

DC 629.3.025.2(045)

DOI:10.18372/1990-5548.70.16771

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## FEATURES OF DESIGNING HIGH-PRECISION SYSTEMS FOR STABILIZATION AND DETERMINATION OF ATTITUDE AND HEADING

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**Abstract**—The paper deals with features of designing high-precision systems for stabilization and determination of attitude and heading reference systems. Features of the parametrical optimization are considered. The necessity to use the robust control laws is grounded. The mathematical model of the studied system in the preliminary levelling mode is represented. Simulink model of the navigation loop is shown. The necessity and features of the model's linearization are explained, Basic concepts of creating models directed for solution of the studied problem are represented. The design process is considered on the example of the precision gimballed navigation system assigned for operation on marine vehicles. The proposed approach to problem solution is accompanied with simulation. The simulation results prove efficiency of the described design procedure. The obtained results can be useful for creating stabilization and motion control systems of the wide class.

**Index Terms**—High-precision navigation system; parametrical optimization; design procedure; mathematical model; robust control; simulation.

### I. INTRODUCTION

Nowadays, urgency of developing new advanced systems for stabilization and determination of the course of moving objects is growing. For some applications, it is necessary to ensure the very high precision of navigation measurements using gimballed navigation systems. In this case, it is vital to ensure high requirements and resistance to external disturbances. Hence, such systems should be synthesized while considering their quality and robustness. Parametric optimization should be based on the system's state-space models [1], [2]. This approach is based on such powerful tools of automated optimal design of control systems as specialized advanced packages of MATLAB, namely, Control System Toolbox and Robust Control Toolbox [3] – [5]. To create the above-mentioned models, it is necessary to perform linearization of the kinematic relations in view smallness of the platform's turns [6], [7]. It is expedient also to ignore the difference of axial moments.

Thus, a major problem of the studied system for stabilization and heading determination is designing navigational loops and optimization of control laws [8].

### II. FEATURES OF PARAMETRIC OPTIMIZATION

To create a computational procedure of the parametric optimization, it is necessary to take into account features of the designed system. Firstly, the

studied system is an astatic system of the second order. So, to perform parametric optimization, it is necessary to determine the minimum realization. As a rule, such systems are characterized by sparse system matrices. Analysis of a system matrix shows that its elements can differ among themselves about three orders of magnitude. With this in mind, it makes sense to create a balanced implementation of the model.

Parametric optimization is performed in two stages. The first stage is synthesis of control laws based on the state-space models using automated design tools. The second phase verifies the results of the synthesis. It is based on mathematical model taking into account all the nonlinearities inherent in real navigation systems.

The procedure of parameters optimization consists of the following steps.

- 1) Creating a full nonlinear model of the system.
- 2) Making a mathematical description of the space of states to obtain the transfer function of the open-loop system.
- 3) Getting advanced model of the stochastic system with the inclusion of the forming filter.
- 4) Obtaining minimum realization of the model.
- 5) Scaling the model based on balanced realization.
- 6) Choosing initial values and carrying out the optimization by the genetic algorithm with cyclic performance of the steps.

7) Making conclusion of the ending parametric optimization, or it will continue with new initial conditions or new weight coefficients in the complex quality criterion.

Consider parametric optimization on the example of gimballed attitude and heading reference system assigned for operation on marine vehicles [9] – [11].

### III. MATHEMATIC MODEL OF SYSTEM IN PRELIMINARY LEVELING MODE

For design of the system, it is proposed to use the model of the external moment acting on the object of stabilization

$$M_{dis} = M_{st} + M_{nom} \text{sign} \omega_0, \quad (1)$$

where  $M_{dist}$  is the disturbance moment;  $M_{st}$  is a constant moment acting on the object of stabilization;  $M_{nom}$  is the nominal moment of stabilization engine;  $\omega_0$  is the external angular velocity acting on the platform [12].

Vector of the moving object's velocity in the geographic coordinate system taking into account the initial azimuth angle  $k_0 = A_0$  is shown in Fig. 1.

