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MATHEMATICAL MODEL OF NONORTHOGONAL MEASURING INSTRUMENT BASED ON TRIAXIAL MEMS GYROSCOPES

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Abstract**—***The paper deals with developing he mathematical model of nonorthogonal inertial measuring instrument based on triaxial MEMS gyroscopes. Both the model of the sensitive element and realization of the algorithm of transforming measurement information are represented. The model of the sensitive element takes into consideration measuring errors of typical MEMS gyroscope. The model is based on characteristics represented of technical description given by manufactures (Analog Device). The algorithm of information processing, which includes the restoration matrix, is developed. The table of direction guides is obtained. The effectiveness of the developed mathematical model is proved by using Simulink model, which takes into consideration elements inherent to real inertial measuring instruments. The results of researching accuracy and the possibility of increasing the dynamic range are represented.*

Index Terms—Cosine guides; inertial sensor; mathematical model; MEMS gyroscope; nonorthogonal measuring instrument; simulation.

I. INTRODUCTION AND PROBLEM STATEMENT

Nonorthogonal configurations of inertial sensors are now widespread [1], [2]. This is due to the ability to make navigational measurements more accurate and reliable with functional redundancy [3], [4]. In this case, it is necessary to convert the information measured by individual gyroscopes in the projection of the angular velocity of a moving object in the inertial coordinate system [1]

$$\mathbf{L} = \mathbf{H}\boldsymbol{\omega}, \tag{1}$$

where $\begin{bmatrix} l_1 & l_2 & \ldots & l_n \end{bmatrix}^{\mathrm{T}}$ is the angular velocity vector measured in an excessive orthogonal coordinate system; $\begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^{\mathrm{T}}$ are projections of the angle velocity of a moving object in the inertial coordinate system; n is the number of inertial sensors in the orthogonal configuration. In other words, the matrix **H** describes the relationship between measuring and inertial coordinate systems. Given expression (1), the ratio for calculating the angular velocity can be represented as [1], [2]

$$\boldsymbol{\omega} = \mathbf{H}^{-1}\mathbf{L}. \tag{2}$$

Expression (2) for determining the pseudo-inverted matrix using the Moore-Penrose algorithm takes the form [1], [2]

$$\mathbf{H}^{-1} = (\mathbf{H}^{\mathrm{T}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{T}} = [\mathbf{H}^{-1}_{\omega_x}\ \mathbf{H}^{-1}_{\omega_y}\ \mathbf{H}^{-1}_{\omega_z}]. \tag{3}$$

Expression (3) is used when modeling excessive nonorthogonal configurations of MEMS sensors in the design of navigation systems. The dynamics of the configuration is taken into account using the relationship (3). Strictly speaking, when creating a model it is necessary to take into account the digital output of modern MEMS sensors. Therefore, the quantization procedure must be performed during modelling [5], [6]. For simulation modeling of the measuring process in the excessive nonorthogonal configuration of several MEMS gyroscopes, it is desirable to develop a mathematical model of the sensor and an algorithm of measuring information processing. In a real measuring instrument, this algorithm is realised by a microprocessor or microcontroller. To create a mathematical model, it is important to choose the kind of inertial measuring instrument and the kind of nonorthogonal configuration [7], [8]. An important issue in creating a mathematical description is the choice of the number of axes of sensitivity of excessive nonorthogonal inertial meter. Inertial meters are divided into uniaxial, biaxial and triaxial according to the number of sensitive axes. It should be noted that the development of inertial sensors has been evolved from the point of view of increasing the number of axes of sensitivity or measuring axes [9], [10]. Nowadays, triaxial inertial meters are the most common. They are called initial measuring units. These devices ensure determination of the full spatial position of moving objects. To create a mathematical model, it is important to choose a device based on the requirements of a particular application. The already mentioned inertial measuring unit ADIS16488 was chosen to present the model. The mathematical model of one ADIS16488 is presented in Fig. 1 [11].

