *System Analysis and Design*, Wiley, 2014.

[6] C. S. Wasson, *System Analysis*, *Design,and Development: Concepts*, *Principles, and Practices***,** A Wiley-Interscience publication, 2005, 832 p. https://doi.org/10.1002/0471728241

[7] I. V. Ostroumov, V. P. Kharchenko, and N. S. Kuzmenko, "An airspace analysis according to area navigation requirements," *Aviation*, vol. 23, issue 2, 2019, pp. 36–42. https://doi.org/10.3846/aviation.2019.10302

[8] I. Ostroumov and N. Kuzmenko, “Configuration Analysis of European Navigational Aids Network,” *2021 Integrated Communications Navigation and Surveillance Conference (ICNS)*. Proceedings, 2021, pp. 1–9, https://doi.org/10.1109/ICNS52807.2021.9441576.

[9] O. A. Sushchenko, Y. M. Bezkorovainyi, and V. O. Golytsin, “Processing of redundant information in airborne electronic systems by means of neural networks,” *IEEE 39th International Conference on Electronics and Nanotechnology, ELNANO-2019*. Proceedings, Kyiv, Ukraine, April 16-18, 2019, pp. 652–655. https://doi.org/10.1109/ELNANO.2019.8783394

[10] R. Beard, *Quadrotor Dynamics and Control Rev 0.1*, 2008. Available at: https://scholarsarchive.byu.edu/facpub/13252.

[11] Swee King Phang, Chenxiao Cai, Ben M. Chen, and Tong Heng Lee, "Design and Mathematical Modeling of a 4-Standard-Propeller (4SP) Quadrotor," *The 10th World Congress on Intelligent Control and Automation.* Proceedings, Beijing, China, 2012, pp. 3270–3275. https://doi.org/10.1109/WCICA.2012.6358437

[12] T.-S. Tsay, "Guidance and Control Laws for Quadrotor UAV," *WSEAS Transactions on Systems and Control*, vol. 9, 2014, pp. 606–613.

[13] V. Kondratiuk, V. Konin, O. Kutsenko, et al., "Testing Static and Kinematic Modes of Precise Point Positioning Service in Ukraine," *Radioelectronic and Communication Systems,* vol. 62, pp. 530–540, 2019. https://doi.org/10.3103/S0735272719100054

[14] R. C. Leishman, J. C. Macdonald, R. W. Beard, T. W. McLain, Quadrotors and Accelerometers. *IEEE Control Systems Magazine*, February 2014, pp. 28–41.

[15] V. B. Larin and A. A. Tunik, "Synthesis of the Quad-Rotor Control Algorithms in the Basic Flight Modes," *TWMS Journal of Pure and Applied Mathematics*", vol. 9, no. 2, 2018, pp. 147–158.

[16] V. B. Larin and A. A. Tunik, "On problem of synthesis of control system for quadrocopter," *International Applied Mechanics*, vol. 53, no. 3, 2017, pp. 342–348. https://doi.org/10.1007/s10778-017-0816-4

[17] A. A. Tunik, "Simplified path tracking control laws for quad-rotor considered as nonholonomic system," *2018 IEEE 5th International Conference on Methods and Systems of Navigation and Motion Control (MSNMC)*. Proceedings, 2018, pp. 83–89, https://doi.org/10.1109/MSNMC.2018.8576316.

[18] P. Castillo, R. Lozano, and A. Dzul, "Stabilization of a mini rotorcraft with four rotors," *IEEE Control Systems Magazine*, December 2005, pp. 45–55. https://doi.org/10.1109/MCS.2005.1550152

[19] A. Tunik, S. Ilnytska and O. Sushchenko, "LMI-based synthesis of quadrotor guidance and control system," *2019 IEEE 5th International Conference Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)*. Proceedings, 2019, pp. 112–116. https://doi.org/10.1109/APUAVD47061.2019.8943927

[20] G. M. Hoffman, S. L. Waslander, and C. J. Tomlin, "Quadrotor helicopter trajectory tracking control," *AIAA Guidance, Navigation and Control Conference and Exhibition*, 18-21 August 2008, Hawaii, Honolulu, pp. 1–14. https://doi.org/10.2514/6.2008-7410

[21] S. Boyd, L. El Ghaoui, E. Feron, and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, Philadelphia: PA SIAM, 1994, 416 p. https://doi.org/10.1137/1.9781611970777

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В. М. Кондратюк, С. І. Ільницька, О. В. Куценко, О. А. Сущенко, М. В. Кондратюк, О. В Семененко. Експериментальне дослідження інтегрованих навігаційних систем в задачах відстеження траєкторій квадрокоптерів

У статті представлено результати експериментальних досліджень інтегрованої системи навігації, наведення та керування квадрокоптера. Представлені математичні основи дослідження системи. Представлено алгоритм функціонування інтегрованої системи. Наведено рівняння оптимального оцінювання. Описано особливості експериментального обладнання. Змодельована траєкторія квадрокоптера в горизонтальній площині. Досліджено координати топоцентричної системи координат. Наведено результати експериментальних досліджень. Представлені компоненти швидкості. Наведено графічні залежності кутів рискання, тангажу та крену. Отримані результати можуть бути корисними для створення перспективних навігаційних систем для квадрокоптерів. Вони дозволяють підвищити якість їх навігації.

Ключові слова: глобальна навігаційна супутникова система; інерціальна навігаційна система; система наведення та управління; квадротор; експериментальні дослідження; льотні випробування.

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