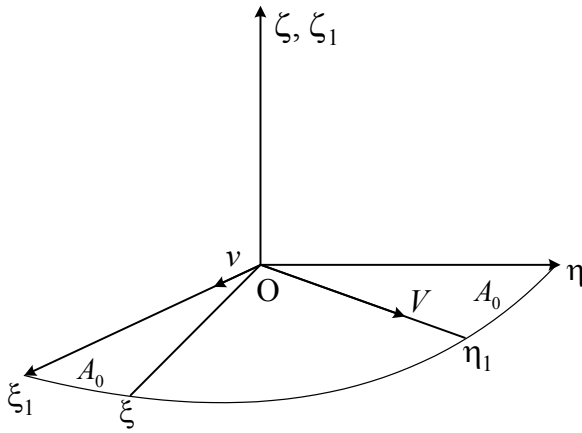


Fig. 1. The vector of velocity in the geographic coordinate system

In view of Fig. 1, expressions for the linear velocity tings take the form

$$\begin{aligned} V_N &= V \cos A_0, & V_E &= V \sin A_0, & v_n &= -v \sin A_0, \\ v_e &= v \cos A_0, & V_n &= V \cos k - \theta_k \omega_k l \cos \omega_k t \sin A_0, \\ V_e &= V \sin k + \theta_k \omega_k l \cos \omega_k t \cos A_0. \end{aligned} \quad (2)$$

In order to create the mathematical model of the system in the preliminary mode, it is necessary to carry out linearization of models taking into consideration small angles of the platform's turns and neglecting axial moments (2). The external influences (1) must be specified by linear dependencies also. Note that the angles of the platform's turns are really small, as in fact the vertical line is simulated in gimballed system. Thus, the linearized model of gimballed system for determination of attitude and heading takes the form:

$$\begin{aligned} \dot{\omega}_{xp} &= [-(f + k_7)\omega_{xp} - k_1\delta_1 - k_5(-\delta_1 + k_3\beta) / T + H\omega_{0x}] / J_x, \\ \dot{\omega}_{yp} &= [-(f + k_8)\omega_{yp} - k_2\delta_1 - k_6(-\delta_2 + k_4\alpha) / T + H\omega_{0y}] / J_y, \\ \dot{\omega}_{zp} &= [-(f + k_9)\omega_{zp} + k_{10}\gamma + H\omega_{0z}] / J_z, \\ \dot{\delta}_1 &= (-\delta_1 + k_3\beta) / T, \\ \dot{\delta}_2 &= (-\delta_2 + k_4\alpha) / T, \\ \dot{\beta} &= \omega_{xp}, \\ \dot{\alpha} &= \omega_{yp}, \\ \dot{\gamma} &= \omega_{zp}, \end{aligned} \quad (3)$$

where  $k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8, k_9, k_{10}$  are coefficients of control laws,  $\omega_{0x}, \omega_{0y}, \omega_{0z}$  are external angular velocities acting on the platform.

The model (3) can be represented in state space in the following way:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{Bu}; \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{Du}, \end{aligned}$$

where  $\mathbf{x}$  is the vector of state variables,  $\mathbf{u}$  is control vector. The matrices of the state-space model are given below.

$$\mathbf{A} = \begin{bmatrix}
 \frac{f+k_7}{J_x} & 0 & 0 & -\frac{k_1-k_5}{J_x} & 0 & -\frac{k_3k_5}{J_x} & 0 & 0 \\
 0 & -\frac{f+k_8}{J_y} & 0 & 0 & -\frac{k_2-k_6}{J_y} & 0 & -\frac{k_4k_6}{J_x} & 0 \\
 0 & 0 & -\frac{f+k_9}{J_z} & 0 & 0 & k_{10} & 0 & 0 \\
 0 & 0 & 0 & -\frac{1}{T} & 0 & \frac{k_3}{T} & 0 & 0 \\
 0 & 0 & 0 & 0 & -\frac{1}{T} & 0 & \frac{k_4}{T} & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix}
 \frac{H}{J_x} & 0 & 0 \\
 0 & \frac{H}{J_x} & 0 \\
 0 & 0 & \frac{H}{J_x} \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0
 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
 \end{bmatrix}.$$

The developed state-space models are adopted for using in procedures of robust parametrical optimization.