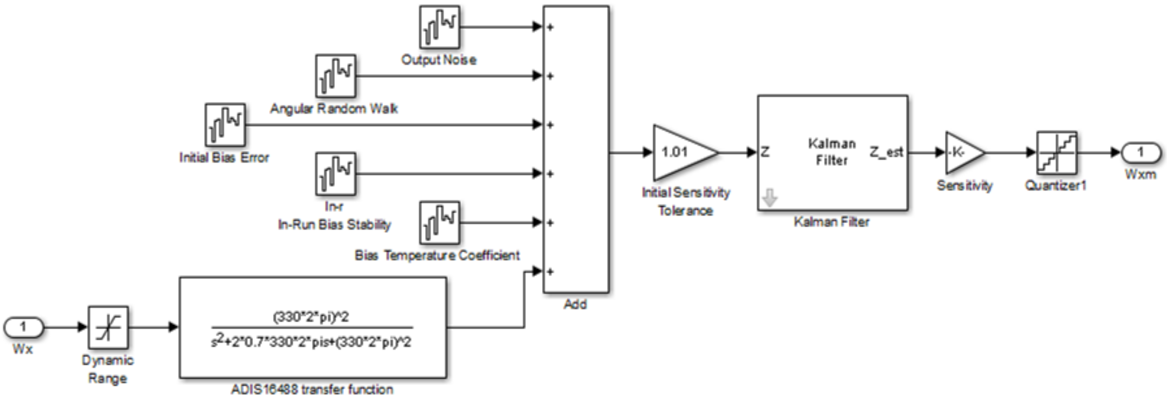


Fig. 1. Simulink model of one measuring channel MEMS gyroscope

The model presented in Fig. 1, contains a linear part in the form of a transfer function and nonlinear components typical of operated measuring units. These non-linearities contain limits on the measurement range (50 deg/s) corresponding to 10 V and the dead zone (0.015 deg/s or 0.00026 rad/s). The description also takes into account the output measurement noise of the sensor $1.5 \cdot 10^{-3}$ rad/s) [12].

The presented description also includes random errors such as output noise ($\sigma =$ 0.16 deg/s), the initial displacement ($\sigma = \pm$ 0.2 deg/s), stability of the primary displacement ($\sigma =$ 6.25 deg/h), temperature coefficient of displacement($\sigma = \pm$ 0.0025 deg/s/°C), which are represented as the white noise [13].

In addition, this model is somewhat simplified compared to that shown in Fig. 1 to avoid unnecessary complications when creating a multi-axis model [14].

II. MATHEMATICAL MODEL OF TRIAXIAL INERTIAL MEASURING UNIT

The inertial measurement unit model is based on a combination of measurement channels. The model of orthogonal excessive inertial meter based on MEMS gyroscopes is presented in Fig. 2.

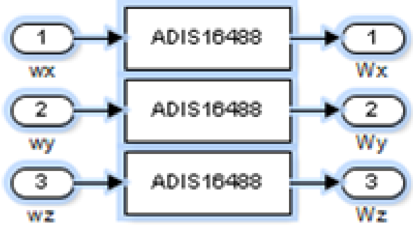


Fig. 2. Simulink model of orthogonal excessive inertial measuring unit

The presented description is grounded on the characteristics of the gyroscope ADIS16488 (manufactured by Analog Device), designed by MEMS technology. It must be recognized that the presented model is all-round and suitable for MEMS gyroscopes of any kind. The preference of the given model is the use of the characteristics of the real sensor provided by the manufacturer. Hence, the creation of this model does not need additional testing and re-examination with special equipment. It is important to note that the gyroscope ADIS16488 has built-in Kalman filters.

The selection of nonorthogonal configuration depends on the requirements for accuracy and weight and dimensions. For light and ultralight unmanned aerial vehicles, it is convenient to use a polyhedron-based configuration. The orientation of the sensitivity axes of the nonorthogonal configuration of the MEMS sensors selected for the study is presented in Fig. 3 [10].

