UDC 629.3.025.2 (045)
DOI:10.18372/1990-5548.69.16427

[1]A. A. Tunik,
[2]V. B. Larin,
[3]O. A. Sushchenko,
[4]S. I. Ilnytska

# ON FEATURES OF PLANNING TRAJECTORIES FOR QUADROTORS

Faculty of Air Navigation, Electronics and Telecommunications, National Aviation University,
Kyiv, Ukraine
E-mails: [1]anatolitunik@gmail.com ORCID 0000-0003-4176-7817,
[2]vblarin@gmail.com ORCID 0000-0001-7702-0251, [3]sushoa@ukr.net ORCID 0000-0002-8837-1521,
[4]ilnytskasv84@gmail.com ORCID 0000-0003-2568-8262

***Abstract*****—***The paper deals with studying the process of planning trajectories during the quadrotor flight. The typical trajectories of the drone flights have been analyzed. Block diagram of the control system in the nonholonomic case and the horizontal flight is given. Comparative analysis of the most widespread trajectories is represented. The possibility to use two types of Dubins trajectory is analysed. Introducing polar coordinates for forming flight trajectories is proposed. The grounded choice of the quadrotor trajectories for different cases of the holonomic and nonholonomic closed-loop control systems was proposed. The advantages and disadvantages of the trajectories planning in each of these cases were analyzed. The Simulink models for generators of the quadrotor trajectories have been developed. The simulation results of generations of these trajectories have been represented. The possibilities of MATLAB for simulating flight trajectories are shown. The obtained results can be applied for unmanned aerial vehicles of different types.*

**Index Terms—**Dubins trajectories; horizontal flight; Simulink-models; quadrotor; holonomic systems; nonholonomic systems; unmanned aerial vehicles.

## I. INTRODUCTION

Nowadays, unmanned aerial vehicles (UAVs) or drones are widespread in many areas of human activity including civil and military applications. Drones have an advantage in comparison with manned vehicles during applications in difficult for human conditions. The improvement and development of new types of UAVs including their guidance, navigation, and control systems are confirmed by the active interest of the world community in this issue. In particular, international scientific conferences on this topic are regularly held in many countries of the world.

Unmanned aerial vehicles are characterized by low cost and ease of operation, which leads to broad perspectives for their use for remote sensing, mapping, monitoring, and other applications. It should be noted that quadrotors are of great interest in some areas of application.

During tests and operations of drones in general and quadrotors in particular, it is necessary to provide the flight by the given trajectory. To solve this problem, it is necessary to consider such tasks as route planning before the flight and during the flight, trajectory flight control, and navigation. The difficult conditions of quadrotor operation and features of their application require solving the problems of ensuring high accuracy of trajectory control and stabilization during stochastic and deterministic perturbations.

All the flight missions of the quadrotors can be divided into indoor and outdoor applications. It should be noted that the problem of planning trajectory for indoor applications is sufficiently simplified. This situation is explained by the closed flight space that imposes restrictions on the position and speed of a quadrotor. Hence, the quantity of possible trajectories is bounded. Therefore, we will pay the main attention to outdoor applications, which are numerous and varied.

The surveys [1], [2] outline the following most popular civil applications for quadrotors: real-time monitoring, providing wireless coverage, remote sensing, search and rescue, delivery of goods, aerial photography of large areas, security and surveillance, precision agriculture, and civil infrastructure inspection. Paper [2] concentrates more on specific applications where the use of drones in swarms may bring additional benefits in comparison to their single use. In addition, it is useful to mention the quadrotors usage to enhance the performance of communication as outlined in tutorial [3].

The goal of the research is the grounded choice of trajectories for quadrotors depending on flight missions, description of ways to give trajectories, and simulation of planning trajectories in Simulink. The last item is a very important part of the Model-in-Loop Simulation (MILS), which is the necessary stage of the Computer-Aided Design (CAD) of the quadrotor [4].

## II. PLANNING TRAJECTORIES OF QUADROTORS

In order to provide all aforementioned flight missions with necessarily planned reference trajectories, it is convenient to choose Dubins trajectories [5], [6], [16] for flight mission implementation. They are universal for the variety of flight missions, and segments of straight lines and arcs of circles define them. It can be explained, that for moving vehicles with constant speed *V*, optimal in time trajectory between two points of the route consists of the aforementioned elements [5]. Such a configuration ensures an optimal transition from one point of planned trajectory to another. Applications of Dubins trajectories have some restrictions such as the necessity to ensure the constant height and speed of the quadrotor flight. The certain disadvantage of Dubins trajectories is some discontinuity at the points of conjugation of two straight lines of the linear piecewise trajectory, which is the typical situation in the waypoint navigation.