This is of great importance for designing systems operating under influence of the external disturbances. Usage of such an approach ensures calculating of  $H_2$  and  $H_\infty$ -norms, which can be used for assessment of precision and robustness of the designed system.

The simulation results of the designed system in the preliminary levelling model are represented in Figs 1 and 2.

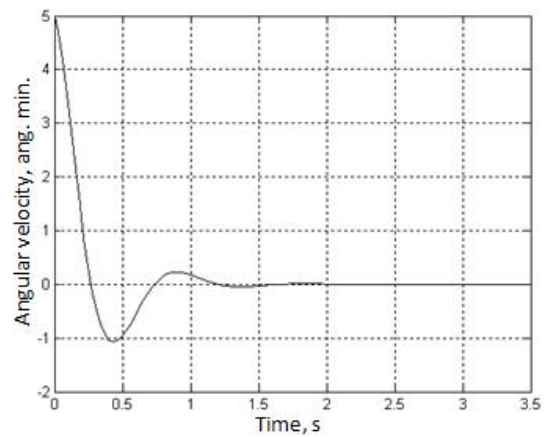


Fig. 1. The transient process of the angle  $\alpha$  in view of damping the angular velocities  $\dot{\delta}_1, \dot{\delta}_2$

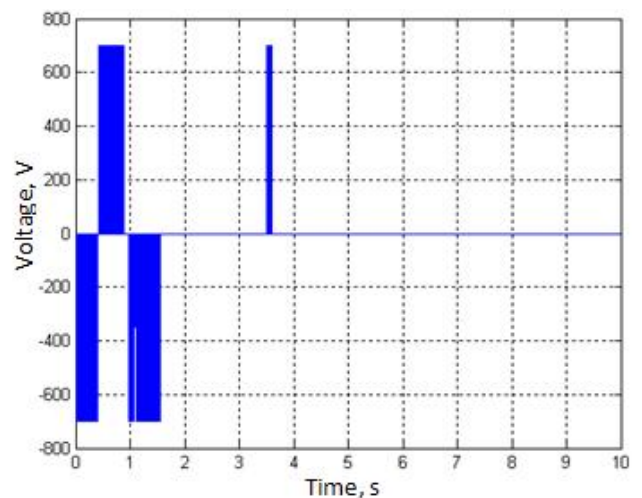


Fig. 2. The output signal of the pulse-width modulator

IV. MATHEMATICAL MODEL OF SYSTEM IN THE MODE OF HEADING DETERMINATION

An important problem in the creation of modern attitude and heading reference systems is the study of navigation contours and the optimization of control laws. For a system of the studied type, it is sufficient to use the parametric optimization, because a priori information about structure of control laws seems quite definite. There are two important tasks, namely: the need to simulate long-

term processes and the need to form complex control laws that can provide the expected accuracy. These circumstances require the maximum simplification of system device models, but all components of control laws are must be fully researched [13]. To simplify the model (see Fig. 2), it is assumed that the equations of motion of gyroscopes with the accuracy of the errors of the stabilization system coincide with the equations of motion of the platform [14].