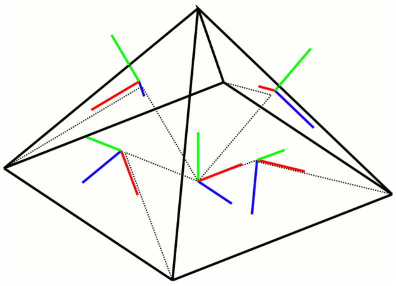


Fig. 3. Orientation of the sensitivity axes of the redundant device using a tetragonal pyramid as a structural uni

Colored lines in Fig. 3 show the direction of the axes of sensitivity of individual triaxial inertial measuring units arranged on the sides of the quadrangular pyramid.

Mathematical model of nonorthogonal excessive inertial meter based on MEMS gyroscopes and structural element in the form of a quadrangular pyramid. The simulink model of such a device is presented in Fig. 4. This model contains both the measurement algorithm and the implementation of the information processing algorithm [15], [16].

Basic peculiarities of nonorthogonal configurations are needs to convert the information about the measurements in the excessive measuring coordinate system into a navigation coordinate system. The measuring coordinate system is created by the axes of sensitivity of individual inertial units.

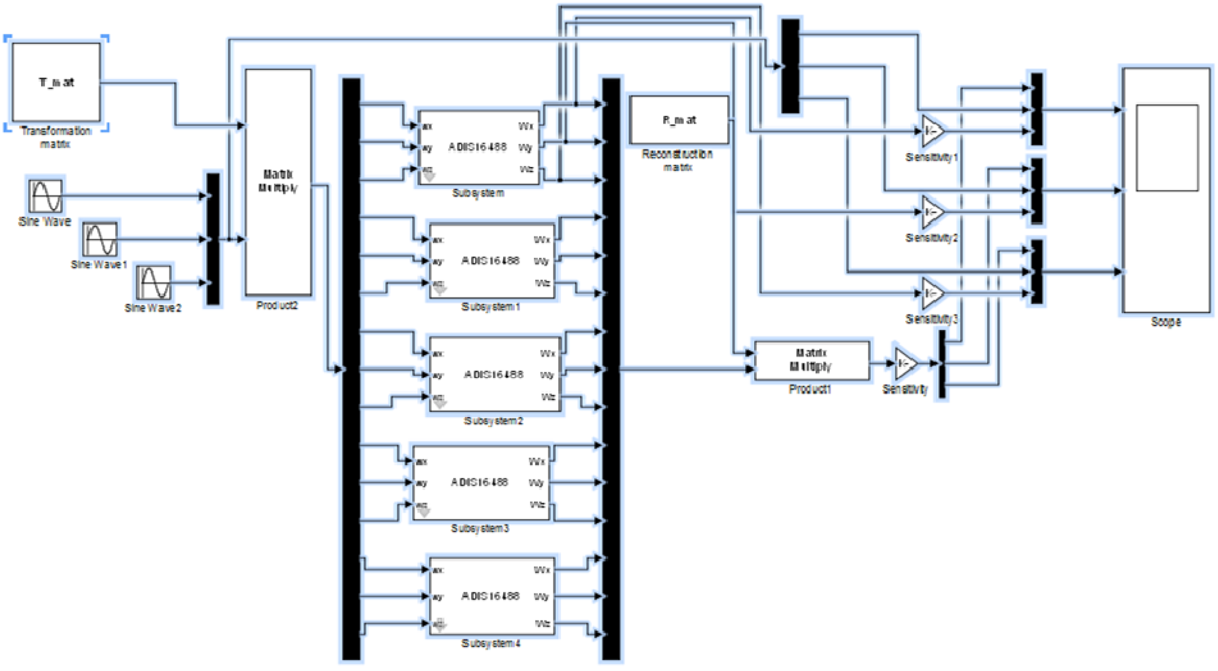


Fig. 4. Simulink model of nonorthogonal excessive inertial meter

A navigation coordinate system has been used to determine the angular velocity of a moving object. Differently, the measurements of individual units with nonorthogonally located axes are converted into angular velocities of a moving object regarding the navigation coordinate system.