It should be noted that all the drones could be divided into the holonomic and nonholonomic systems. For example, a UAV with fixed wings is a nonholonomic system. On contrary, quadrotors can be used as both nonholonomic and holonomic systems. It depends on the type of feedback in the flight control system [7] – [11], [14], [16]. The basic difference between approaches to planning trajectories of nonholonomic and holonomic systems is as follows. Planning a trajectory for a holonomic system is carried out by the positions of the quadrotor in the space and the time marker corresponding to this position. That is why these trajectories are called double-index trajectories [8], [9]. Planning a trajectory for a nonholonomic system is implemented by components of the vector of the spatial speed in some inertial reference frame. For planning trajectories of nonholonomic systems, it is sufficient to give the sequence of intermediate waypoints (WP) in space only. In this case, the piece-linear trajectory is defined by the WP set. While the circular trajectory is defined by the center of the circle, the radius of the circle, and points of input in the circle at output from it. That is why these trajectories are called one-index (or space-index) trajectories [8], [12], [14], and [15]. Block-diagram of the system for planning the trajectory of the quadrotor as the nonholonomic system in the horizontal plane is represented in Fig. 1, where the reference trajectory unit (RTU) determines signals for planning the trajectory by the speed command *V* and heading command $\psi$ for the guidance system (GS) and the heading control system "$\psi$-con" respectively [10], [11]. The guidance system determines an error between the given trajectory and the real position of the quadrotor. It also produces reference signals $X_c$, $Y_c$ for the position in the outer contours and $V_x$, $V_y$ for speed control in the inner contours, driving the systems of the longitudinal motion control *X*-con and the lateral motion control *Y*-con. Signals $X_q$, $Y_q$ denote the actual position of the quadrotor in the horizontal plane.

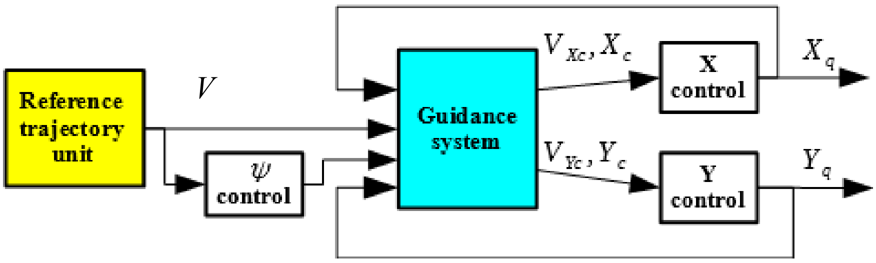


Fig. 1. Block diagram of the control system in the nonholonomic case and the horizontal flight: GS is the guidance system; RTU is the reference trajectory unit

Block diagram of the system for planning trajectory of the quadrotor as the holonomic system is given in Fig. 2. In this Figure, the blocks "*X*-con" and "*Y*-con" are the closed-loop systems for the quadrotor position control on axes "*X*" and "*Y*" respectively. "Reference trajectory unit" is the generator of the reference track, which produces reference commands "$X_{\text{ref}}$" and "$Y_{\text{ref}}$" for these closed-loop systems in accordance with the sequence of waypoints $\text{WP}_i(X_i, Y_i)$.

The position control is preferable for indoor applications, and then the quadrotor is considered as a holonomic system. In this case, such property is the essential advantage of the holonomic systems, because it is possible to achieve better accuracy of the quadrotor positioning. The last circumstance is very important for short-distance flights with obstacles. However, for the quadrotor motion in a holonomic case, the position-based path following requires the trajectory, defined by not only space indices, but time indices as well, requiring in this case the two-index (space and time) trajectory planning. Therefore, in the case of outdoor usage of the quadrotor, when a customer can frequently change the flight missions (routes, velocities, etc.) one-index path planning is more preferable due to its simplicity. Finally, we can make the conclusion that in cases of the great number of flight missions, it is convenient to apply quadrotors as nonholonomic systems because planning trajectory by WP only is simpler in this case. Therefore, the customer has a choice to apply the quadrotor as a holonomic system at the cost of the more complicated trajectory planning or to use the same drone as a nonholonomic system with simpler trajectory planning.