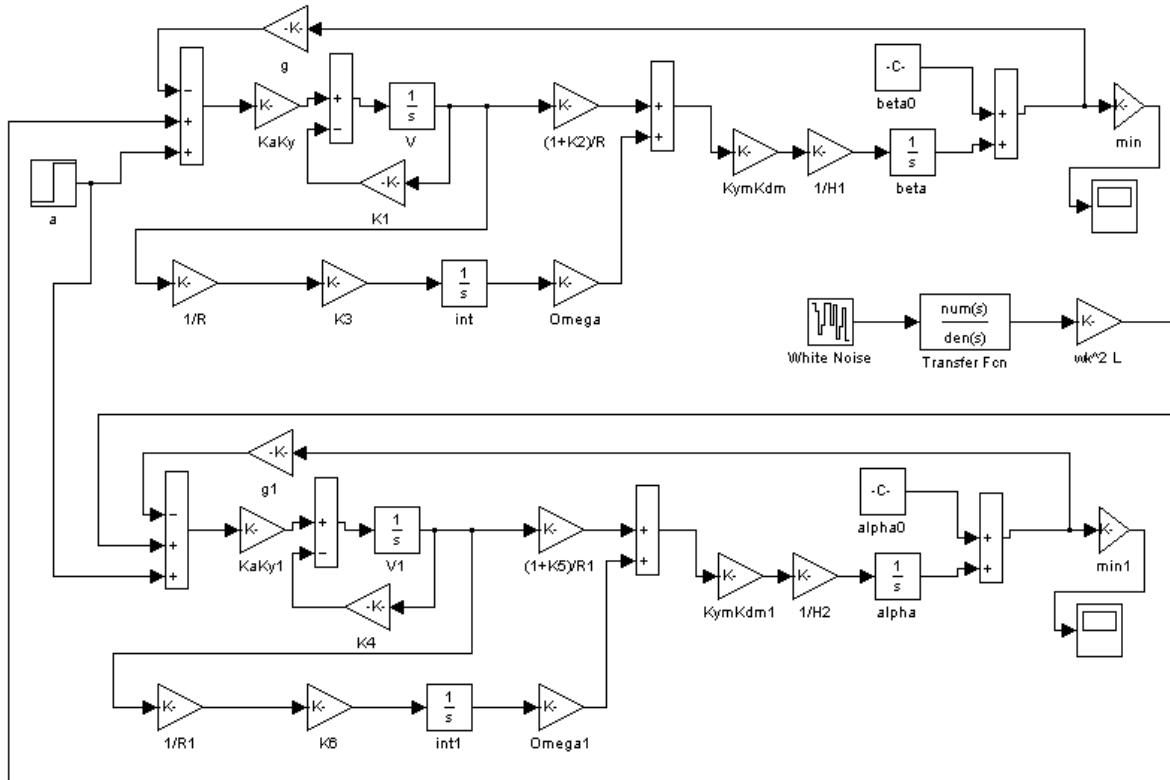


Fig. 2. Simulink-model of the navigation loop

In the navigation loop of the gyroscopic compass, control can be performed by means of integral correction, adjustment according to the information from the lag, as well as corrections to take into account the angular motion of the Earth and the moving object.

To simplify the parametric optimization of the system for stabilization and determining of the course, we consider the model taking into account channels of azimuth and roll. Results of modelling are represented in Fig. 3.

Obviously, the characteristics of the system will be somewhat worse in terms of real operation, as is proved by simulation using a model close to real systems.

To design the system able for functioning in conditions of influence of the external disturbances,

it is convenient to apply the robust parametrical optimization [4].

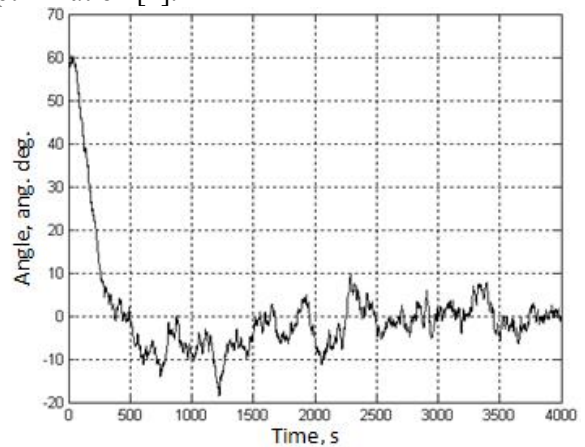


Fig. 3. Results of simulation

Efficiency of the robust parametric optimization is illustrated in Figs 4 and 5, where transient processes of non-optimized and optimized systems. The optimization has been based on using the complex criterion with  $H_2$  and  $H_\infty$ -norms of the deterministic and stochastic systems [4].