The information processing algorithm contains transformations in measuring and navigation coordinate systems. Given this fact, the mathematical model includes the following components.

1) Setting the input signals when the sinusoidal signals are multiplied by the conversion matrix, which ensures the input of signals appropriate to the signals of individual inertial sensors.

2) Projections ω_x, ω_y, ω_z, kept in the first stage, are included in the description of the gyroscope ADIS16488, and the outputs Ω_x, Ω_y, Ω_z create excessive data of information measured by sensors.

3) Calculating projections of the angle velocity of a moving object is grounded on multiplexing the excess data of the measured signals, multiplied by the transformation matrix between the measuring and navigation coordinate systems.

4) The projections of angle velocities on the *x*, *y*, *z* axes are fed to the demultiplexer and then to the block of presentation of results. For appropriate processing the measuring results, the primary sine waves of angle velocities and 1st gyroscope outputs are also sent to the block of presentation of results. It is important to note that the matrix of guiding cosines of the measuring and navigation coordinate systems for 1st gyroscope is a diagonal identity matrix. This case ensures that the outputs of 1st gyroscope coincide with the angle velocities defined in the navigation coordinate system. Represent the conversion matrices in detail. Angle velocities measured by a nonorthogonal redundant measuring instrument can be represented in the form

$$\boldsymbol{\omega} = \begin{bmatrix} \omega_x^1 & \omega_y^2 & \omega_z^3 & \ldots & \omega_x^{n-2} & \omega_y^{n-1} & \omega_z^n \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

5) Angular velocities determined by a nonorthogonal redundant instrument can be represented in the form of projections **W** onto axes of the navigation coordinate system, you can use the ratio

$$\mathbf{W} = \begin{bmatrix} W_x & W_y & W_z \end{bmatrix} = \mathbf{H}^{\mathrm{T}} \boldsymbol{\omega}, \quad (5)$$

where **H** is the matrix of guiding cosines, which determines the relative position of the nonorthogonal measuring coordinate system and the orthogonal navigation coordinate system. Matrix **H** is characterized by dimension $n \times m$, matrix $\mathbf{H}^{\mathrm{T}}$ – $m \times n$ respectively.

The process of establishing the initial signals of an excessive nonorthogonal inertial meter is described by the expression

$$\omega_{in} = \mathbf{HS}, \tag{6}$$

where **S** is the vector of primary data, for example, sine waves.

Based on expressions (4) – (6), the recovery matrix can be represented as

$$\mathbf{D} = [\mathbf{H}^T\mathbf{H}]^{-1}\mathbf{H}^T, \tag{7}$$

The elements of the matrix of guide cosines (7) are presented in Table I.

It should be noted that the first three rows of Table I show the accordance of the measuring axes of 1st MEMS gyroscope to the navigation coordinate system. Output signals **Ω** are defined as

$$\mathbf{\Omega} = \mathbf{D}\omega, \tag{8}$$

III. SIMULATION RESULTS

The results of modeling the measuring signals (8) using the proposed mathematical model of excessive nonorthogonal meter based on inertial sensors (high-speed MEMS gyroscopes) are presented in Figs 5–7. Here are the measurement results for the three measurement channels.

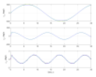
Each figure is represented by three diagrams that correspond to the inputs; measurement by a three-axis sensor and measurement by an excessive nonorthogonal meter. The results of the corresponding simulation based on the given sinusoidal signals are presented in Fig. 5.

Simulation based on given signals as a sequence of rectangular pulses is shown in Fig. 6.

The results presented in Figs 5 and 6 show an increase in the accuracy of measurements of excessive nonorthogonal meter compared to the measurements of a single inertial sensor.

The developed model also allows to study the possibility of improving the dynamic measuring range. The corresponding results are presented in Fig. 7. They correspond to a sufficient and moderate excess of the measuring range of a single inertial MEMS sensor. The study of increasing the dynamic range is of great importance for stochastic motion caused by external perturbations and deviations from the set values.