Figure 3 illustrates processes of planning the piece-linear and circular trajectory for nonholonomic and holonomic systems. It should be noted that only four points ($X_0$, $Y_0$), ($X_1$, $Y_1$), ($X_2$, $Y_2$), ($X_3$, $Y_3$) must be given for planning piece-linear trajectory for a nonholonomic system. The necessary information for planning trajectory in the case of the holonomic system looks like ($X_{00}$, $Y_{00}$), $t_{00}$; ($X_{11}$, $Y_{11}$), $t_{11}$; ($X_{22}$, $Y_{22}$), $t_{22}$; ($X_{33}$, $Y_{33}$), $t_{33}$. For planning the circular trajectory of a nonholonomic system, it is necessary to give coordinates ($X_0$, $Y_0$), ($X_1$, $Y_1$), $R$. The necessary information for the circular trajectory for the holonomic system is ($X_0$, $Y_0$), $t_0$; ($X_1$, $Y_1$), $t_1$; ($X_k$, $Y_k$), $t_k$; ($X_N$, $Y_N$), $t_N$.

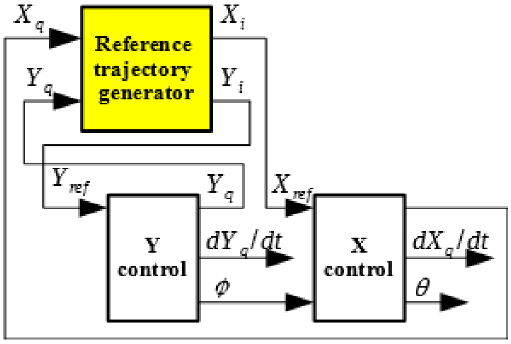
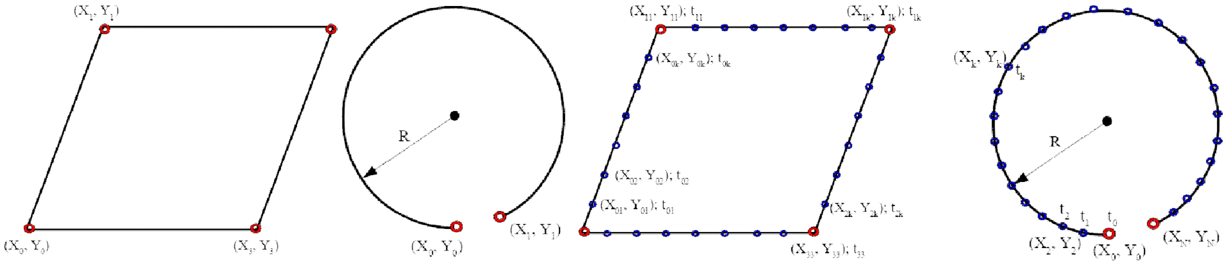


Fig. 2. Block diagram of planning trajectory in the case of the holonomic system

Figure 3 demonstrates that in the holonomic case, we have to define piece-linear and circular trajectories by means of both given time and space coordinates. The disadvantage of this approach is the changed speed of motion because motion can accelerate and become slow. To ensure smooth motion along the given trajectory, it is expedient to design a special dynamic system, which generates continuously moving WP in the space (one can choose the speed of the WP motion depending on characteristics of the given quadrotor [9]).



a)………………… ……b)…………… ….c)………………… …d)

Fig. 3. Planning trajectory: (a) is the piece-linear trajectory for the nonholonomic system; (b) is the circular trajectory for the nonholonomic system; (c) is the piece-linear trajectory for theholonomic system; (d) is the circular trajectory for the holonomic system

## III. SIMULATION OF TRAJECTORIES

Analysis of the known quadrotor applications for practical cases shows, that for the time being the usage of the holonomic systems prevails over the nonholonomic ones [9], [15], although the amount of the last cases is increasing [10] – [13], and [16]. That is why, in this item, we will describe the dynamic systems for the generation of the given reference trajectories with continuously moving WP for holonomic systems. It is expedient to use for models of such systems the Simulink package.

The Simulink model of the linear-piecewise trajectory reference unit for the holonomic system is represented in Fig. 4.

The block "Step" implements the setting of the given speed. The consequent WP$_i$ ("$X_i$", "$Y_i$") along with correspondent values of the headings "$\psi_i$" are stored in the special block. They are consequently applied to the inputs "InPSI", "InX", "InY". The operator 1/$s$ provides the dynamical properties of the reference track generator.