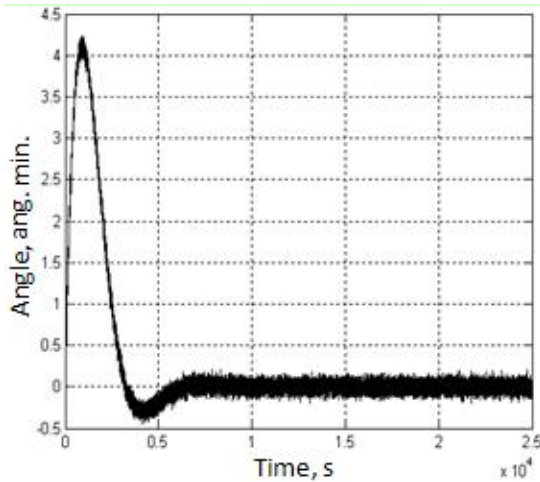


Fig. 4. Simulation of non-optimized system

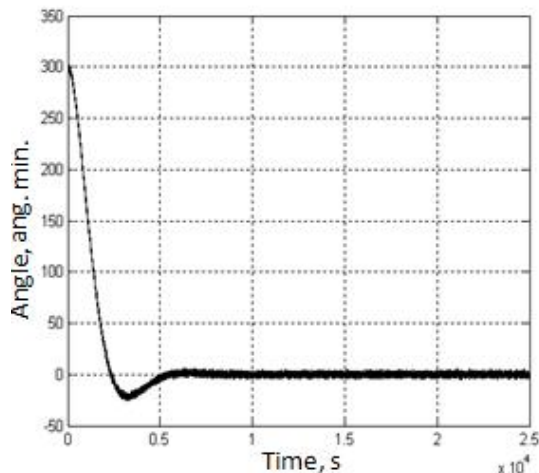


Fig. 5. Simulation of optimized system

Results of parametric optimization such as  $H_\infty$  and  $H_2$  norms for non-optimized and optimized systems are 17.24, 2.78 and 0.81, 0.08, respectively.

The obtained results can be useful for designing navigations systems of the wide class.

## V. CONCLUSIONS

The approaches to parametrical optimization of the gimbaled systems for stabilization and determination of a moving objects attitude and heading are represented.

Mathematical models of the system in the preliminary levelling mode is represented.

Simulink model of the system in the mode of attitude determination is represented.

Simulation results in the form of the transient processes are given. Results of the robust parametrical, optimization are represented.

## REFERENCES

- [1] O. A. Sushchenko, "Mathematical model of attitude and heading reference system with biaxial platform," *Proceedings of the National Aviation University*, 2017, no. 1, pp. 31–41. <https://doi.org/10.18372/2306-1472.71.11745>.
- [2] O. A. Sushchenko, Mathematical model of triaxial multimode attitude and heading reference system. *Proceedings of the National Aviation University*, 2017, no. 2, pp. 31–41. <https://doi.org/10.18372/2306-1472.71.11745>.
- [3] O. A. Sushchenko, Design of robust navigation laws and stabilization contours of precision attitude and heading reference system. *Proceedings of the National Aviation University*, 2017, no. 3, pp. 48–56, <https://doi.org/10.18372/2306-1472.72.11981>.
- [4] A. A. Tunik and O. A. Sushchenko, "Usage of vector parametric optimization for robust stabilization of ground vehicles information-measuring devices," *Proceedings of the National Aviation University*, 2013, no. 4, pp. 23–32. <https://doi.org/10.18372/2306-1472.57.5530>.
- [5] J. M. Hilkert, Inertially stabilized platform technology. *IEEE Control Systems Magazine*, 2008, vol. 28, no. 1, pp. 26–46. <https://doi.org/10.1109/MCS.2007.910256>.
- [6] H. G. Wang, T.G. Williams, "Strategic inertial navigation systems. *IEEE Control Systems Magazine*," 2008, vol. 28, no. 1, pp. 65 – 85. <https://doi.org/10.1109/MCS.2007.910206>.
- [7] D.W. Gu, P. Petkov, M. Konstantinov, "Robust control design with MATLAB," Berlin, Springer, 2003, 465 p.
- [8] A.D. Aleksandrov, *Yndykatornie hyroskopycheskye platformi* [Indicated gyroscopic platforms. Moscow, Nauka Publ., 1979, 239 p.
- [9] T. Perez, *Ship Motion Control*. London: Springer-Verlag, 2005, 300 p.
- [10] S. S. Rivkin, *Stabylyzatsyya yzmary-tel'nikh ustroystv na kachayushchetsya osnovanyy* [Stabilization of measuring devices on swinging base]. Moscow, Nauka, 1978, 239 p.
- [11] V. B. Larin and A. A. Tunik, "On inertial-navigation system without angular-rate sensors," *International Applied Mechanics*, 2013, vol. 49 (4), pp. 488–499. <https://doi.org/10.1007/s10778-013-0582-x>
- [12] V. Chikovani, O. Sushchenko, and H. Tsiruk, "Redundant information processing techniques comparison for differential vibratory gyroscope," *Eastern-European Journal of Enterprise Technologies*, vol. 4 (7/82), pp. 45–52. <https://doi.org/10.15587/1729-4061.2016.75206>