TABLE I. GUIDING COSINES

	ω_x	ω_y	ω_z
$d_1 = \omega_x^1$	1	 0	0
$d_2 = \omega_y^1$	0	1	0
$d_3 = \omega_z^1$	0	0	1
$d_4 = \omega_x^2$	– 0.289	– 0.817	– 0.5
$d_5 = \omega_y^2$	– 0.408	0.577	0.707
$d_6 = \omega_z^2$	0.866	0	0.5
$d_7 = \omega_x^3$	0.866	0	0.5
$d_8 = \omega_y^3$	– 0.408	0.577	– 0.707
$d_9 = \omega_z^3$	0.289	0.817	0.5
$d_{10} = \omega_x^4$	0.866	0	0.5
$d_{11} = \omega_y^4$	– 0.408	0.577	– 0.707
$d_{12} = \omega_z^4$	0.2887	0.817	0.5
$d_{13} = \omega_x^5$	– 0.866	0	0.5
$d_{14} = \omega_y^5$	– 0.408	0.577	– 0.707
$d_{15} = \omega_x^5$	– 0.2887	– 0.817	– 0.5

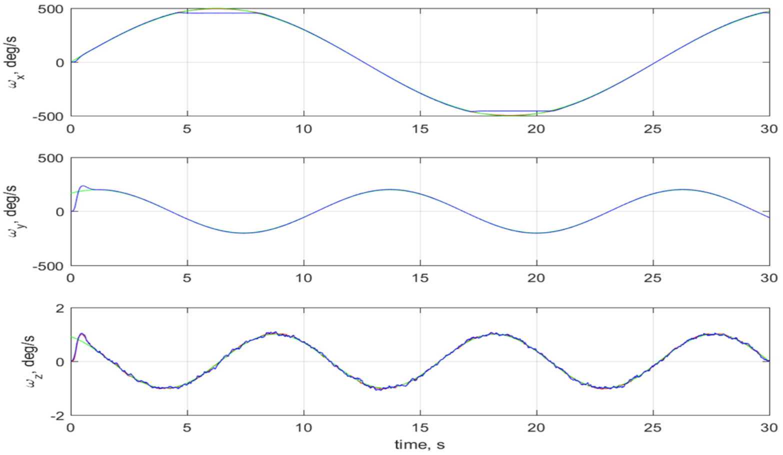


Fig. 5. Simulation of sinusoidal signal measurement process

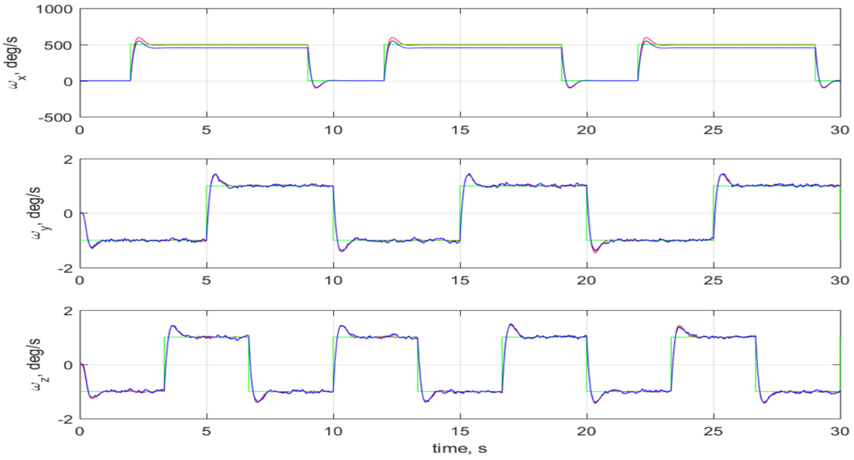


Fig. 6. Simulation of the process of measuring rectangular signal pulses

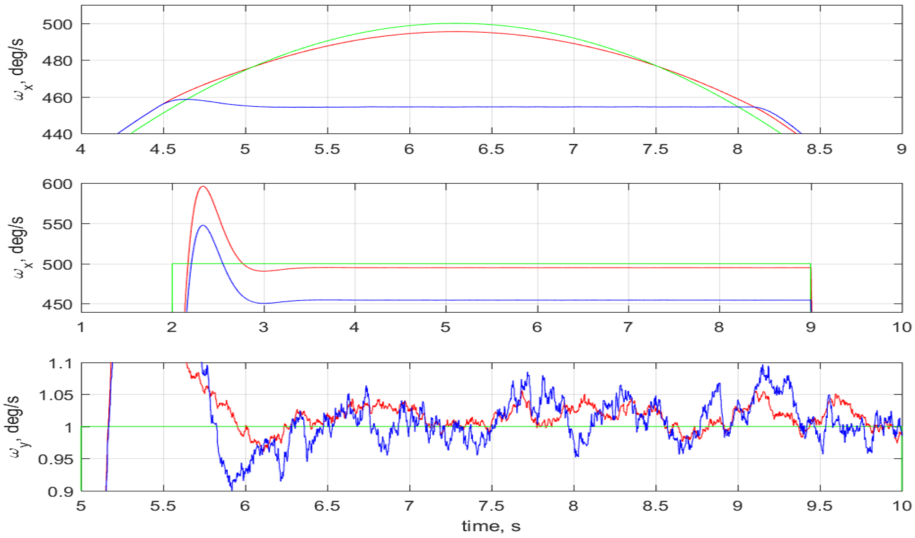


Fig. 7. Simulation of the possibility of increasing the dynamic range

IV. CONCLUSIONS

The mathematical model of nonorthogonal inertial measurement instruments using triaxial MEMS units has been developed and the appropriate conversion matrix using guide cosines has been developed.

Research of accuracy and improving dynamic range of nonorthogonal measurement devices based on three-axis MEMS sensors using the developed mathematical model has been carried out.

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В. О. Голіцин. Математична модель неортогонального вимірювача на основі триосних МЕМС-гіроскопів

У статті розглянуто розробку математичної моделі неортогонального інерціального вимірювального приладу на основі тривісних MEMS-гіроскопів. Представлено як модель чутливого елемента, так і реалізація алгоритму перетворення вимірювальної інформації. Модель чутливого елемента враховує похибки вимірювання типового MEMS-гіроскопа. Модель