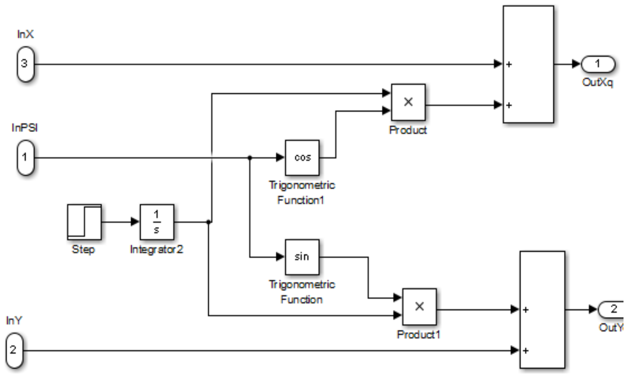


Fig. 4. Linear-piecewise reference trajectory generator

The simplest system for generating the circular reference trajectory can be described by equations:

$$X(t)=\int_{t_0}^{t} R\cos(\omega\tau)d\tau,\ \ Y(t)=\int_{t_0}^{t} R\sin(\omega\tau)d\tau . \quad (1)$$

In the expressions (1) radius $R$ and angular velocity $\omega$ define the linear velocity $V=\omega R$, which must be feasible for the given quadrotor.

The generalized approaches for generating Dubins trajectories as functions of space and time are given in [5], [17]. In article [17] such algorithms are called “nonholonomic Dubins’ car”. It should be noted that planning Dubins trajectory is accompanied by difficulty caused by the necessity to connect segments of straight lines and arcs of the circle [5], [16], [17]. To avoid these difficulties, we propose to generate in polar coordinates smooth pseudo-Dubins trajectories. As examples, we used two types of such trajectories calculated by formulas:

$$R(t)=1+0.1\cdot\sin\left(4\omega t-\frac{\pi}{2}\right), \quad (2)$$

for the 1st case, and

$$R(t)=1+0.1\sin\left(4\omega t+\frac{\pi}{2}\right)+0.5\left(\sin 8\omega t+\frac{\pi}{2}\right), \quad (3)$$

for the 2nd case, and $\omega t=(0...2\pi)$ for both cases.

Figures 5 and 6 show the Simulink models, which represent processes of generating Dubins trajectories of the first (2) and second (3) types. One can make sure that computational blocks for forming Dubins trajectories of the first and second types correspond to expressions (2), (3).

Figures 7 and 8 show the results of generating the planned pseudo-Dubins trajectory in correspondence to formulas (2), (3) in the Cartesian frame.

Changing the parameters in expressions (2), (3), such as the constant terms and the amplitudes, we can widen or narrow these closed-loop reference tracks. Simulation results, represented in Figures 7 and 8, prove the efficiency of the proposed approach because these tracks are very similar to the combination of the linear piecewise and circular trajectories. At the same time, their generation does not require complications connected with the conjugation of the linear and circular segments. Note, that one can generate these trajectories in MATLAB also.