- [13] A. P. Parshin and Y. A. Nemshilov, "Development of measurement UAV attitude control unit with non-collinear arrangement of sensing elements," *Modern technics and technologies*, 2016, vol. 3. Available at: <http://technology.snauka.ru/2016/03/9697>
- [14] Y. P. Petrov, *Optymyzatsyya upravlyaemikh system, yspityvayushchykh vozdeystviye vetra morskoho volnenyya* [Optimization of controlled systems disturbed by the wind of sea irregular waves]. Saint-Petersburg, Sudostroenie, 1973, 214 p.
- [15] O. A. Sushchenko, Y. N. Bezkorovainyi, and N. D. Novytska, "Nonorthogonal redundant measurement devices of inertial sensors," *Proceedings of 2017 IEEE 4th International Conference Actual Problems of Unmanned Aerial Vehicles Developments (APUAVD)*, 2017 October, Kyiv, Ukraine, pp. 73–78. <https://doi.org/10.1109/apuavd.2017.8308780>

Received November 03, 2021.

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### **О. А. Сущенко. Особливості проєктування високоточних систем стабілізації та визначення просторової орієнтації та курсу**

У статті розглянуто особливості проєктування високоточних систем стабілізації та визначення просторової орієнтації та курсу. Розглянуто особливості параметричної оптимізації. Обґрунтовано необхідність використання робастних законів управління. Представлено математичну модель досліджуваної системи в режимі попереднього горизонтування. Показано Simulink модель навігаційного контуру. Пояснені необхідність та особливості лінеаризації моделі, представлені основні концепції створення моделей, спрямованих на вирішення досліджуваної проблеми. Розглянуто процес проєктування на прикладі точної платформної навігаційної системи, призначеної для експлуатації на морських рухомих об'єктах. Запропонований підхід до вирішення проблеми супроводжується моделюванням. Результати моделювання підтверджують ефективність описаної процедури проєктування. Отримані результати можуть бути корисними для створення систем стабілізації та керування рухом широкого класу.

**Ключові слова:** високоточна навігаційна система; параметрична оптимізація; процедура проєктування; математична модель; робастне керування; імітаційне моделювання.

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### **О. А. Сущенко. Особенности проектирования высокоточных систем стабилизации и определения пространственной ориентации и курса**

В статье рассмотрены особенности проектирования высокоточных систем стабилизации и определения пространственной ориентации и курса. Рассмотрены особенности параметрической оптимизации. Обоснована необходимость использования робастных законов управления. Представлена математическая модель исследуемой системы в режиме предварительного горизонтирования. Показана модель Simulink навигационного контура. Объяснены необходимость и особенности линейаризации модели, представлены главные концепции создания моделей, направленных на решение изучаемой проблемы. Рассмотрен процесс проектирования на примере высокоточной платформенной навигационной системы, предназначенной для эксплуатации на движущихся морских объектах. Предлагаемый подход к решению проблемы сопровождается моделированием. Результаты моделирования подтверждают эффективность описанной процедуры проектирования. Полученные результаты могут оказаться полезными для создания систем стабилизации и управления движением широкого класса.

**Ключевые слова:** высокоточная навигационная система; параметрическая оптимизация; процедура проектирования; математическая модель; робастное управление; имитационное моделирование.

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