базується на характеристиках, представлених у технічних описах виробників (Analog Device). Розроблено алгоритм обробки інформації, що включає матрицю відновлення. Отримана таблиця напрямних косинусів. Ефективність розробленої математичної моделі доведена використанням моделі Simulink, яка враховує елементи, властиві реальним інерціальним вимірювальним приладам. представлено результати дослідження точності та можливості збільшення динамічного діапазону.

Ключові слова: спрямовуючі косинуси; інерціальний датчик; математична модель; MEMS-гіроскоп; неортогональний вимірювач; моделювання.

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В. О. Голицин. Математическая модель неортогонального измерителя на основе трехосных МЭМС-гироскопов

В статье рассмотрена разработка математической модели неортогонального измерительного инерциального прибора на основе трехосных MEMS-гироскопов. Представлены как модель чувствительного элемента, так и реализация алгоритма преобразования измерительной информации. Модель чувствительного элемента учитывает ошибки измерения типичного MEMS-гироскопа. Модель основана на характеристиках, представленных в технических описаниях производителей (Analog Device). Разработан алгоритм обработки информации, включающий матрицу обновления. Получена таблица направляющих косинусов. Эффективность разработанной математической модели доказана использованием модели Simulink, учитывающей элементы, присущие реальным инерциальным измерительным приборам. Представлены результаты исследования точности и возможности увеличения динамического диапазона.

Ключевые слова: направляющие косинусы; инерциальный датчик; математическая модель; MEMS-гироскоп; неортогональный измеритель; моделирование.

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