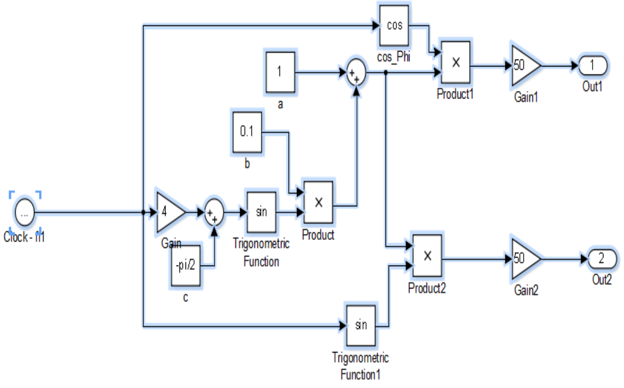


Fig. 5. Simulink-model for forming pseudo-Dubins trajectory of the first type

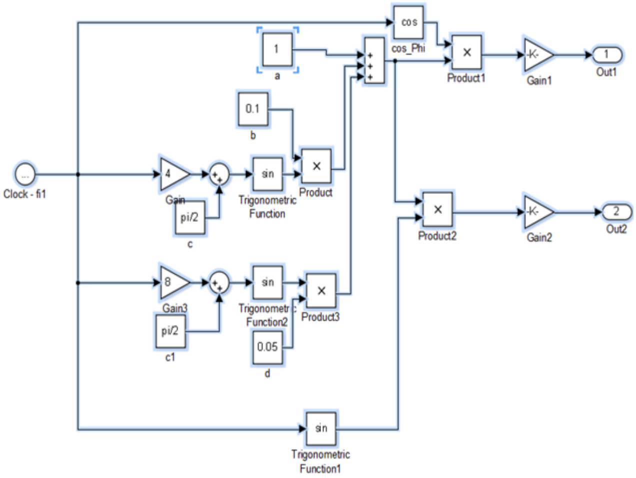
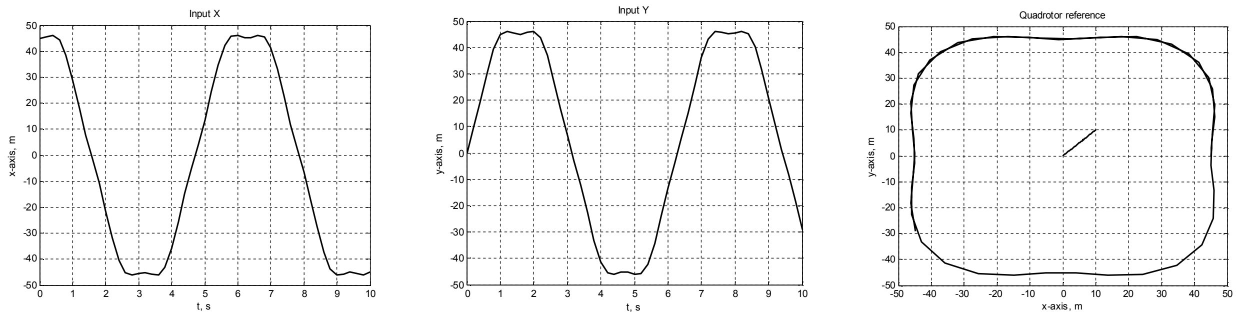
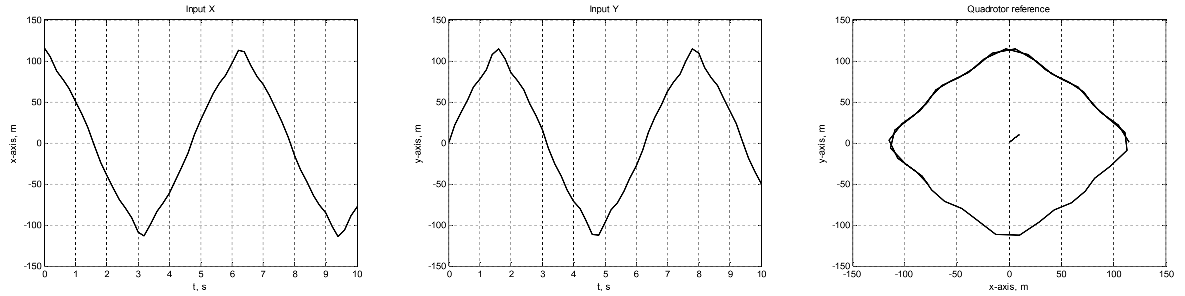


Fig. 6. Simulink-model for forming pseudo-Dubins trajectory of the second type



a) b) c)

Fig. 7. Forming pseudo-Dubins trajectories of the first type: (a) $x$-coordinate in time; (b) $y$-coordinate in time; (c) resulting trajectory in horizontal plane



a) b) c)

Fig. 8 Forming pseudo-Dubins trajectories of the second type: (a) $x$-coordinate in time; (b) $y$-coordinate in time; (c) resulting trajectory in horizontal plane

## IV. CONCLUSIONS

The trajectory planning for the quadrotor strongly depends on its control system, which can give the closed-loop system properties of the holonomic or the nonholonomic systems.

For the holonomic case, trajectory planning is more complex because it requires double indexing in both time and space. However, the trajectory control system, in this case, is simpler because it contains one positional control loop only [9]. In addition, the holonomic control system uses directly the position of the centre of a mass of the drone on a given trajectory as the main feedback. Therefore, the tracking of the trajectory is more accurate in comparison with a nonholonomic system

For the non-holonomic case, trajectory planning is much simpler because it requires the use of space indices only. At the same time, the trajectory control system is more complicated because it consists of two loops: internal for velocity control and external for correction of the direction of the current velocity vector respectively the given reference trajectory.

The generally accepted and the most convenient type of reference trajectory is the Dubins trajectory consisting of the straight lines segments and the arcs of circles. For the nonholonomic systems, it is quite enough to define the waypoints in the space only for the linear-piecewise trajectory and the position of the circle, its radius, and the input-output points.

Meanwhile, for the holonomic system, it is possible to use the special dynamic system, generating the continuous temporal-spatial trajectory. In the paper, such generators of Dubins-type trajectories are considered. However, there are some difficulties for conjugation of the line segments and the circle arcs in one dynamic system.

To avoid these difficulties, we propose to use the pseudo-Dubins trajectories generators, which create the smooth trajectories in the polar frame.

## REFERENCES

[1] H. Shakhatreh et al., "Unmanned Aerial Vehicles (UAVs): A Survey on Civil Applications and Key Research Challenges," *in IEEE Access*, vol. 7, pp. 48572–48634, 2019, https://doi.org/10.1109/ACCESS.2019.2909530.

[2] M. Abdelkader, S. Guler, H. Jaleel, J.S. Shamma. “Aerial Swarms: Recent Applications and Challenges”. Current Robotics Report, Issue 2, 309–320, (2021). https://doi.org/10.1007/s43154-021-00063-4

[3] M. Mozaffari et al., “A Tutorial on UAVs for Wireless Networks: Applications, Challenges, and Open Problems.”, IEEE Communications Surveys

and Tutorials, 2019, 21(3): pp. 2334–60. https://doi.org/10.1109/COMST.2019.2902862.

[4] A. Tunik, K. Rydlo, O.Savchenko, K. Melnik . Computer-Aided Design of the Stability and Controllability Augmentation System. Proc. of Intern. Conf. "Actual Problems of Unmanned Aerial Vehicles Developments" , Kyiv, NAU.-Oct.13-15, 2015, pp. 73–79. https://doi.org/10.1109/APUAVD.2015.7346563

[5] R. W. Beard and T. W. McLain, "Small Unmanned Aircraft. Theory and Practice," Princeton University Press, 2012. https://doi.org/10.1515/9781400840601

[6] L .E. Dubins, "On curves of minimal length with a constraint on average curvature and with prescribed initial and terminal positions and tangents," *American Journ. of Math*., vol 79, no. 3, July 1957, pp. 497–516. https://doi.org/10.2307/2372560

[7] A. A. Tunik, S. I. Ilnytska, and O. A. Sushchenko, "Hierarchical and Heterogeneous Leader-follower Formation for Quadrotor's Outdoor Application," *Proc. of IEEE 5th Intern. Conf. "Actual Problems of Unmanned Aerial Vehicles Developments*," Kyiv, NAU, Oct. 19-21, 2021, pp. 11–16.

[8] G. M. Hoffman, S. L. Waslander, and C. J. Tomlin, "Quadrotor Helicopter Trajectory Tracking Control," *Proc. of the AIAA Guidance, Navigation and Control Conference and Exhibition*, 18-21 August 2008, Hawaii, Honolulu, pp. 1–14. https://doi.org/10.2514/6.2008-7410

[9] A. A. Tunik, S. I. Ilnytska, O. A. Sushchenko, "Synthesis of Quadrotor Robust Guidance and Control System via Parametrization of all Stabilizing H-infinity State Feedback Gains," *Electronics and Control Systems,* Kyiv: Osvita Ukrainy, no. 4(62), pp. 33–41, 2019. https://doi.org/10.18372/1990-5548.62.14379

[10] V. B. Larin and A. A.Tunik, "Two-stage Procedure of Hinf – Parameterization of Stabilizing Controllers Applied to Quadrotor Flight Control," *TWMS Journal of Pure and Applied Mathematics*, vol. 12, no.2, pp. 199–208, 2021.

[11] A. A. Tunik, "Simplified Path Tracking Control Laws for Quad-rotor Considered as Nonholonomic System," *Proceedings of 2018 IEEE 5th International Conference "Methods and Systems of Navigation and Motion Control"*, Kyiv, Ukraine, 2018, pp. 83–89. https://doi.org/10.1109/MSNMC.2018.8576316

[12] Y. Bouzid, Y. Bestaoui, H. Siguerdidjane and M. Zareb, "Quadrotor Guidance-Control for flight like nonholonomic vehicles," *2018 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2018, pp. 980-988, https://doi.org/10.1109/ICUAS.2018.8453367.

[13] A. Sánchez-Orta *et al.*, "Aerial Following of a Non-holonomic Mobile Robot subject to Velocity Fields: A Case Study for Autonomous Vehicles Surveillance," *2020 International Conference on Unmanned Aircraft Systems (ICUAS)*, 2020, pp. 1093–1102, https://doi.org/10.1109/ICUAS48674.2020.9214053.

[14] H. Bouadi, F. Mora-Camino, and D. Choukroun, "Space-Index Control for Aircraft Vertical Guidance with Time Constraint", *Journal for Guidance, Control and Dynamics*, vol. 37, no. 4, 2014, https://doi.org/10.2514/1.62118.

[15] V. B. Larin and A. A. Tunik, "Synthesis of the Quad-Rotor Control Algorithms in the Basic Flight Modes," *TWMS Journal of Pure and Applied Mathematics*, vol. 9, no. 2, pp. 147–158, 2018.

[16] Y.Ding, X. Wang, Y. Bai and N. Cu. "High-precision prescribed-time path following for quadrotor," *International Journal of Advanced Robotic Systems*. March-April, 2020, pp. 1–18. https://doi.org/10.1177/1729881420920058

[17] C. Yong and E. J. Barth, "Real-time Dynamic Path Planning for Dubins' Nonholonomic Robot," *Proc. of the 45th IEEE Conf. on Decision & Control*, San Diego, CA, USA, December 13–15, 2006, pp. 2418 – 2423. https://doi.org/10.1109/CDC.2006.377829

Received August 30, 2021.

**Tunik Anatolii.** ORCID 0000-0003-4176-7817. Doctor of Engineering Sciences. Professor.
Research and Training "Aerospace Center", National Aviation University, Kyiv, Ukraine.
Education: Kharkiv Polytechnic Institute, Kharkiv, Ukraine, (1961).
Research area: robust control theory, navigation and flight control systems.
Publications: 370.
E-mail: anatolitunik@gmail.com

**Larin Volodymyr.** ORCID 0000-0001-7702-0251. Doctor of Physics and Mathematics Sciences. Professor.
Institute of Mechanics of National Academy of Sciences, Kyiv, Ukraine.
Education: Kyiv Polytechnic Institute, Kyiv, Ukraine, (1958).
Research area: control of the complex dynamic systems
Publications: 470.
E-mail: vblarin@gmail.com

**Sushchenko Olha.** ORCID 0000-0002-8837-1521. Doctor of Engineering Sciences. Professor.
Faculty of Air Navigation, Electronics and Telecommunications, National Aviation University, Kyiv, Ukraine.
Education: Kyiv Polytechnic Institute, Kyiv, Ukraine, (1980).
Research area: systems for stabilization of information-measurement devices.
Publications: 250.
E-mail: sushoa@ukr.net

**Ilnytska Svitlana**. ORCID 0000-0003-2568-8262. Candidate of Sciences (Engineering). Senior Researcher.
Research and Training "Aerospace Center," National Aviation University, Kyiv, Ukraine, Kyiv, Ukraine.
Education: National Aviation University, Kyiv, (2007).
Research area: navigation and control, integrated navigation systems, satellite and inertial navigation systems, sensor calibration technologies, computer simulation.
Publications: 45.
E-mail: ilnytskasv84@gmail.com

**А. А. Тунік, В. Б. Ларін, О. А. Сущенко, С. І. Ільницька. Особливості планування траєкторій для квадрокоптерів**

У статті досліджується процес планування траєкторій під час польоту квадрокоптера. Проаналізовано типові траєкторії польотів дронів. Наведено структурну схему системи керування в неголономному випадку для горизонтального польоту. Представлено порівняльний аналіз найбільш поширених траєкторій. Проаналізовано можливість використання двох типів траєкторії Дубінса. Запропоновано введення полярних координат для формування траєкторій польоту. Запропоновано обґрунтований вибір траєкторій квадрокоптера для різних випадків голономної та неголономної замкнутих систем керування. Проаналізовано переваги та недоліки планування траєкторій у кожному з цих випадків. Розроблено Simulink-моделі для генераторів траєкторій квадрокоптера. Наведено результати моделювання процесу генерації цих траєкторій. Показано можливості MATLAB для моделювання траєкторій польоту. Отримані результати можуть бути застосовані для безпілотних літальних апаратів різних типів.

**Ключові слова**: траєкторії Дубінса; горизонтальний політ; Simulink-моделі; квадрокоптер; голономні системи; неголономні системи; безпілотні літальні апарати.

**Тунік Анатолій Азарійович.** ORCID 0000-0003-4176-7817. Доктор технічних наук. Професор.
Науково-навчально центр «Аерокосмічний центр», Національний авіаційний університет, Київ, Україна.
Освіта: Харківський політехнічний інститут, Харків, Україна, (1961).
Напрям наукової діяльності: теорія робастного управління, системи навігації та управління польотом.
Кількість публікацій: 370.
E-mail: anatolitunik@gmail.com

**Ларін Володимир Борисович.** ORCID 0000-0001-7702-0251
Доктор фізико-математичних наук. Професор.
Інститут механіки ім. С.П. Тимошенка НАН України, Національний авіаційний університет, Київ, Україна.
Освіта: Київський політехнічний інститут, Київ, Україна, (1958).
Напрям наукової діяльності: управління складними динамічними системами.
Кількість публікацій: 470.
E-mail: vblarin@gmail.com

**Сущенко Ольга Андріївна.** ORCID 0000-0002-8837-1521
Доктор технічних наук. Професор.
Факультет аеронавігації, електроніки та телекомунікацій, Національний авіаційний університет, Київ, Україна.
Освіта: Київський політехнічний інститут, Київ, Україна, (1980).
Напрям наукової діяльності: системи стабілізації інформаційно-вимірювальних пристроїв.
Кількість публікацій: 250.
E-mail: sushoa@ukr.net

**Ільницька Світлана Іванівна.** ORCID 0000-0003-2568-8262
Кандидат технічних наук. Старший науковий співробітник.
Науково-навчально центр «Аерокосмічний центр», Національний авіаційний університет, Київ, Україна.
Освіта: Національний авіаційний університет, Київ (2007).
Напрямок наукової діяльності: навігація та управління, інтегровані навігаційні системи, супутникові та інерціальні навігаційні системи, технології калібрування датчиків, комп'ютерне моделювання
Кількість публікацій: 45.
E-mail: ilnytskasv84@gmail.com

**А. А. Туник, В. Б. Ларин, О. А. Сущенко, С. И. Ильницькая. Особенности планирования траекторий для квадрокоптеров**

Статья посвящена изучению процесса планирования траекторий полета квадрокоптера. Проанализированы типичные траектории полета дронов. Приведена блок-схема системы управления в неголономном случае для горизонтального полета. Представлен сравнительный анализ наиболее распространенных траекторий. Анализируется возможность использования двух типов траекторий Дубинса. Предлагается введение полярных координат для формирования траекторий полета. Предложен обоснованный выбор траекторий квадрокоптера для различных случаев голономной и неголономной замкнутой системы управления. Проанализированы преимущества и недостатки планирования траекторий в каждом из этих случаев. Разработаны Simulink-модели генераторов траекторий квадрокоптера. Представлены результаты моделирования процесса генерирования этих траекторий. Показаны возможности MATLAB для моделирования траекторий полета. Полученные результаты могут быть применены для беспилотных летательных аппаратов различных типов.

**Ключевые слова**: траектории Дуббинса; горизонтальный полет; Simulink-модели; квадрокоптер; голономные системы; неголономные системы; беспилотные летательные аппараты.

**Туник Анатолий Азариевич.** ORCID 0000-0003-4176-7817. Доктор технических наук. Профессор.
Научно-учебный центр «Аэрокосмический центр», Национальный авиационный университет, Киев, Украина.
Образование: Харьковский политехнический институт, Харьков, Украина, (1961).
Направление научной деятельности: теория робастного управления, системы навигации и управления полётом.
Количество публикаций: 370.
E-mail: anatolitunik@gmail.com

**Ларин Владимир Борисович.** ORCID 0000-0001-7702-0251. Доктор физико-математических наук. Профессор.
Інститут механики им. С.П. Тимошенко НАН Украины, Национальный авиационный университет, Киев, Украина.
Направление научной деятельности: управление сложными динамическими системами.
Количество публикаций: 470 .
E-mail: vblarin@gmail.com

**Сущенко Ольга Андреевна.** ORCID 0000-0002-8837-1521. Доктор технических наук. Профессор.
Факультет аэронавигации, электроники и телекоммуникаций, Национальный авиационный университет, Киев, Украина.
Образование: Киевский политехнический институт, Киев, Украина, (1980).
Направление научной деятельности: системы стабилизации информационно-измерительных устройств.
Количество публикаций: 250.
E-mail: sushoa@ukr.net

**Ильницкая Светлана Ивановна**. ORCID 0000-0003-2568-8262. Кандидат технических наук. Старший научный сотрудник.
Научно-учебный центр «Аэрокосмический центр», Национальный авиационный университет, Киев, Украина.
Образование: Национальный авиационный университет, Киев, Украина, (2007).
Направление научной деятельности: навигация и управления, интегрированные навигационные системы, спутниковые и инерциальные навигационные системы, технологии калибровки датчиков, компьютерное моделирование
Количество публикаций: 45.
E-mail: ilnytskasv84@gmail.com